Controlled Random Search Optimization For Linear Antenna Arrays

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Abstract. *An optimization problem for designing non-uniformly spaced, linear antenna's arrays is formulated and solved by means of a controlled random search algorithm.*

The proposed iterative method aims at a linear array and the optimization of element positions and excitations coefficients by minimizing the side-lobes level and respecting a beam pattern shape. Selected examples are included, which demonstrate the effectiveness and the design flexibility of the proposed method in the framework of the electromagnetic synthesis of linear antenna arrays.

Keywords

Antenna array, pattern shape, synthesis, optimization, controlled random search.

1. Introduction

An antenna array with certain radiation characteristics is often asked to be designed. Necessarily, the nulls have to be in a certain direction [1], or the main lobe has to be directed in a certain direction; also other requirements for the direction and the level of the side lobes [2] might be stated.

The global synthesis of antenna arrays that generate a desired radiation pattern is a highly nonlinear optimization problem. Many analytical methods have been proposed for its solution. Examples of analytical techniques include the well-known Taylor method and the Chebyshev method [3]. In many applications, the synthesis problem of an antenna array consist of finding an appropriate set of amplitude and phase weights that will yield the desired far-field pattern with an equally spaced linear array [4]. However, it is well known that the antenna performance related to the beam width and side lobes levels can be improved by choosing both the best position and the best set of the amplitude and phase for each element of an unequally spaced array [5].

The paper is aimed to present a modular method, based on a controlled random search (CRS) algorithm, which is able to simultaneously optimize the excitation coefficients (amplitude and phase) and the best position, according to different constraints, such as side lobes peak minimization, and beam pattern (BP) shape modeling. Although the CRS has proven to be very robust in many applications [6], [7], it is somewhat less known to the engineering community. The CRS is characteristic by finding good near-optimal solutions early in the optimization run. The CRS does not use derivatives, and is also independent on the complexity of the objective function under consideration.

Two examples are used to demonstrate the effectiveness of the proposed controlled random search–based procedure for the antenna array optimization. The CRS-simulated results are also compared with those obtained by genetic algorithms in [4].

2. Antenna Array Pattern Formulation

The far field factor of a linear array with an even number of uniformly spaced isotropic elements ($2N$) can be written in the form:

$$F(\theta) = \sum_{i=1}^{N} a_i \cos(k_0 d_i \sin\theta + \psi_i) \ , \tag{1}$$

where $k_0 = 2\pi/\lambda$ is the wave number, θ denotes the angular direction, a_i and ψ_i are the amplitude and the phase of the excitation complex feed.

The position d_i can be computed from the inter-element spacing (Fig. 1), according to the following formula:

$$d_i = \sum_{m=1}^{i} \Delta d_m - \frac{\Delta d_1}{2} \ . \tag{2}$$

In order to synthesize the desired pattern, element excitations (a_i and ψ_i) and positions d_i are be optimally determined by the CRS. In order to control the actual pattern both in the shaping region and in the predetermined side lobe regions, we have adopted the cost function given as:

$$C_{cost} = \sum_{\theta} L(\theta) \ , \tag{3}$$

where

$$L(\theta)=\frac{k(\theta)+|k(\theta)|}{2}\ , \tag{4}$$

$$k(\theta)=\left[SR_{max}(\theta)-|F(\theta)|\right]\left[SR_{min}(\theta)-|F(\theta)|\right]. \tag{5}$$

Here, SR_{min} and SR_{max} represent the minimum and the maximum shaping region, respectively (Fig. 2), and $L(\theta)$ does not equal to 0 only if $|F(\theta)|$ is situated inside the shaping region.

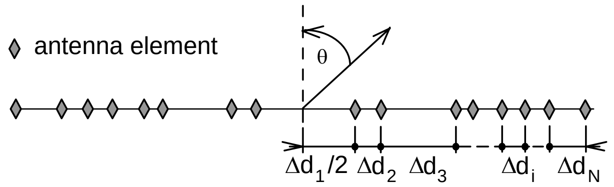

Fig. 1. Non-equidistant elements symmetrical linear array.

In order to obtain the desired pattern, the cost function given by (3) is minimized by the CRS, which is described in the following section.

3. Control Random Search Algorithm

The idea of the Controlled Random Search (CRS) algorithm was developed by W. L. Price [8]. The algorithm requires a minimum preparation of data to operate, and can be applied to constrained as well as to unconstrained optimization problems where the gradient of the objective function is unavailable.

In the preliminary stage, a pool of randomly selected points is generated and the function values are calculated for each point. Then the main routine starts which computes subsequent trial points and updates pool accordingly.

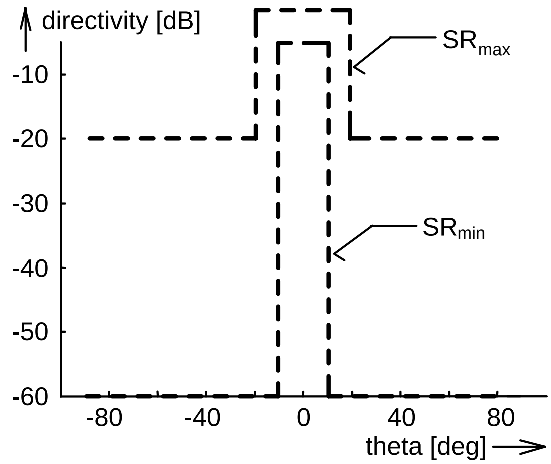

Fig. 2. Desired pattern shape.

CRS, although simple, has several disadvantages in its initial form. Primarily, the convergence rate worsens while approaching neighborhood of the solution. Secondary, the unconditional rejection of a trial point that falls off the constraints makes it difficult to find optimal points that are located on the optimization domain boundaries. The CRS algorithm used in this paper is presented below:

- **Step 0:**Let k be the step counter; set $k = 0$. Randomly choose M points from the optimization domain D. The chosen points constitute the initial pool $P_k = \{X_1, ..., X_M\}$. The pool size should be sufficiently large, usually $M = 10(n+1)$. Evaluate the objective function for the chosen points $f(X_j)$, $j = 1 ... M$.

  Here, X is the array parameters being searched, D is the optimization domain (e.g., amplitude $a_i \in [0, 1]$, phase $\psi_i \in [0, 2\pi]$ and inter-elements spacing $\Delta d_i \in [0.25\,\lambda,\ 1.5\,\lambda]$, where $\lambda = 0.06$ m corresponding to the frequency 5 GHz), and n is the dimension of the cost function defined in (3), $n = 3N$.

- **Step 1:** Find the point $X_{k,l}$ in the current pool P_k that provides the lowest cost function value, and point $X_{k,h}$ providing the highest cost function value.

- **Step 2:** Create n+1 dimensional subset of P_k called the simplex S_k. The simplex has to contain the point $X_{k,l}$. The remaining n points are randomly chosen from P_k (without duplications; choosing $X_{k,h}$ is allowed). Compute the centre of the simplex:

$$c_k=\sum_{x_j\in S_k}\frac{X_j}{n+1}\ . \tag{6}$$

- **Step 3:** Compute a trial point X_t by reflecting the current worst simplex element $X_{k,h}$ from the simplex centre S_k: $X_t = 2\,c_k - X_{k,h}$.

- **Step 4:** If any coordinate of X_t violates constraints, bounce the point back into the domain using the following scheme:

$$X_{t,j}=\begin{cases}2X_j^{min}-X_{t,j} & if \quad X_{t,j}<X_j^{min}\\ 2X_j^{max}-X_{t,j} & if \quad X_{t,j}>X_j^{max}\\ X_{t,j} & \text{else}\end{cases}\ . \tag{7}$$

- **Step 5:** If the test point is better than the worst point in the pool, i.e. if $f(X_t) < f(X_{k,h})$, then create a new pool P_{k+1} that is P_k with $X_{k,h}$ replaced by X_t **,** and increase the step counter. Otherwise, go to the step 2 in order to repeat the creation of a simplex S_k once more.

- **Step 6:** Stop if the stop criterion is satisfied, i.e. if the objective function value has not decreased below ε in a prescribed number of steps h:

$$\begin{aligned}&f(X_{k-h,l})-f(X_{k,h})<\varepsilon,\\ &f(X_{k-h,l})=f(X_{0,h}) \quad \text{if} \quad k-h<0\ .\end{aligned} \tag{8}$$

- **Step 7:** Go to the step 1.

The total number of function evaluations made can be applied as an additional stop criterion. There are two reasons for introducing this criterion:

- We obtain a general control of costly function evaluations (in terms of the CPU-time used);
- We can prevent the algorithm from falling accidentally into infinite loops.

The implementation of CRS concerns two its parts: a pool initialization and a trial point computation. In the initialization phase, the index of the first point in the initial pool P_0 that has not been yet processed is available for all the threads. A single thread starts to process the point (i.e. starts to compute the cost function value) and increments the index. Therefore, the pool is not divided between threads in advance, but is processed accordingly to a thread computation capability.

When the initialization is done, each thread enters its own trial point computation loop. There are several possibilities of interactions between loops in order to improve convergence. W. L. Price in [8] proposes all threads to operate on the same pool that is updated as soon as a better point is found. In this scheme, all threads reflect the same point X_h , but with regard to various simplex centers.

4. Numerical Results

In order to illustrate the capabilities of the CRS for the shaped beam pattern synthesis of a linear array, two examples are considered. In the first example, an unequally spaced linear array consisting of 12 isotropic elements is used, and in the second one, we consider an unequally spaced linear array consisting of 8 isotropic elements. In the optimization process, the value of n is 18 and 12 for the two examples, respectively. The total number of function evaluations is fixed to 3000 and ε to 10^{-4}, which is found to be sufficient to obtain satisfactory patterns with a desired performance. The calculations are performed on a personal computer with Pentium IV processor running at 2.26 GHz, and for the two examples considered here, the optimization results are obtained within 485 s and 325 s, respectively.

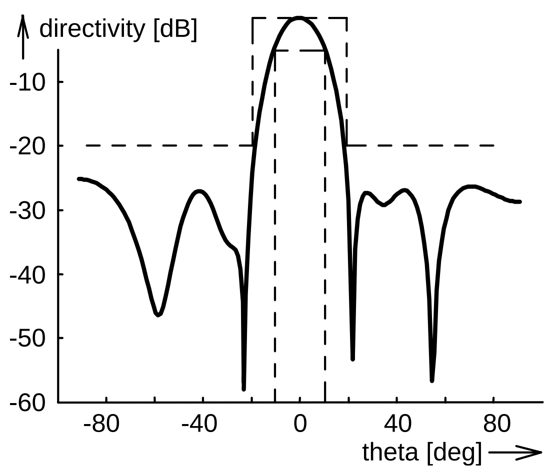

Fig. 3. The mask of desired pattern (dashed line) and the radiation pattern obtained by the CRS using 12 elements (solid line).

As the first example, a pattern with a main beam width defined in the sector $\theta \in$ [–20°, 20°] and a side lobe level lower than –20 dB were chosen as the desired pattern. The mask of the desired pattern is shown in Fig. 3. In order to obtain the desired pattern, the values of the cost function parameters given in (5) are used as follows:

$$SR_{max}(\theta)=\begin{cases}0\,\text{dB} & \text{for} \quad -20^0 \le \theta \le 20^0 \\ -20\,\text{dB} & \text{elsewhere ,}\end{cases} \tag{9}$$

$$SR_{min}(\theta)=\begin{cases}-5\,\text{dB} & \text{for} \quad -10^0 \le \theta \le 10^0 \\ -50\,\text{dB} & \text{elsewhere .}\end{cases} \tag{10}$$

In Fig. 3, the pattern obtained from the CRS by determining both the amplitude, the phase and the position of each element of the array is illustrated. Clearly, the pattern shows a good performance in the shape region, and there are no side lobes exceeding the specified value –20 dB.

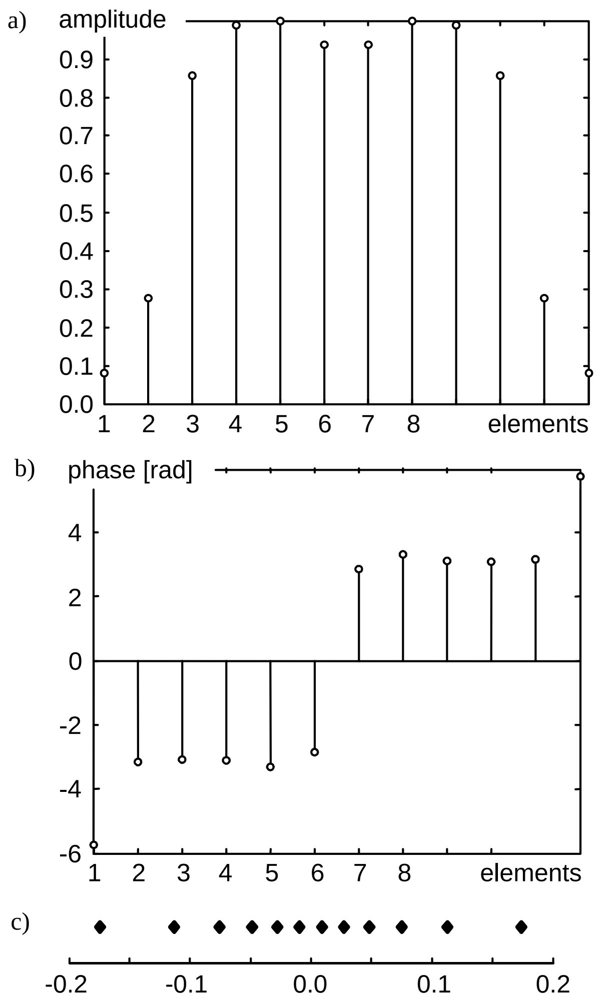

Fig. 4. Optimized parameters of array: a) optimized amplitude excitation, b) optimized phase excitation, c) position of each element in the array.

The required element amplitude, the phase and the position of each array element for the pattern given in Fig. 3 are depicted in Fig. 4.

In the second example, we are going to produce the array pattern with a large main beam width defined in the sector $\theta \in [0°, 60°]$. The SR_{max} and SR_{min} are given below:

$$SR_{max}(\theta) = \begin{cases} 0\,\mathrm{dB} & \text{for} \quad 0^0 \le \theta \le 60^0 \\ -20\,\mathrm{dB} & \text{elsewhere}\,, \end{cases} \tag{11}$$

$$SR_{min}(\theta) = \begin{cases} -5\,\mathrm{dB} & \text{for} \quad 10^0 \le \theta \le 50^0 \\ -50\,\mathrm{dB} & \text{elsewhere}\,. \end{cases} \tag{12}$$

Fig. 5 shows the CRS is capable to synthesize an unequally spaced linear array producing a shaped beam pattern with the good performance both in the shaped region and in the side lobe region. The algorithm can easily consider any parameter of interest (side lobe level, beam width, etc.) by including it in the cost function.

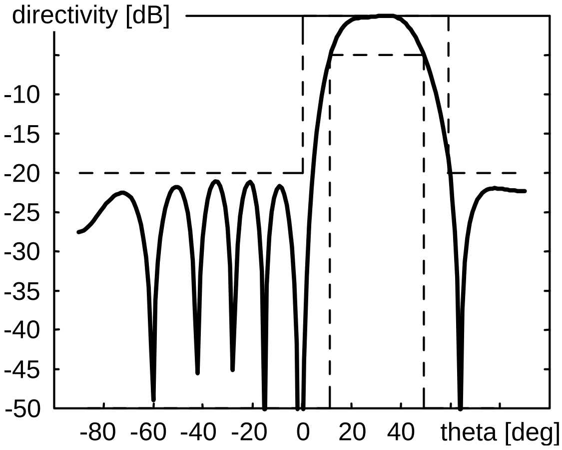

Fig. 5. The mask of desired pattern (dashed line) and the radiation pattern obtained by the CRS using 8 elements (solid line).

The elements amplitude, phase and positions obtained by the CRS to produce the pattern depicted in Fig. 5 are shown in Fig. 6.

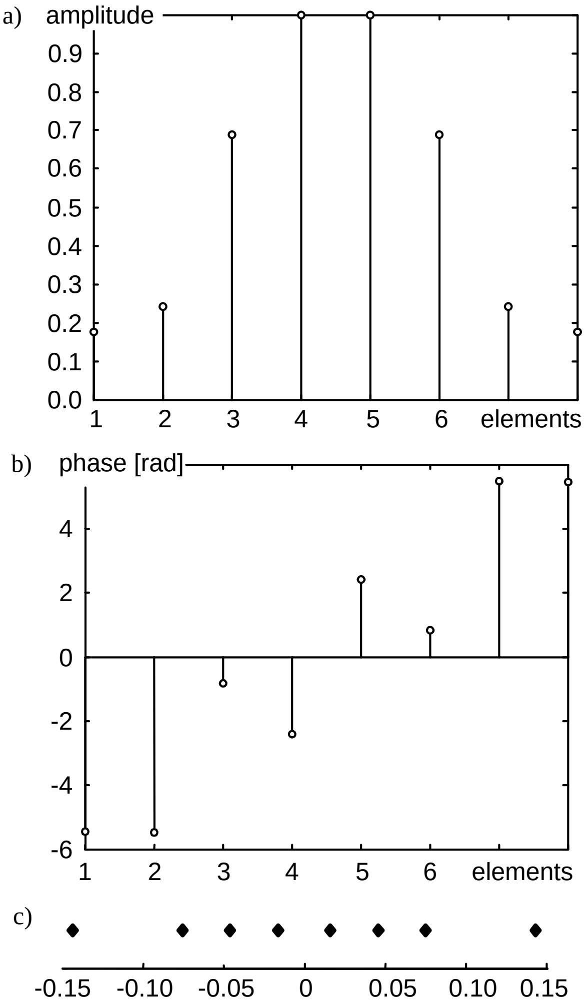

Fig. 6. Optimized parameters of array: a) optimized amplitude excitations, b) optimized phase excitations, c) position of each element in the array.

For the simulation speed comparison between CRS and GA [4], the GA simulation takes about 120 s on 486/33 MHz PC [4]. The CRS simulation takes 142 s on Pentium IV processor running at 2.26 GHz. Obviously, the GA simulation is much faster than CRS for array-pattern synthesis.

5. Comparative Study

In order to evaluate the performance of the proposed CRS algorithm, this section compares the numerical results calculated by CRS and the Genetic Algorithm (GA). For comparison, a linear array of 30 isotropic elements at half wavelength spacing is considered. The array excitation amplitude is symmetric with null phase, the number of iterations (generations) is set to 100 [4].

Fig. 7 shows the comparison of the far-field patterns among the CRS-simulated results, and the GA-simulated results in [4]. Obviously, the side lobes close to the main beam are lowered, and the levelled side lobes in Fig. 7 indicate that the result is close to the optimum solution for that particular beam width. This fact can be confirmed by comparing the solid line GA pattern [4] and the dashed line CRS one. Although the CRS side lobe level is –35.15 dB, this result remains comparable to the GA one: –36.02 dB.

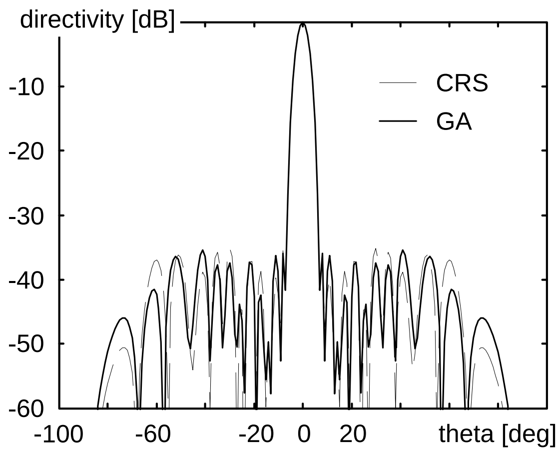

Fig. 7. Comparison of the radiation patterns of the broad side linear array with excitation coefficients by the CRS (dashed line) and the GA method (solid line).

However, the GA requires a good adjustment of these parameters in order to obtain good results. Fig. 8 shows the elements amplitude excitations generated by the CRS algorithm and GA method. All elements amplitude excitations are almost identical.

6. Conclusion

A new optimization method for the synthesis of linear array pattern functions has been proposed and assessed. The shaped beam pattern, the constrained side-lobes level, and the main-lobe width are contemporarily taken into the account by minimizing a cost function by means of an innovative improved Controlled Random Search procedure.

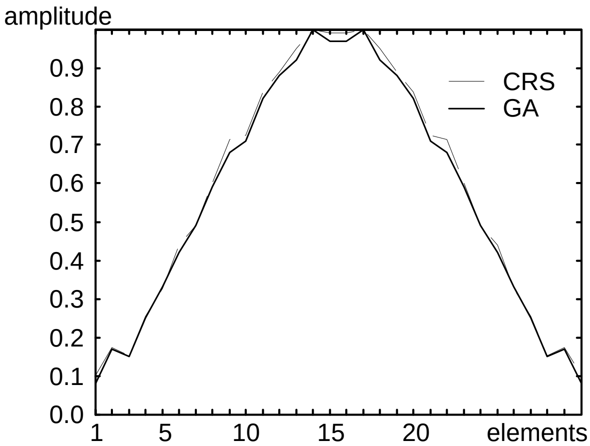

Fig. 8. Comparison of the optimized amplitude excitations computed by CRS (dashed line) and GA (solid line).

The different results show the great flexibility of the proposed approach. Many additional extensions of the CRS method could be also easily implemented both in term of the cost function definition, the optimization methodology and applications, and antenna types.

References

[1] YANG, S., BENG GAN, Y., QING, A. Antenna-array pattern nulling using a differential evolution algorithm. *International Journal on RF and Microwave*.2004, vol. CAE 14, p. 57–63.

[2] LUE, W. C., HSU, F. Use of B-spline curves and genetic algorithms to reduce the sidelobe level in array-patterns. *Microwave and Optical Technology Letters*, 2003, vol. 38, no. 4, p. 308–311.

[3] BALANIS, C. *Antenna Theory – Analysis and Design*. New York: Wiley, 1982.

[4] YAN, K., LU, Y. Sidelobe reduction in array-pattern synthesis using genetic algorithms. *IEEE Transactions on Antennas and Propagation*, 1997, vol. 45, no. 7, p. 1117–1122.

[5] AKDAGLI, A., GUNEY, K. Shaped beam pattern synthesis of equally and unequally spaced linear antenna arrays using modified tabu search algorithm. *Microwave and Optical Technology Letters*, 2003, vol. 36, No. 1, p. 16–20.

[6] ALI, M. *Some Modified Global Optimization Algorithms with Applications*. Loughborough University of Technology, PH.D Thesis, 1994.

[7] ALI, M. M., STOREY, C., TORN, A. Application of some stochastic global optimization algorithms to practical problems. *Journal of Optimization Theory and Applications*, 1997, vol. 95, no. 3, p. 545 to 563.

[8] PRICE, W.L. Global optimization by controlled random search. *Journal of Optimization Theory and Applications*, 1983, vol. 40, p. 333–348.

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