

# Design of Dual Bandpass and Bandreject LC Ladder Filters

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**Abstract.** This paper deals with the design of two-passband bandpass and two-stopband bandreject LC ladder filters. The design method is based on the special dual frequency transformation that transforms normalized lowpass to either bandpass with two passbands or to bandreject with two stopbands that are specified by four cutoff frequencies. The paper shows analytical solution relating these four cutoff frequencies to parameters of dual frequency transformation. It enables a direct computation of dual band LC filter elements from a normalized lowpass filter by means of simple relations. These relations have been implemented in the mathematical program Maple™ as new user functions. They are supposed to be used as an enhancement of Syntfil package which is intended for analog filter design in program Maple. Specific application is shown on an example of the two-passband bandpass LC filter design.

## Keywords

Frequency transformation, two-passbands bandpass filter, two-stopbands bandreject filter, Syntfil package.

## 1. Introduction

Frequency transformation is a frequently used tool for the design of basic types of transfer characteristic, such as highpass bandpass and bandreject functions, from a known normalized lowpass transfer function – lowpass prototype [1], [2]. A similar way can be used for obtaining dual band filters.

Dual band filters are useful when it is necessary to transmit or attenuate signals in two separated frequency bands. Spectral plans of VDSL (Very high speed Digital Subscriber Lines) technology illustrated on Fig. 1 (see [3]) can serve as an example. There is a need to obtain two passbands or stopbands in the frequency axis (band in low frequencies is optional).

Design of these filters could be realized in several ways. Optimization algorithms and methods are also used for the design of special filters. These methods are based on Differential Evolutionary Algorithms and seem to be

very powerful for solving these types of tasks [4], [5]. Analytical solution based on special frequency transformation was applied in this case.

### FDD Duplexing: spectral plans

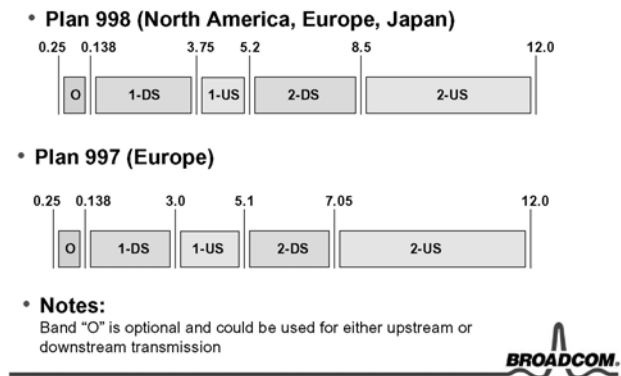


Fig. 1. Spectral plans of VDSL technology (see [3]).

## 2. Dual Frequency Transformations

The ordinary idea behind the frequency transformation is to find a function

$$s_{NLP} = F(s) \quad (1)$$

such that the passband of the normalized lowpass (NLP), i.e.  $0 \leq |\omega_{NLP}| \leq 1$  is transformed into the passband(s) of the target filter and the stopband of the NLP ( $|\omega_{NLP}| > 1$ ) is transformed into stopband(s) of the target filter. Two-passband bandpass filter can be combined of a bandpass (BP) with a bandreject (BS) with stopband within passband of the bandpass. This consideration leads to the elementary transformation of complex frequency, i.e.

$$s_{NLP} = s_{BP} + \frac{1}{s_{BP}} \quad \text{and} \quad s_{BP} = s_{2BP} + \frac{1}{s_{2BP}}. \quad (2)$$

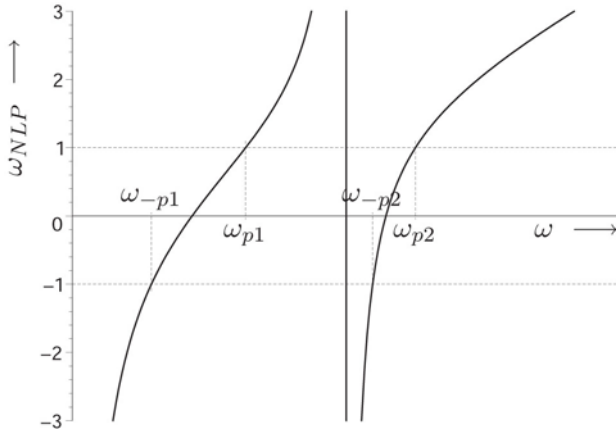
Hence for two-passband bandpass transformation from NLP we find the general relation

$$s_{NLP} = \frac{1}{\omega_A} \left( s_{2BP} + \frac{\omega_B^2}{s_{2BP}} \right) + \frac{1}{\frac{1}{\omega_C} \left( s_{2BP} + \frac{\omega_D^2}{s_{2BP}} \right)} \quad (3)$$

which is the mentioned combination of a bandpass and bandreject. Because equation (3) transforms the  $j\omega_{2BP}$ -axis (dual bandpass) into the  $j\omega_{NLP}$  (normalized lowpass) and, because the transfer function of the target function is also a real rational function of frequency it follows that the function (1) must be an odd real rational function, so that

$$S_{NLP} = j\omega_{NLP} = F(j\omega) \equiv jf(\omega). \quad (4)$$

The function  $\omega_{NLP} = f(\omega)$  transforms then both passbands of the target filter into the range  $-1 \leq \omega_{NLP} \leq 1$  and stopbands into  $|\omega_{NLP}| > 1$  as was mentioned. Fig. 2 shows this function for dual bandpass transformation given by relation (3). There are two passbands in intervals  $\omega_{-p1} \leq \omega \leq \omega_{p1}$  and  $\omega_{p2} \leq \omega \leq \omega_{-p2}$ , and stopbands outside of these ranges.



**Fig. 2.** Transformation function for a two-passband bandpass into a lowpass prototype.

It is necessary to find relations between transformation constants  $\omega_A, \omega_B, \omega_C, \omega_D$  and frequencies of the target filter specification – cutoff frequencies  $\omega_{-p1}, \omega_{p1}, \omega_{p2}, \omega_{-p2}$ . The relations between these two sets of frequencies can be derived from following equations compiled according to mentioned transformation.

$$-1 = \frac{1}{\omega_A} \left( \omega_{-p1} + \frac{\omega_B^2}{\omega_{-p1}} \right) + \frac{1}{\omega_C} \left( \omega_{-p1} + \frac{\omega_D^2}{\omega_{-p1}} \right)$$

$$1 = \frac{1}{\omega_A} \left( \omega_{p1} + \frac{\omega_B^2}{\omega_{p1}} \right) + \frac{1}{\omega_C} \left( \omega_{p1} + \frac{\omega_D^2}{\omega_{p1}} \right)$$

$$-1 = \frac{1}{\omega_A} \left( \omega_{-p2} + \frac{\omega_B^2}{\omega_{-p2}} \right) + \frac{1}{\omega_C} \left( \omega_{-p2} + \frac{\omega_D^2}{\omega_{-p2}} \right)$$

$$1 = \frac{1}{\omega_A} \left( \omega_{p2} + \frac{\omega_B^2}{\omega_{p2}} \right) + \frac{1}{\omega_C} \left( \omega_{p2} + \frac{\omega_D^2}{\omega_{p2}} \right)$$

Solving these equations we get relations for determination

of frequency constants of the equation (3) from frequencies of the required filter specifications. Let us define:

$$\begin{aligned} \Delta f_{p2} &= f_{p2} - f_{-p2} \\ \Delta f_{p1} &= f_{p1} - f_{-p1} \\ K_f &= f_{p1} f_{-p1} \Delta f_{p2} + f_{p2} f_{-p2} \Delta f_{p1} \end{aligned} \quad (5)$$

where  $\omega_{p1} = 2\pi f_{p1}$ , etc. Then we get relations for frequency constants  $\omega_A, \omega_B, \omega_C, \omega_D$  using known cutoff frequencies

$$\begin{aligned} f_A &= \Delta f_{p2} + \Delta f_{p1} \\ f_C &= \frac{\Delta f_{p1} (f_{-p2} - f_{p1}) \Delta f_{p2} (f_{p2} - f_{-p1})}{K_f} \cdot \\ &\quad \cdot \frac{(f_{-p1} + f_{-p2})(f_{p1} + f_{p2})}{f_A^2}. \end{aligned} \quad (6)$$

$$f_B = \frac{\sqrt{f_{-p1} f_{p1} f_{-p2} f_{p2}}}{f_C}$$

$$f_D = \sqrt{\frac{K_f}{f_C}}$$

It is possible to derive the transformation for two-stopband bandreject by the same way. It is an inverse transformation – the basic equation is in following form

$$S_{NLP} = \frac{1}{\frac{1}{\omega'_A} \left( s_{2BS} + \frac{\omega_B'^2}{s_{2BS}} \right) + \frac{1}{\omega'_C} \left( s_{2BS} + \frac{\omega_D'^2}{s_{2BS}} \right)} \quad (7)$$

where transformation constants  $\omega'_A, \omega'_B, \omega'_C, \omega'_D$  can be obtained in the same way as in previous case. It is evident that if the cutoff frequencies of dual bandreject filter are equal to the cutoff frequencies of dual bandpass filter then the transformation constants  $\omega'_A, \omega'_B, \omega'_C$  and  $\omega'_D$  will be equal to transformation constants  $\omega_A, \omega_B, \omega_C, \omega_D$ .

For complete filter design it is also necessary to solve the transformation of dual bandpass (bandreject) filter specification to the specification of normalized lowpass filter. We can apply following process to find the cutoff frequency of NLP filter from the known dual filter specification. We substitute all stopband frequencies step by step to transformation (3) (eventually (7)) and find the corresponding NLP frequencies. Then most strict case (the frequency which is closest to the 1) is chosen as the stopband frequency of NLP.

### 3. Transformation of LC Filter Elements

The equation (3) and relation (6) determine the transformation of NLP transfer function to transfer function of two-passband bandpass. These relations can be also used

for the transformation of NLP LC ladder filter elements. An inductor  $l$  is transformed to a series connection of a series and a parallel resonant circuit because

$$lS_{NLP} \rightarrow L_1S_{2BP} + \frac{1}{C_1S_{2BP}} + \frac{1}{C_2S_{2BP} + \frac{1}{L_2S_{2BP}}} \quad (8)$$

where

$$L_1 = \frac{lR}{\omega_A}, C_1 = \frac{\omega_A}{lR\omega_B^2}, L_2 = \frac{lR\omega_C}{\omega_D^2}, C_2 = \frac{1}{lR\omega_C} \quad (9)$$

and  $R$  represents an impedance scaling. A capacitor is transformed to a parallel connection of a parallel and a series resonant circuit because

$$cS_{NLP} \rightarrow C_1S_{2BP} + \frac{1}{L_1S_{2BP}} + \frac{1}{L_2S_{2BP} + \frac{1}{C_2S_{2BP}}} \quad (10)$$

where

$$C_1 = \frac{c}{R\omega_A}, L_1 = \frac{R\omega_A}{c\omega_B^2}, C_2 = \frac{c\omega_C}{R\omega_D^2}, L_2 = \frac{R}{c\omega_C} \quad (11)$$

and complete scheme of normalized lowpass filter is transformed using this process.

The equation (7) and relation (6) determine the transformation of NLP transfer function to transfer function of two-stopband bandreject. These relations can be used again for transformation of normalized lowpass LC ladder filter elements. An inductor  $l$  is transformed to a parallel connection of a parallel and a series resonant circuit because

$$lS_{NLP} \rightarrow \frac{1}{C_1S_{2BS} + \frac{1}{L_1S_{2BS}} + \frac{1}{L_2S_{2BS} + \frac{1}{C_2S_{2BS}}}} \quad (12)$$

where

$$C_1 = \frac{1}{lR\omega_A}, L_1 = \frac{lR\omega_A}{\omega_B^2}, C_2 = \frac{\omega_C}{lR\omega_D^2}, L_2 = \frac{lR}{\omega_C} \quad (13)$$

A capacitor is transformed to a series connection of a series and a parallel resonant circuit because

$$cS_{NLP} \rightarrow \frac{1}{L_1S_{2BS} + \frac{1}{C_1S_{2BS}} + \frac{1}{C_2S_{2BS} + \frac{1}{L_2S_{2BS}}}} \quad (14)$$

where

$$L_1 = \frac{R}{c\omega_A}, C_1 = \frac{c\omega_A}{R\omega_B^2}, L_2 = \frac{R\omega_C}{c\omega_D^2}, C_2 = \frac{c}{R\omega_C} \quad (15)$$

## 4. Example of Calculation

The relations presented in previous section have been implemented in program Maple as new user functions. These functions can be used together with Syntfil package which is intended for analog filter design in program Maple (see [6-8]). The Syntfil package and the above mentioned new functions were used to solve following example.

A design of two-passband bandpass filter with Caer approximation of third order with stopband NLP frequency  $\omega_{S,NLP} = 1.2$  and passband ripple 1dB was chosen as an example. Cutoff frequencies of dual bandpass were chosen according to Fig. 1, i.e.  $f_{p1} = 3.75$  MHz,  $f_{p2} = 5.2$  MHz,  $f_{p2} = 8.5$  MHz and  $f_{p2} = 12$  MHz. Structure of NLP LC ladder filter was calculated first using standard functions of Syntfil package; see help of the package in [8]. Output of the Syntfil function for calculation of NLP filter structure follows:

```
table([1 = table([orientation = direct,
elements = {L1 = 1.181}, Z = s L1]),
2 = table([orientation = shunt,
elements = {L1 = 0.9287, C1 = 0.6335}, Z = s L1 + \frac{1}{s C1}],
3 = table([orientation = direct, elements = {L1 = 0.5057},
Z = s L1]), R2 = 1., R1 = 0., type = LC_NLP_short])
```

This structure defines NLP LC ladder filter description together with element values. The corresponding filter structure is shown in Fig. 3.

The obtained structure was transformed using new functions based on the introduced transformation. Output of the function for dual bandpass calculation from program Maple follows:

```
table([1 = table([orientation = direct, elements =
{L1 = 0.1899 10^-5, L2 = 0.1184 10^-5, C1 = 0.2928 10^-9,
C2 = 0.4901 10^-9}, Z = s L1 + \frac{1}{s C1} + \frac{1}{s C2 + \frac{1}{s L2}},
2 = table([orientation = shunt, elements = {L3 = 0.1365 10^-5,
L4 = 0.2285 10^-5, L2 = 0.9303 10^-6, C4 = 0.2538 10^-9,
L1 = 0.1493 10^-5, C3 = 0.4074 10^-9, C1 = 0.3725 10^-9,
C2 = 0.6235 10^-9},
Z = s L1 + \frac{1}{s C1} + \frac{1}{s C2 + \frac{1}{s L2}} + \frac{1}{s C3 + \frac{1}{s L3}} + \frac{1}{s L4 + \frac{1}{s C4}},
3 = table([orientation = direct, elements = {L1 = 0.8130 10^-6,
C2 = 0.1145 10^-8, L2 = 0.5066 10^-6, C1 = 0.6840 10^-9},
Z = s L1 + \frac{1}{s C1} + \frac{1}{s C2 + \frac{1}{s L2}}]), R2 = 50., R1 = 0.,
type = LC_BPBP_short])
```

The table based structure description corresponds with the standard LC ladder description of Syntfil package [5]. The resulted LC ladder structure of two-passband bandpass filter is illustrated in Fig. 4.

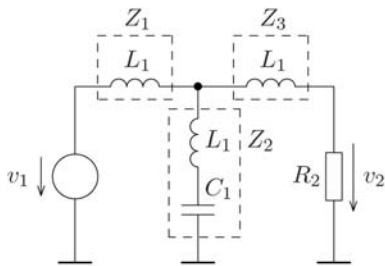


Fig. 3. Resulted structure of NLP LC ladder prototype.

The similar structure can be derived for two-stopband bandreject LC filter realization. However the impedances  $Z_1$  and  $Z_3$  are, of course, changed in this case. Fig. 5 shows the structure of the  $Z_1$  and  $Z_3$  impedances for realization two-stopband bandreject. Resulted element values of the dual bandreject filter are skipped.

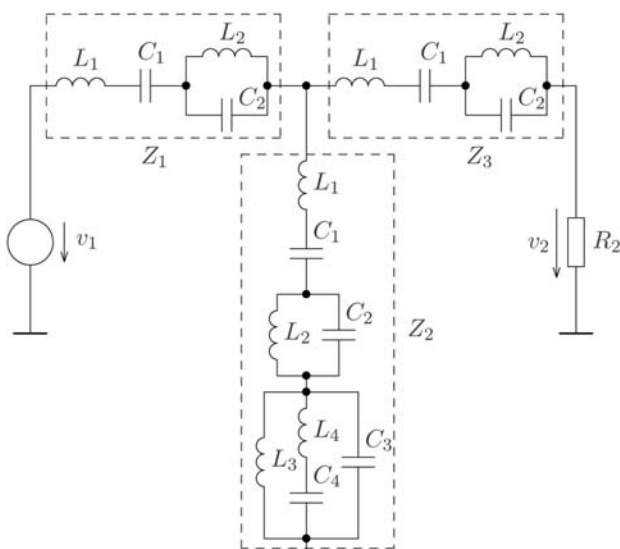


Fig. 4. Calculated structure of two-passband bandpass LC ladder filter.

The designs have been verified by analysis of resulting LC ladder structures. Fig. 6 shows plot of magnitude responses of synthesized structures. The red line represents the magnitude response for the two-passband bandpass filter and the blue one is the magnitude response of two-stopband bandreject filter with the same cutoff frequencies.

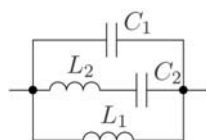


Fig. 5. Structure of  $Z_1$  and  $Z_3$  impedances for realization of two-stopband bandreject.

It is clear from the plot that the design was correct and designed transformation is useful for design of these filters.

### 5. Conclusion

The frequency transformation represents standard tool for design of basic types of filters (lowpass, highpass band-

pass and band reject). We used the idea of the transformation for design of dual band filters – two-passband bandpass and two-stopband bandreject filters. These filters can be useful for example in modern communication technology, for example VDSL modems. The new transformation was derived and implemented in Maple program environment. The designed functions can be used together with standard functions of Syntfil package for analog filter design in program Maple. Usability of this process was verified on the example of filter design.

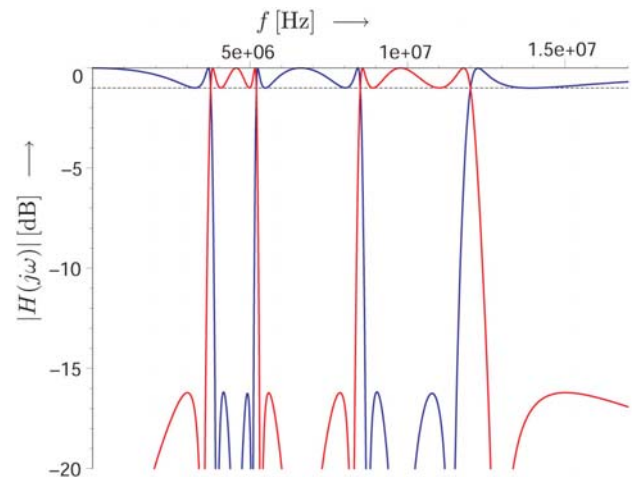


Fig. 6. Transfer function magnitudes both filters.

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