Optimization of Dynamic Range of Cascade Filter Realization

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Abstract. *This paper deals with a dynamic range optimization procedure for active filters based on the cascade realization. Dynamic characteristics of the cascade filter depend on many factors, mainly on pole-zero pairing, section ordering and gain assignment. Just the procedure for an optimal gain assignment for particular biquadratic sections is discussed in this paper. The input parameters of the procedure are parameters of particular biquads i.e. pole frequency* ω_0*, quality factor Q, eventually zero frequency* ω_n *for elliptic section and the transfer function type of the section. The gain is distributed so that output signal limitation of particular biquads occurs for the same level of the filter input signal. The procedure is versatile – can be used for analog as well as for digital filters with the cascade structure. The presented algorithm is fully universal (doesn't suppose any simplification). It has been used in Syntfil package for the filter design in the mathematical program Maple.*

Keywords

Gain assignment, cascade filter realization, dynamic range optimization.

1. Introduction

The gain assignment belongs to one of the final steps in the realization of a cascade filter. Generally, the gain selection based on an optimal dynamic range concerns with the goal of keeping the signals below amplifier saturation limits (overflow) and above the system noise [1], [3], [4]. The structure of the cascade filter is illustrated in Fig. 1.

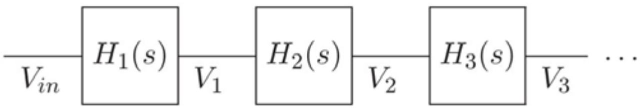

Fig. 1. Cascade structure of a filter.

The transfer function of the individual k^{th} biquad is labeled $H_k(s)$ and can be expressed as:

$$H_k(s) = \frac{V_k}{V_{k-1}} \ . \tag{1}$$

Further, we can label the transfer function after the k^{th} biquad $H_{1\to k}(s)$ which is defined as:

$$H_{1\to k}(s) = \frac{V_k}{V_{in}} = \prod_{i=1}^{k} H_i(s) \ . \tag{2}$$

The transfer function of the whole filter containing n sections is then given by the following expression:

$$H(s) = \frac{V_n}{V_{in}} = H_{1\to n}(s) = = \prod_{k=1}^{n} h_k \frac{a_{2k}s^2 + a_{1k}s + a_{0k}}{s^2 + \frac{\omega_{0k}}{Q}s + \omega_{0k}^2} \ , \tag{3}$$

where h_k are gain constants of the k^{th} biquad transfer function and coefficients a_{2k}, a_{1k} and a_{0k} determine the type of the transfer function $H_k(s)$. The relation (3) is valid for even order transfer function. The first order section of odd order transfer function is usually the last section of the filter. This section has usually gain 1 (passive circuit) and this is not subject to optimization.

The goal of the optimal gain distribution is to find gain constants h_k so that the output signal limitation of the particular biquads occurred for the same level of the filter input signal i.e. modulus maxima of the transfer functions $|H_{1\to k}(j\omega)|$ after each section will be the same:

$$\max |H_{1\to k}(j\omega)| = M \qquad 0 < \omega < \infty, \tag{4}$$

where M is constant (usually 1) for $k = 1,\dots,n$.

2. Procedure of the Gain Assignment

It is necessary to find maxima of transfer function magnitudes after the biquads, i.e.

$$\max |H_{1\to k}(j\omega)| = |H_{1\to k}(j\omega_{qmk})| = = h_{qmk} \ \text{ for } \ k = 1,\dots n \ . \tag{5}$$

The maxima h_{qmk} occur at frequencies ω_{qmk}. Consequently, ratios of individual biquad gain constants h_i can be found using these maxima h_{qmk} – gain constant has to be distributed, so that maxima $h_{qmk} = M$ for all $k = 1,\dots, n$. The calculation of maxima h_{qmk} leads to calculations of maxima frequencies ω_{qmk}, which is not generally a trivial task. This task is solved numerically.

The procedure starts with solving frequencies ω_{qk} of the local magnitude maxima for particular biquads $\max|H_k(j\omega)| = |H_k(j\omega_{qk})| = h_{qk}$ if they exist. Fig. 2 shows labeling of ω_q and h_q (frequency and module elevation) on magnitude plot of elliptic lowpass transfer function.

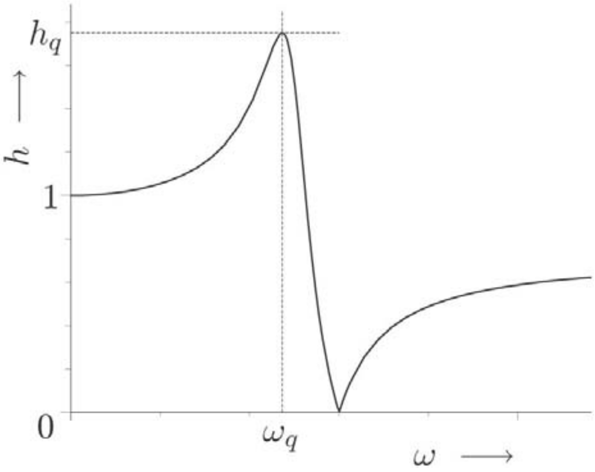

Fig. 2. Characteristic of elliptic lowpass biquad.

In the case of a non-elliptic transfer function, it is easy to derive that a local maximum occurs if

$$Q > \frac{1}{\sqrt{2}}\ .$$

Then

$$\omega_q = \begin{cases} \dfrac{\omega_0\sqrt{4Q^2-2}}{2Q} & \text{for the lowpass (LP)}, \\ \dfrac{2Q\omega_0}{\sqrt{4Q^2-2}} & \text{for the highpass (HP)}, \\ \omega_0 & \text{for the bandpass (BP)}. \end{cases} \tag{6}$$

Two necessary conditions have to be satisfied for an of an occurrence module elevation of the elliptic section. For lowpass and highpass, following non-equivalencies have to be valid:

$$\begin{aligned} Q > \frac{1}{\sqrt{2}}\ \&\ \frac{2Q^2}{2Q^2-1} < \frac{\omega_n^2}{\omega_0^2} \quad \text{for LP}, \\ Q > \frac{1}{\sqrt{2}}\ \&\ \frac{2Q^2}{2Q^2-1} < \frac{\omega_0^2}{\omega_n^2} \quad \text{for HP}, \end{aligned} \tag{7}$$

then the local maximum frequency is given by

$$\omega_q = \omega_0\sqrt{\frac{2Q^2(\omega_n^2-\omega_0^2)-\omega_n^2}{2Q^2(\omega_n^2-\omega_0^2)+\omega_0^2}}, \tag{8}$$

for both types of transfer function. The module of notch biquadratic transfer function is flat in the passband, i.e. there is not any local maximum on the magnitude plot.

The local maximum frequencies ω_{qk} of individual biquads are not the same as the local maximum frequencies $\omega_{qi,k}$ of transfer function magnitudes after the biquads. However the frequencies ω_{qk} are used for numerical calculations of frequencies $\omega_{qi,k}$ for all transfer function magnitudes $|H_{1\to k}(j\omega)|$ after the biquads. The positions of frequencies ω_{qk} are shifted for each transfer function $H_{1\to k}(j\omega)$ as can be seen in the Fig. 3, i.e. $\omega_{qi} \to \omega_{qi,k}$.

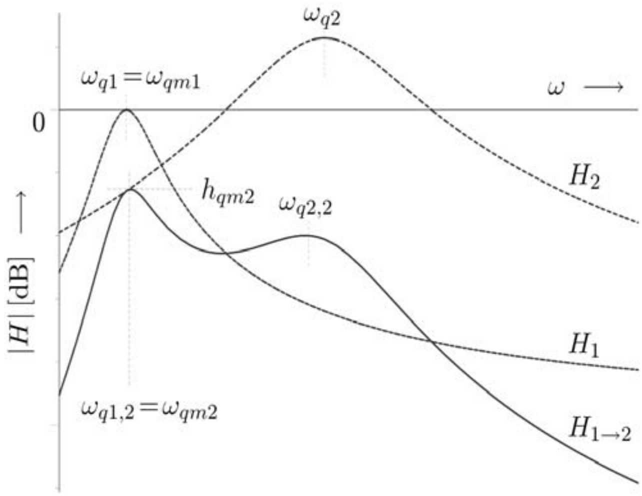

Fig. 3. Example of magnitude plot of a first two sections of a cascade filter.

Clearly from the Fig. 3,

$$\max\left|H_{1\to k}(j\omega_{qi,k})\right| = \left|H_{1\to k}(j\omega_{qmk})\right| \ \text{for all } i. \tag{9}$$

The frequencies $\omega_{qi,k}$ are solved for the zero derivation of the transfer function square – it is not necessary to calculate them from magnitude (absolute value) which is more complicated and gives the same results. As was mentioned above, these frequencies $\omega_{qi,k}$ are solved numerically in sequence for each function $H^2_{1\to k}(j\omega)$ in neighborhood of all points ω_{qk}. At most k number ($i \le k$) of frequencies $\omega_{qi,k}$ is calculated for the transfer function $H_{1\to k}(j\omega)$, because some function $H_k(j\omega)$ needs not have module elevation.

Some of the module elevation can be canceled by next sections (for example of inverse Chebyshev approximation). Numerical calculation of this maximum frequency fails in that case – the frequency is then determined as a frequency of transmission zero (in case of elliptic transfer function) or $\omega_{qi,k} = 0$ or $\omega_{qi,k} \to \infty$. Hence the algorithm development of the optimal gain distribution has to consider also this case.

It is necessary to verify, whether each frequency $\omega_{qi,k}$ for each function $H_{1\to k}(j\omega)$ lies within the passband of the filter. However the task is solved generally, only parameters of particular biquadratic section and their type has to be known (filter specification doesn't need to know). The verification can be done with respect to known frequencies ω_{qk}. In case of the lowpass filter, frequencies $\omega_{qi,k}$ must lie within interval $\langle 0,\omega_h \rangle$, where ω_h is the maximal frequency ω_{qk} for all $k = 1,...,n$. Similar rules can be found for highpass and bandpass filter. In case of the highpass filter, frequencies $\omega_{qi,k}$ must lie within the interval $\langle \omega_d,\infty \rangle$, where ω_d is the minimal frequency ω_{qk} for all $k = 1,\ldots, n$. In case of the bandpass filter, frequencies $\omega_{qi,k}$ must lie within the interval $\langle \omega_d,\omega_h \rangle$, where ω_d is the minimal frequency and ω_h is the maximal frequency from the set ω_{qk} for all $k = 1,\ldots, n$.

The band-rejection filter is the most complicated case. Firstly, it is necessary to find the central frequency ω_m of stopband. This frequency can be easily calculated from the pole frequencies ω_0 of two sections with the same quality factor Q

$$\omega_m = \sqrt{\omega_{0k}\omega_{0l}} \quad \text{where} \quad Q_k = Q_l \text{ and} \tag{10}$$

where Q_k, ω_{0k} are parameters of k^{th} section and Q_l, ω_{0l} are parameters of the l^{th} section.

Consequently, frequencies ω_{qk} are divided into two sets: $\omega_{qk} < \omega_m$ and $\omega_{qk} > \omega_m$. The correct calculated frequency $\omega_{qi,k}$ must lie within one of the intervals $\langle 0,\omega_d \rangle$ and $\langle \omega_h,\infty \rangle$, where ω_d is the highest frequency of the first set and ω_h is the lowest frequency of the 2nd set of frequencies.

k	ω_0 [rad/s]	Q	ω_n [rad/s]
1	12566.37062	0.80205	12566.37062
2	23589.75869	1.93286	17893.96496
3	6694.16218	1.93286	8824.96813

Tab. 1. Particular biquads parameters of a cascade filter with inverse Chebyshev magnitude approximation.

All frequencies $\omega_{qi,k}$ for each function $H_{1\to k}(j\omega)$ can be calculated this way. Following process is used for optimal gain distribution:

1. Solve frequencies ω_{qk} of the local magnitude maxima for all biquads (sections) which have module elevation.
2. Start with the transfer function $H_{1\to 2}(j\omega)$ and find all local magnitude maxima frequencies $\omega_{qi,k}$ (for $H_{1\to 2}(j\omega)$ is $i \le 2$). It is necessary to use the above-mentioned numerical computation with respect to the known frequencies ω_{qk}.
3. Find $\max|H_{1\to k}(j\omega_{qi,k})| = h_{qmk}$ for all $i \le k$.
4. Divide the transfer function $H_{1\to k}(s)$ by the constant h_{qmk} and multiply the transfer function $H_1(s)$ by the same constant h_{qmk} at once to keep the general gain of the transfer function $H(s)$ unchanged.
5. Apply the same technique (item 2 to 5) step by step to the remaining transfer functions $H_{1\to k}(s)$, i.e. firstly for $k = 3$ then $k = 4$, until $k = n$.

The optimal gain distribution is achieved using this algorithm. An additional gain is necessary to apply only to the first block, i.e. to transfer function $H_1(s)$ to keep dynamic characteristic of the whole cascade optimized.

3. Example of Calculation

The described algorithm is implemented in the ARCBlock function of Syntfil package [5-7]. Syntfil is a package for analog filter design in Maple™ program. The ARCBlock function calculates the optimal gain distribution of a cascade filter and allows a user to put additional gain constants individually for each block. These constants are entered in a form of optional parameters of the function; see help of the package in [7].

The application of the algorithm (ARCBlock function) is shown in an example. It is the example of a band-rejection filter with the inverse Chebyshev magnitude approximation. The approximation task has been solved using Syntfil package in Maple program.

The process of calculation of the optimal gain distribution can be monitored in ARCBlock function print-out. It is call out by setting **`infolevel[syntfil]:=2`**. The print-out follows:

```
ARCBlock:
Frequencies of modulus peaks of particular
blocks: [[2, 4931.1984], [3, 811.16180]].
Frequencies of modulus peaks after
particular blocks: [[2, [6336.6400]], [3,
[0.,0.]]].
```

```
ARCBlock:
Amplification of particular blocks
h0 = [1.000000, 0.981521, 1.018827].
```

The first biquad is notch hence there is not module elevation (ω_{q1} doesn't exist) in the module $|H_1(j\omega)|$. The frequencies ω_{qk} of the local magnitude maxima for two following biquads are written up. Next print indicates the calculated local maximum frequencies $\omega_{qi,k}$ of the transfer function magnitudes $|H_{1\to k}(j\omega)|$ after the biquads. It is clear that $\omega_{q1,1} = \omega_{q1}$ does not exist. The transfer function after the second biquad $|H_{1\to 2}(j\omega)|$ has one local maxima in frequency $\omega_{q1,2} = 6336.64$ rad/s, which is shifted with respect to $\omega_{q2} = 4931.2$ rad/s. The module of the transfer function after the third biquad $|H_{1\to 3}(j\omega)|$ doesn't have any elevation − approximation type is inverse Chebyshev and algorithm of ARCBlock function did not fail. The resulting gain distribution (amplification of particular biquads) is written at the end of the listing. Figure 3 and 4 illustrate plots of particular transfer function magnitudes. There is a plot of the transfer function magnitudes of individual biquads $|H_k(j\omega)|$ in Fig. 3, while Fig. 4 shows the transfer function magnitudes after the biquads $|H_{1\to k}(j\omega)|$. It is clear from the Fig. 4 that gain has been assigned optimally.

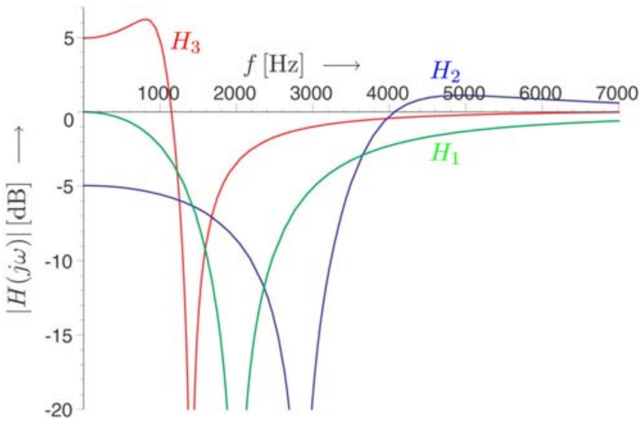

Fig. 3. Plot of the transfer function magnitudes of individual biquads.

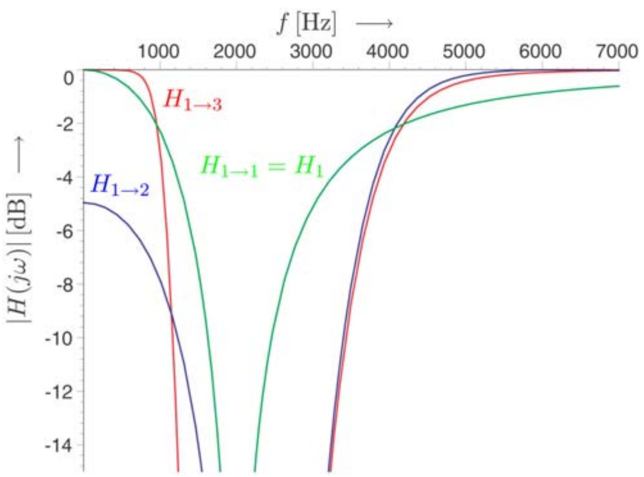

Fig. 4. Plot of the transfer function magnitudes after biquads.

4. Conclusion

The gain assignment is one of the final steps in the design of a cascade filter. The optimal dynamic range of a filter structure is achieved solving this task. The described algorithm solves the problem generally on the basis of parameters of particular biquads (ω_0, Q, ω_n) and transfer function type of the section. The procedure is fully universal, it doesn't suppose any simplification. The algorithm has been used in ARCBlock function of Syntfil package for filter design in mathematical program Maple.

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About Author

Jiří HOSPODKA was born in Havlíčkův Brod, Czech Republic in 1967. He received the Ing. (M.Sc.) and Ph.D. degree 1991 and 1995 in CTU Prague, Czech Republic. Research interests: circuit theory, analog electronics, filter design, switched capacitor and switched current circuits. He is one of two authors of Syntfil package for design of analog filters in Maple program.