Modeling Delays of Microwave Transistors and Transmission Lines by the 2nd-Order Bessel Function

Josef DOBEŠ, Karel ULOVEC

Dept. of Radio Electronics, Czech Technical University, Technická 2, 166 27 Praha, Czech Republic

dobes@feld.cvut.cz, xulovec@feld.cvut.cz

Abstract. *At present, most of simulation programs can characterize gate delays of microwave transistors. However, the delay is mostly approximated by means of first-order differential equations. In the paper, a more accurate way is suggested which is based on an appropriate second-order differential equation. Concerning the transmission line delay, majority of the simulation programs use both Branin (for lossless lines) and LCRG (for lossy lines) models. However, the first causes extreme simulation times, and the second causes well-known spurious oscillations in the simulation results. In the paper, an unusual way for modeling the transmission line delay is defined, which is also based on the second-order Bessel function. The proposed model does not create the spurious oscillations and the simulation times are comparable with those obtained with the classical models. Properties of the implementation of the second-order Bessel function are demonstrated by analyses of both digital and analog microwave circuits.*

Keywords

Bessel function, ordinary differential equations, group delay, MESFET, gate delay, transmission line.

1. Introduction

The second-order Bessel function [1] appears to be an efficient mathematical tool for a general-purpose delay modeling. In the paper, an appropriate mathematical form has been chosen for an approximation of both MESFET and transmission line delays. The models have been implemented into and checked by our software tool CIA (Circuit Interactive Analyzer). Properties of both models have been checked by the analysis of microwave analog and digital circuits.

2. General Properties of the 2nd-Order Bessel Function

Distributed phenomena are described by the partial differential equations in the exact way. However, we should use an appropriate approximation of the exact solution of the partial differential equations in both MESFET and transmission line modeling due to convenient model complexity. In characterizing the bipolar junction transistor [2], the second-order Bessel function is efficiently used for modeling the delay of collector current at high frequencies. However, that function can also be used for modeling the MESFET and transmission line in a similar precise way.

Consider a linear second-order differential equation

$$\frac{1}{3\omega_0^2}\frac{d^2y(t)}{dt^2}+\frac{1}{\omega_0}\frac{dy(t)}{dt}+y(t)=x(t) \tag{1}$$

(which is used in [1] for the model of collector current of the microwave bipolar junction transistors) to be solved for $y(t)$ as a result—the parameter ω_0 will be defined later. Using the Laplace transform of (1), we get

$$Y(s)=X(s)\,\Phi(s), \tag{2}$$

where s is the Laplace operator and $\Phi(s)$ is the second-order Bessel function

$$\Phi(s)=\frac{3\omega_0^2}{s^2+3\omega_0 s+3\omega_0^2}. \tag{3}$$

To examine the frequency properties of (3), let us set $s=j\omega$, and formulate the argument $\varphi(\omega)$ of $\Phi(j\omega)$, i.e.,

$$\begin{aligned}\varphi(\omega)&=\arg\Phi(j\omega)=\arg(-\omega^2+3\omega_0^2-j3\omega_0\omega)\\&=\arctan\frac{-3\omega_0\omega}{3\omega_0^2-\omega^2}.\end{aligned} \tag{4}$$

The group delay is the value of the derivative of $\varphi(\omega)$ with respect to ω, i.e., it is possible to write

$$\begin{aligned}-\frac{d\varphi(\omega)}{d\omega}&=\frac{1}{1+\left(\dfrac{3\omega_0\omega}{3\omega_0^2-\omega^2}\right)^2}\\&\quad\times\frac{3\omega_0\left(3\omega_0^2-\omega^2\right)-(-2\omega)3\omega_0\omega}{\left(3\omega_0^2-\omega^2\right)^2}\\&=\frac{9\omega_0^3+3\omega_0\omega^2}{9\omega_0^4+3\omega_0^2\omega^2+\omega^4}.\end{aligned} \tag{5}$$

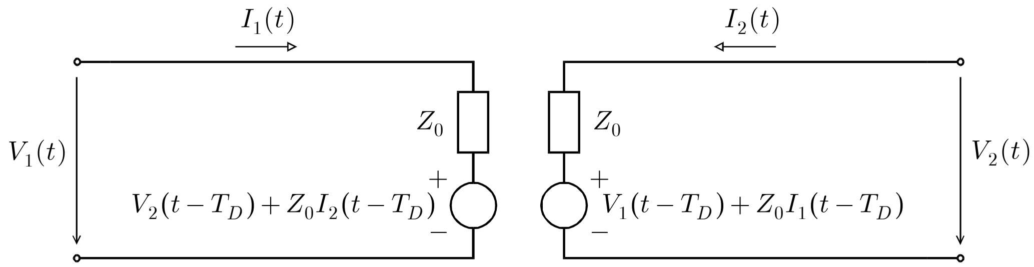

Fig. 1. Classical Branin model of the lossless transmission line, which will be approximated by means of the 2nd-order Bessel function.

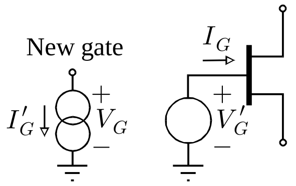

Fig. 2. Novel way of MESFET or pHEMT gate delay modeling.

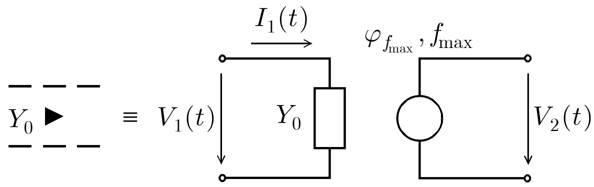

Fig. 3. New CIA built-in sub-element called φ-shift.

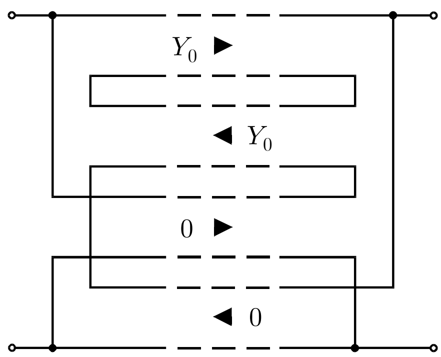

Fig. 4. One element of the new transmission line model created by the four φ-shift sub-elements.

Consider now ω to be appreciably lesser than ω_0. For example, for $\omega = \frac{3}{4}\omega_0$, $\omega^4 \big/ \left(9\omega_0^4 + 3\omega_0^2\omega^2\right) < 0.03$—therefore, the last member in the denominator in (5) (i.e., ω^4) may be neglected, resulting to the simple formula

$$-\frac{d\varphi(\omega)}{d\omega} \simeq \frac{1}{\omega_0}. \tag{6}$$

Thus, the group delay is approximately constant, and (1) can therefore be considered to be a very accurate tool for the delay modeling.

If the phase delay φ_τ is determined in degrees at $1/(2\pi\tau)$, where τ represents, e.g., a new MESFET model parameter, the time delay at this frequency can be expressed by the formula

$$\frac{\varphi_\tau}{360^\circ} 2\pi\tau\,. \tag{7}$$

Comparing (6) and (7), we obtain the final equation for the model parameter ω_0 (in fact, ω_0 is *not* a primary model parameter; it is derived from the primary parameters φ_τ and τ, the phase delay in degrees and reciprocal limit frequency, respectively):

$$\omega_0 = \frac{360^\circ}{2\pi} \frac{1}{\varphi_\tau\,\tau} \tag{8}$$

—however, if the phase delay φ'_τ is determined in radians at $1/(2\pi\tau)$, we obtain the more natural form

$$\omega_0 = \frac{1}{\varphi'_\tau\,\tau}. \tag{9}$$

2.1 Limit of Approximating by the 2nd-Order Bessel Function

If the function (3) is used for the delay modeling, the magnitude of $Y(s)$ has to be also checked. For $s = j\omega$, the magnitude of $\Phi(j\omega)$ can easily be expressed as

$$\left|\Phi(j\omega)\right| = \frac{1}{\sqrt{1 + \frac{1}{3}\left(\frac{\omega}{\omega_0}\right)^2 + \frac{1}{9}\left(\frac{\omega}{\omega_0}\right)^4}}. \tag{10}$$

This magnitude decreases to $95.77\ \%$ for $\omega = \frac{1}{2}\omega_0$. Therefore, the angular frequency ω should not be greater than $\frac{1}{2}\omega_0$—in other case, the φ-shift element also changes the amplitude of the signal (not only the phase).

Substituting $\omega \leftarrow \frac{1}{2}\omega_0$ in (4), we obtain the shift

$$\varphi(\tfrac{1}{2}\omega_0) = \arctan(-\tfrac{12}{22}) = -28.61^\circ. \tag{11}$$

—therefore, the phase shifts greater than approximately 30° cannot be modeled by one equation ((1) or (3)) only.

3. Modeling the MESFET Gate Delay

During the transient operations in a FET, the electron depletion width under the gate must be changed by charge transport [2]. Hence, a change in the gate voltage does not cause an instantaneous change in the drain-source current.

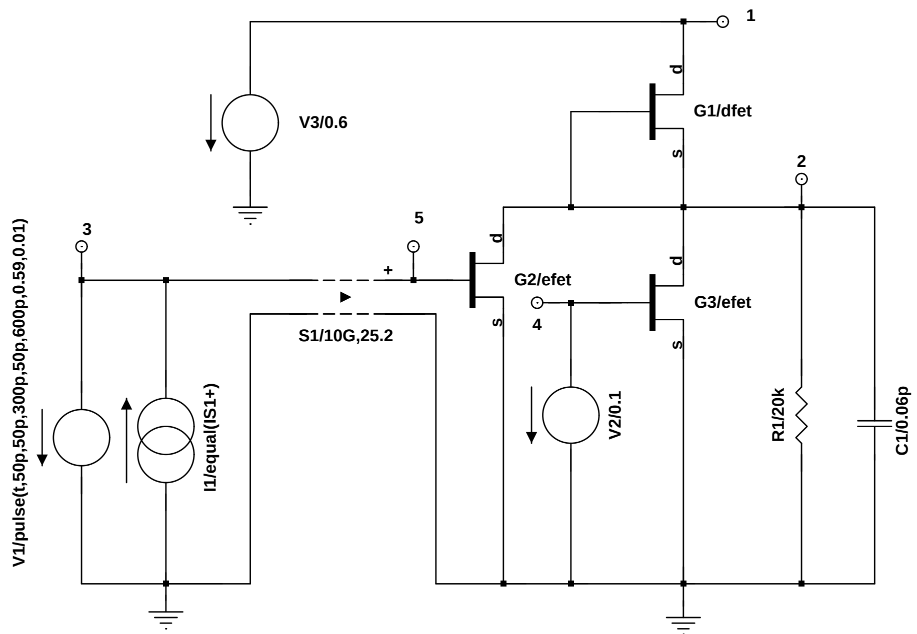

Fig. 5. Digital circuit with both enhancement- and depletion-mode MESFETs used as a test of the model with the 2nd-order Bessel function.

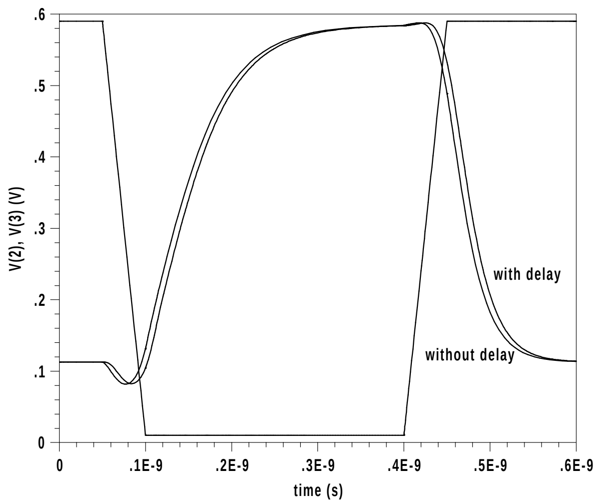

Fig. 6. Additional delay given by the 2nd-order Bessel function.

This delay can be accurately defined by means of an appropriate differential equation (see (1) and Fig. 2)

$$\frac{1}{3\omega_0^2}\frac{d^2V_G'(t)}{dt^2}+\frac{1}{\omega_0}\frac{dV_G'(t)}{dt}+V_G'(t)=V_G(t) \quad (12)$$

with the following natural initial conditions:

$$V_G'(0)=V_G(0),\ dV_G'/dt(0)=0. \quad (13)$$

As the V_G' source is unilateral and DC conditions cannot change, the I_G' is also necessary (see Fig. 2 and [3], e.g.)

$$I_G'=I_G. \quad (14)$$

The solution of (12) can be performed in a numerical way by means of a circuit analyzer's integration algorithm. However, a mixed analytical-numerical solution is also possible, because the affiliated homogenous differential equation to (12) is *linear*.

4. Modeling the Transmission Delay

Let us consider the classical Branin model in Fig. 1 [2], where Z_0 is the characteristic impedance of the transmission line. The main problem of this model is introduced by the incompatibility of the $(t-T_D)$ delays with the numerical integration methods. Therefore, another approximation of the delay model is necessary [4]. An unusual suggestion can also be created by the differential equations

$$\frac{1}{3\omega_0^2}\frac{d^2V_{1,2}'(t)}{dt^2}+\frac{1}{\omega_0}\frac{dV_{1,2}'(t)}{dt}+V_{1,2}'(t)=V_{1,2}(t), \quad (15)$$

$$\frac{1}{3\omega_0^2}\frac{d^2I_{1,2}'(t)}{dt^2}+\frac{1}{\omega_0}\frac{dI_{1,2}'(t)}{dt}+I_{1,2}'(t)=I_{1,2}(t). \quad (16)$$

The solutions of (15) and (16) are used as the approximation of the delayed voltages and currents, i.e.,

$$V_{1,2}(t-T_D):=V_{1,2}'(t),\ I_{1,2}(t-T_D):=I_{1,2}'(t). \quad (17)$$

In fact, a novel unusual model is created in this way. First, a new circuit element called φ-shift is implemented with the symbol, which is drawn in the left part of Fig. 3. Second, one element of the new transmission line model is created in the way, which is shown in Fig. 4—this is an unusual analogy of the Branin model.

Let us emphasize again that the phase shift must not be greater than approximately $30°$ — see (11). Therefore, several Branin subcircuits (the circuit in Fig. 4 with the "Bessel" approximation of the delays) must be serially connected in the case of greater phase shifts.

The model of transmission line based on (15)–(17) has quite different properties than LCRG one. The LCRG model generates spurious oscillations—on the contrary, the new model generates smoothed shapes of all the signals.

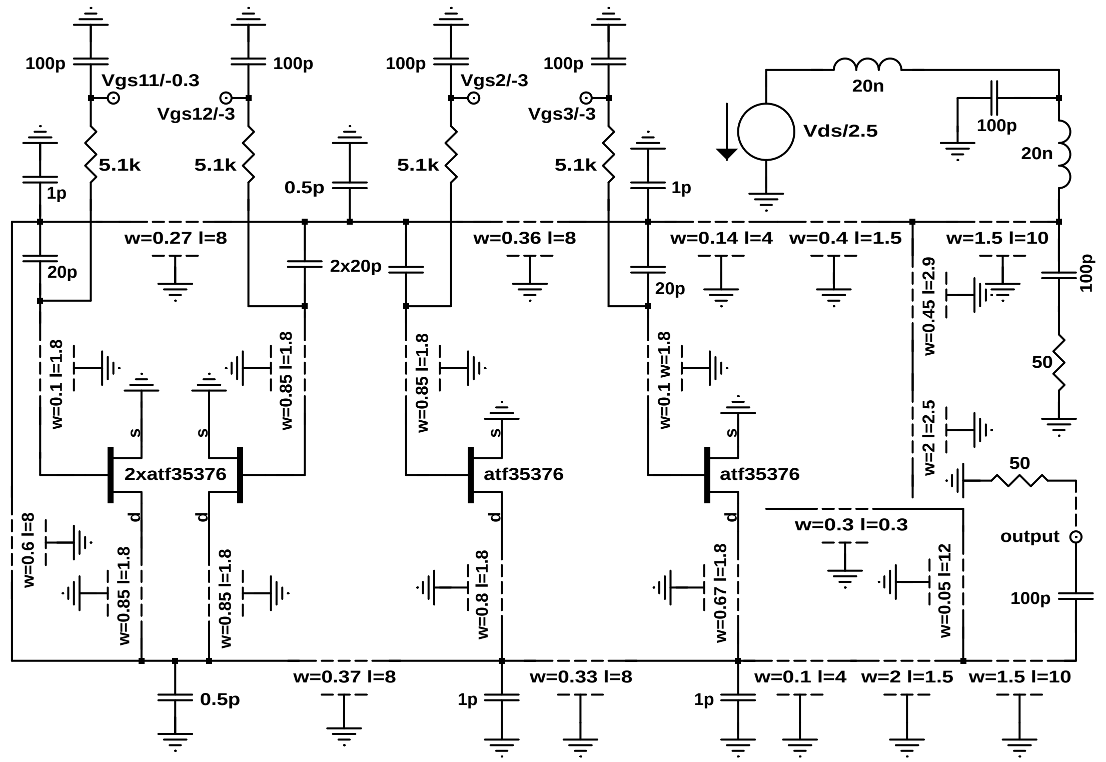

Fig. 7. Microwave voltage-tunable distributed oscillator used as a test of the transistor gate delay model based on the 2nd-order Bessel function.

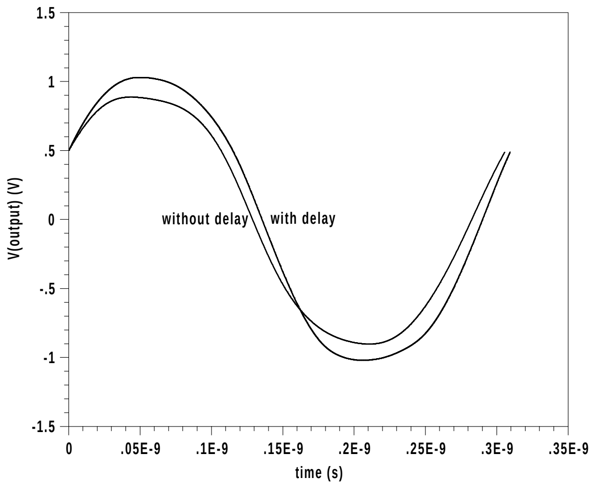

Fig. 8. Comparison of the two output voltages for the zero and nonzero gate delays, 3.266 versus 3.228 GHz detected.

5. Testing the MESFET Gate Delay

5.1 Testing Digital Circuit with MESFETs

Consider a fast digital circuit in Fig. 5 [7] with the trapezoidal pulse at the input—see Fig. 6. A substantial part of the delay is caused by the MESFET nonlinear capacitances, of course. However, another part of the delay is caused by the distributed circuit phenomena, which is also shown in Fig. 6. This additional part of the delay has been modeled in the way described in Fig. 2. In this simple circuit, the additional delay is not fundamental. However, for more complicated digital circuits, accurate modeling is necessary.

5.2 Testing Distributed Microwave Oscillator

As another more sophisticated example, consider a distributed *tunable* oscillator in Fig. 7 [5]. The oscillator has been developed in our university and the analyses with CAD tools have been performed by the classical Berkeley's SPICE 3 system and our CIA program, because the tools based on harmonic balance did not converge for this circuit.

Formerly, the gate delay has not been modeled (the SPICE 3 model is unable to define it)—the importance of the gate delay modeling is demonstrated here. The results are shown in Fig. 8. The curve marked "without delay" can be obtained by any of the standard circuit simulators; the curve marked "with delay" has been computed by the CIA program—the $\mathrm{atf35376}$ transistor has had the gate phase shift $25.2°$ at 10 GHz (i.e., the limit $30°$ was not exceeded).

As observed, the period determined by the new model is slightly greater, which is expected. However, the phase shifts induced by the transistor delays cause somewhat greater magnitude, which is also important. Note that the steady-state has been detected by the CIA extrapolation algorithm monitoring the intersection of the output voltage with the level 0.5 V (automatically). As the oscillator is tunable, a number of analyses had to be performed—therefore, the automatic period detection is very important.

The measured results [5] confirm the simulated ones. For the bias voltages shown in Fig. 7, the oscillation frequency 3.004 GHz has been measured. In the tuning range, the output power 3–11 dBm has been measured (the level 10 dBm is related to the output voltage 1 V).

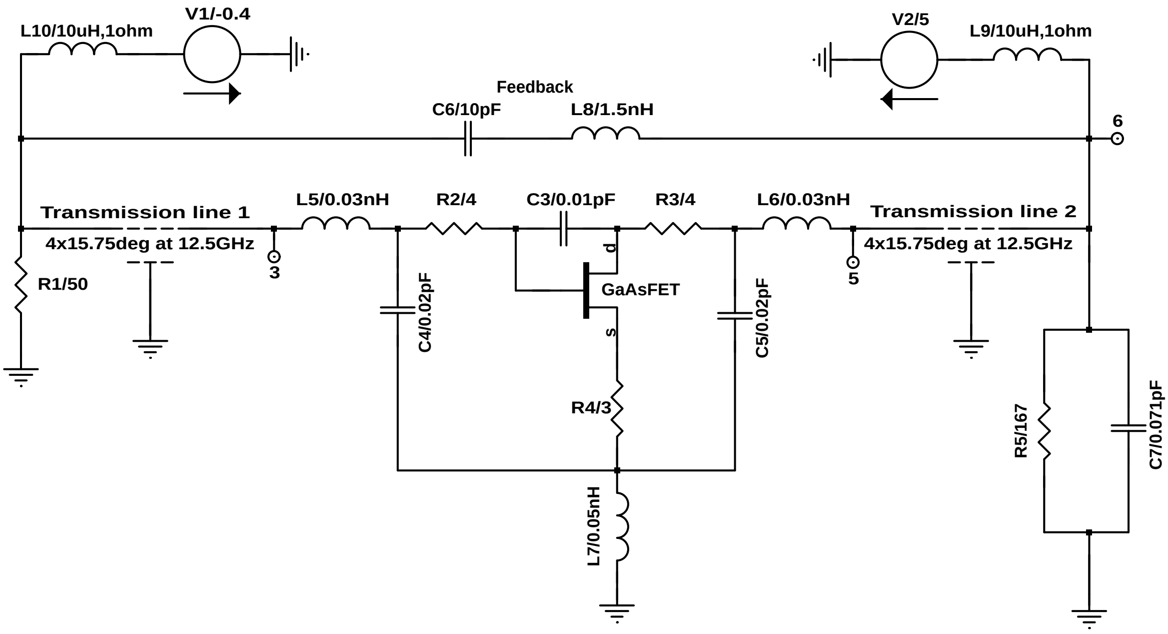

Fig. 9. Feedback microwave oscillator with great amplitude of output used as a test of both “Bessel” and LC transmission line models.

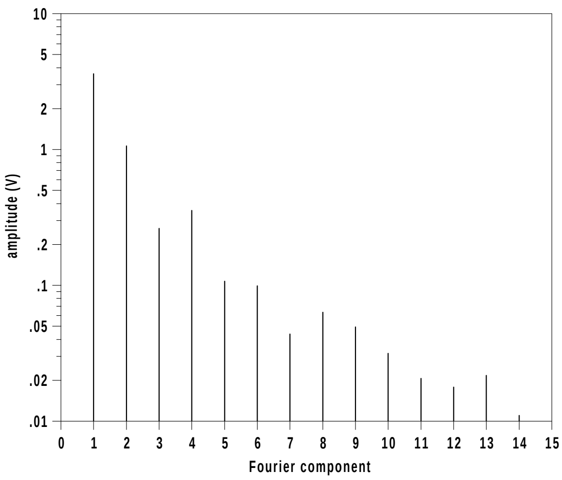

Fig. 10. Harmonic analysis of the output waveform created by LC model (detected period is 5.649 GHz).

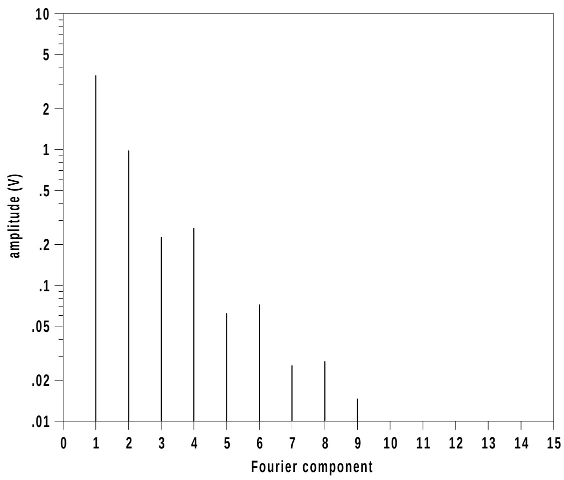

Fig. 11. Harmonic analysis of the output waveform created by “Bessel” model (detected period is 5.653 GHz).

6. Testing the Transmission Delay

Consider a feedback microwave oscillator in Fig. 9 [6]. The oscillator has a number of harmonic components, and therefore it is very appropriate for testing the transmission line model. The pins 3 and 5 mark the exterior gate and drain, respectively; the pin 6 marks the output.

The first analysis has been performed by the classical LC model of the two transmission lines (4 LC sections were used, the inductance and capacitance per section were $l = 0.175$ nH and $c = 0.07$ pF, respectively). The second analysis has been performed with the 2^{nd}-order “Bessel” model of the two transmission lines (as defined in Fig. 9, four sections has been used, each of them with the phase shift $15.75°$ at 12.5 GHz—for the accuracy purposes, it is better to use more sections with lesser values of shifts to protect the amplitude of the signal).

The two results are compared using the Fourier components of the outputs—see Figs. 10 and 11. The proposed model suppresses the spurious oscillations (see the higher harmonic components of the signals). Moreover, the new model has claimed a lesser number of integration steps—275 in comparison with 376 for the LC model.

7. Conclusions

The second-order Bessel function has been proposed as an efficient tool for modeling the delays in microwave and RF circuits. A practical way for using this function for modeling the MESFET gate and transmission line delays was outlined. The models have been checked using both digital and analog microwave circuits. The results show that the modeling of the delays caused by the distributed circuit phenomenon could be very important for the overall precision of the simulations, and therefore it should be implemented into the CAD tools.

Acknowledgements

This paper has been supported by the Grant of the European Commission “TARGET” (6th Framework Programme), by the Grant Agency of the Czech Republic, grant Nº 102/05/0277, and by the Research Program of the Czech Technical University in Prague MSM6840770014.

References

[1] MASSOBRIO, G., ANTOGNETTI, P. *Semiconductor Device Modeling With SPICE*. 2nd ed. New York: McGraw-Hill, 1993.

[2] VLADIMIRESCU, A. *The Spice Book*. 1st ed. New York: John Wiley & Sons, 1994.

[3] MADJAR, A. A fully analytical AC large-signal model of the GaAs MESFET for nonlinear network analysis and design. *IEEE Transactions on Microwave Theory and Techniques,* 1988, vol. 36, no. 1, p. 61 - 67.

[4] GUO, Y.-S. Transient simulation of high-speed interconnects based on the semidiscretization of telegrapher's equations. *IEEE Transactions on Computer-Aided Design,* 2002, vol. 21, no. 7, p. 799 - 809.

[5] DIVINA, L., SKVOR, Z. The distributed oscillator at 4 GHz. *IEEE Transactions on Microwave Theory and Techniques,* 1998, vol. 46, no. 12, p. 2240 - 2243.

[6] TAJIMA, Y., WRONA, B., MISHIMA, K. GaAs FET large-signal model and its application to circuit designs. *IEEE Transactions on Electron Devices,* 1981, vol. 28, no. 2, p. 171 - 175.

[7] SUSSMAN-FORT, S. E., HANTGAN, J. C., HUANG, F. L. A SPICE model for enhancement- and depletion-mode GaAs FET’s. *IEEE Transactions on Microwave Theory and Techniques,* 1986, vol. 34, no. 11, p. 1115 - 1119.

About Authors...

Josef DOBEŠ received the Ph.D. degree in microelectronics at the Czech Technical University in Prague in 1986. From 1986 to 1992, he was a researcher of the TESLA Research Institute, where he performed analyses on algorithms for CMOS technology simulators. Currently, he works at the Department of Radio Electronics of the Czech Technical University in Prague. His research interests include the physical modeling of circuit elements for radio engineering, especially RF and microwave transistors and transmission lines, creating or improving special algorithms for the circuit analysis and optimization, such as time- and frequency-domain sensitivity, pole-zero intermodulation, or steady-state analyses, and creating a comprehensive CAD tool for the analysis and optimization of RF and microwave circuits CIA (Circuit Interactive Analyzer).

Karel ULOVEC was born in 1977 in Prague, Czech Republic. He received his master's degree in electrical engineering from the Faculty of Electrical Engineering, Czech Technical University in Prague, in 2001. At present, he is an assistant professor at the same faculty and he is working on the Ph.D. thesis. His research interests include measurement of radio transmitter and receivers and radio signal processing.