Practice Utilization of Algorithms for Analog Filter Group Delay Optimization

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Abstract. *This contribution deals with an application of three different algorithms which optimize a group delay of analog filters. One of them is a part of the professional circuit simulator Micro Cap 7 and the others two original algorithms are developed in the MATLAB environment. An all-pass network is used to optimize the group delay of an arbitrary analog filter. Introduced algorithms look for an optimal order and optimal coefficients of an all-pass network transfer function. Theoretical foundations are introduced and all algorithms are tested using the optimization of testing analog filter. The optimization is always run three times because the second, third and fourth-order all-pass network is used. An equalization of the original group delay is a main objective of these optimizations. All outputs of all algorithms are critically evaluated and also described.*

Keywords

Analog filters, group delay, optimization, all-pass networks, MATLAB, Micro-Cap, evolutionary algorithms.

1. Introduction

Analog filters belong to the most widely used dynamic systems. A design, analysis and optimization of the filters have been developing for a long time. Results of the development have been widely published in many countries; see [2], [5], [9], [12] and [17]. Standard design and realization methods are mastered at present. Development works are currently oriented to various areas, e.g. current-mode networks, monolithic integration, multi-criteria design using neural networks or evolutionary algorithms [7] and [18]. New network structures have been found. They use modern active elements and they also extend a usable frequency bandwidth [3], [22].

It is a well-known fact that required magnitude frequency response of the analog filter is a basic design criterion. Selective properties of the filter are concentrated in the response, see [5], [17]. It is necessary to consider also other frequency responses, e.g. a group delay. The group delay response of the analog filter can create so-called linear distortion of the processed signal in a pass-band together with the magnitude response [17], [20]. An optimal course of the analog filter group delay response is rigorously required in some areas of electronics, e.g. in video signal processing or during the TV signal transmission [13]. A lot of integrated circuit producers include group delay equalizers directly into their chips [25], [26].

In the past, many special papers were published about the group delay optimization [8], [10], [20]. Our main idea was to develop simply applicable, fast and robust algorithms. Both described original algorithms are used for personal computers (PC) and they were developed in the MATLAB environment [21]. The results of the algorithms are compared with the algorithm which is integrated in the Micro Cap 7, see [24]. All outputs of our algorithms are practically usable.

2. Basic Optimization Idea

The analog filter designed using the magnitude frequency response requirements often has an unsuitable course of the group delay response. There is a big ripple at a limit of the pass-band. To optimize the group delay response and to keep the optimal magnitude response are the main objectives of the optimization task. It can be realized using the all-pass network which is connected behind the optimized analog filter. The all-pass magnitude frequency response is constant. That is why the optimization task is oriented to its phase properties.

A basic optimization idea is shown in Fig. 1.

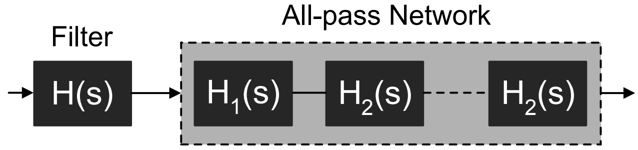

Fig. 1. Basic optimization idea.

To optimize the analog filter group delay means to find the suitable order and coefficients of the all-pass network transfer function. The group delay of the all-pass network is added with the group delay of the analog filter. A total group delay will be equalized. A designer can specify a few restrictive conditions or requirements to the optimization outputs are realizable. The main optimization result often is to reduce the analog filter group delay ripple. There can be other requirements, e.g. concrete distribution of the group delay along the frequency axis or the all-pass order maximum value given. A mean value of the total group delay response rises during the optimization, because both of the circuits, the analog filter and the all-pass network are connected in cascade, see Fig. 1. It is necessary to explore this effect but there are no problems in many practice applications.

2.1 The First and Second-Order All-Pass Networks

Let us repeat that the all-pass networks are linear dynamic systems having the constant magnitude frequency response. Their phase properties are specified during the optimization. The main optimization result is the transfer function in a semi-symbolic form. It is suitable to decompose the filter transfer function into the first and second-order sections, because of possible later cascade ARC realization. Then a cut-off or resonant frequency and a Q-factor of each section are the typical outputs of the algorithm [17], see Fig. 1.

The all-pass networks are uniquely determined by their transfer functions. The transfer function of the first-order all-pass network has the following form

$$H_1(s) = \pm \frac{s - \omega_0}{s + \omega_0}, \tag{1}$$

where s denotes a so-called complex frequency and ω_0 is a cut-off cyclic frequency. It is possible to derive that the group delay response has the form

$$\tau_1(\omega) = \frac{2.\omega_0}{\omega_0^2 + \omega^2}, \tag{2}$$

where ω is an independent variable as a cyclic frequency axis.

The group delay response of the first-order all-pass networks is uniquely determined by the cut-off frequency ω_0. Its course is a downward-sloping curve from an initial value

$$\tau(0) = \frac{2}{\omega_0}[s]. \tag{3}$$

The transfer function of the second-order all-pass network has the following form

$$H_2(s) = \frac{s^2 - \dfrac{\omega_r}{Q}.s + \omega_r^2}{s^2 + \dfrac{\omega_r}{Q}.s + \omega_r^2}, \tag{4}$$

where ω_r denotes a resonant frequency and Q is a quality factor.

The group delay has the following course

$$\tau_2(\omega) = \frac{2.Q.\omega_r.\left(\omega^2 + \omega_r^2\right)}{Q^2.\left(\omega_r^2 - \omega^2\right)^2 + \omega_r^2.\omega^2}. \tag{5}$$

The courses have one maximum which depends on the Q-factor and ω_r according to the formula

$$\tau_{\max} = \frac{4.Q}{\omega_r}[s]. \tag{6}$$

Many other details can be found in [17].

If we design the first-order all-pass network then we optimize one parameter ω_0. There are two parameters: the Q-factor and ω_r in case of the second-order all-pass network design. An optimization space has a high and variable dimension in a general case.

3. Optimization Algorithms

In this chapter three optimization algorithms are described. The description is oriented especially to their practice utilization and comparison. The first two algorithms are the original functions, the third one is a comparative method used in the Micro Cap 7.

The calculation of the all-pass network group delay course is simple because of equations (2) and (5).

3.1 Original Optimized Analog Filter

All three optimization algorithms were tested by using one common analog filter. It was the fifth-order low-pass filter designed by the Chebyshev approximation. There was the 3dB ripple in the pass-band. The transfer function coefficients can be obtained by several possibilities, e.g. using the tables of active ARC realization or by a direct design using the computer.

Testing filter design results are shown in Tab. 1. The cut-off frequency is 1 Hz.

5th order low-pass filter	f_0, f_r [Hz]	Q[-]
1st order section	0,1775	---
2nd order section	0,6140	2,1375
2nd order section	0,9675	8,8178

Tab. 1. Coefficients of the testing filter transfer function.

The testing filter group delay course is shown in Fig. 2. There is the group delay course and also the group delay ripple $\Delta\tau$. The group delay ripple is defined as the difference between a maximum and a minimum of the group delay course.

All group delay courses shown below were obtained by the simulation in the Micro Cap 7. The analog filter and all-pass networks were simulated by using Laplace blocks denoted in MC-7 as *LFVofV*.

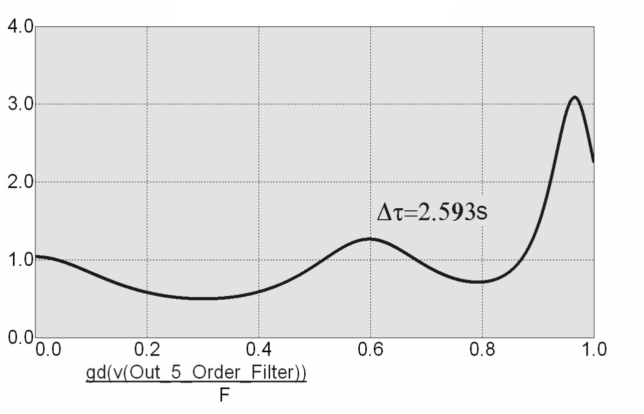

Fig. 2. Group delay course of the initial 5th-order analog filter.

3.2 Initial Optimization Requirements

The main objective of all optimization algorithms is to reduce the initial group delay ripple $\Delta\tau$ of the testing fifth-order analog filter. The algorithms will look for optimal values of f_0, f_r and Q of all the first and second-order sections. The second, third and fourth-order all-pass networks will be used for this task. It could be predicted that the resulting group delay ripple will be decreased.

3.3 Required Outputs

The analog filter group delay course will be optimized by using the three all-pass networks mentioned above. The group delay with the reduced ripple will be the main output for each used all-pass network. Both ripples in percents, initial and resulting will be compared. The user specifies how many percents it is necessary to reduce the ripple.

3.4 Simple Iterative Algorithm

The first introduced algorithm is based on simple iterative approach with respect to the analog filter properties. The initial group delay course (simulated or measured), the analog filter transfer function and user requirements are the possible input parameters. The user specifies his requirements in these forms:

- Concrete required reduction of the ripple in percent.
- Concrete all-pass network order if necessary.
- Maximum all-pass network order if necessary.
- Definition of restrictive intervals of a frequency axis.
- Definition of restrictive intervals of all-pass network transfer function parameters.

If the all-pass network order is not specified by a user, then the algorithm will look for its optimal value. A simplified block diagram of the algorithm is shown in Fig. 3.

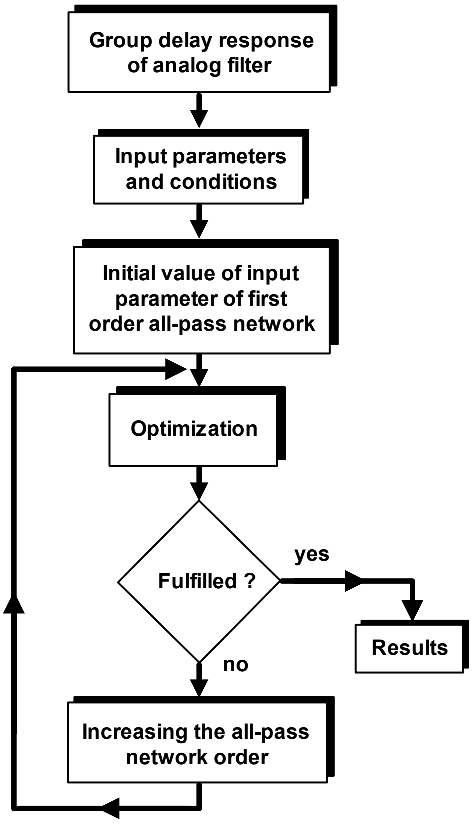

Fig. 3. Simplified block diagram of the iterative algorithm.

The algorithm works in these steps:

- Loading the initial group delay course.
- Setting the initial values of the all-pass network transfer function (the first-order or defined by user).
- Optimization of these parameters.
- Increasing the all-pass network order if necessary.
- Generating all outputs.

3.4.1 Objective Function of Optimization

The initial values of the all-pass network parameters can be set according to various criteria, e.g. as a centre of a parameter interval or according to the ARC normalized filter coefficients. The initial values of f_0, f_r and Q are set. Then a new group delay course is calculated and also compared with required result. If the user main requirement is not fulfilled then sensitivity of variables f_0, f_r and Q is determined by using a little change of them. After it a new vector of mentioned parameters is set and the optimization continues with the new values. The new values are set by using a step set before.

An objective function is very important part of the algorithm. It had to be chosen suitable with regard to analog filters properties. We explored a few possibilities how to choose the objective function. Finally we chose so-called *sum of square deviation* which had to be normalized by using the frequency axis. An actual course of the group delay is compared with an ideal constant course during the optimization.

The concrete form of the objective function is as follows in the MATLAB environment:

```
function DEVI=Sumdeviation(Vector1,Vector2,omega)
    DEVI=sum((omega.*Vector1-omega.*Vektor2).^2);
```

The first row denotes calling the objective function and the second one is the objective function. The symbol ω is an independent variable as the cyclic frequency axis, the symbol *Vector1* denotes a vector of the actual group delay course and *Vector2* is the comparative constant. In this way, a very good convergence of the algorithm is provided also for higher-order analog filters.

3.4.2 Testing of the Algorithm

The described algorithm was widely tested. The results of the optimization are shown in Fig. 4, 5, 6. The resulting optimal coefficients of the all-pass networks transfer function are grouped in Tab. 2. The results are valid for the normalized frequency axis 1 Hz. A few algorithm properties can be controlled by using some initial settings, e.g. speed and accuracy. The algorithm works with the values of f_0, f_r and Q. These sections coefficients have a concrete physical meaning. The algorithm can automatically change the all-pass network order during the optimization. The whole algorithm consists of one m-file.

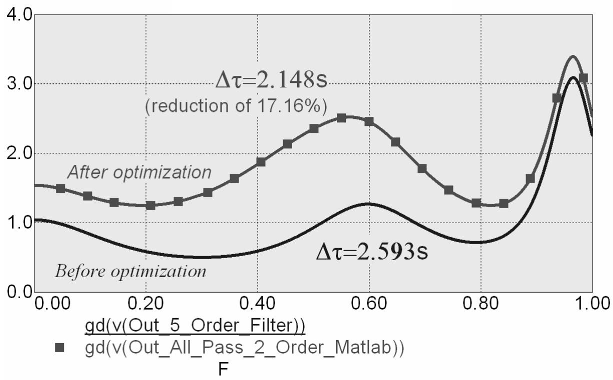

Fig. 4. Filter optimization by using 2nd order all-pass network.

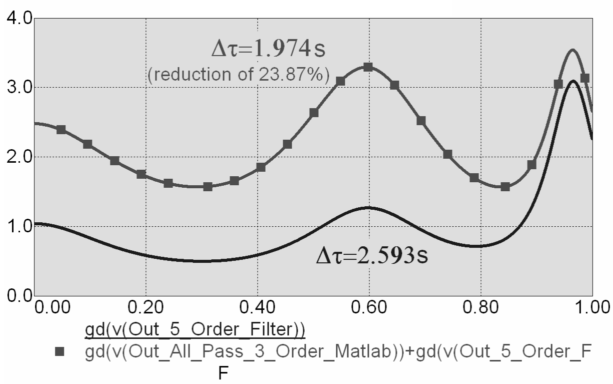

Fig. 5. Filter optimization by using 3rd order all-pass network.

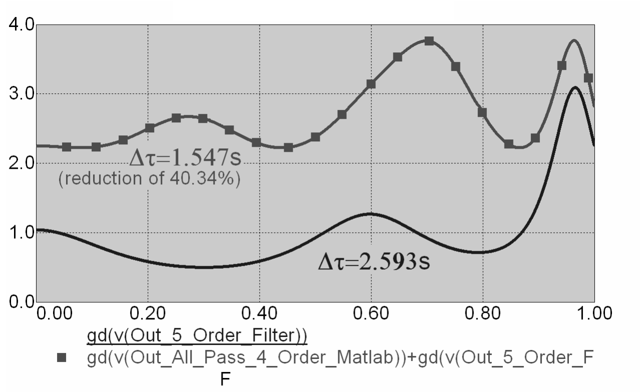

Fig. 6. Filter optimization by using 4th order all-pass network.

All-pass order	$f_0, f_r [Hz]$	$Q[-]$
2	0.5477	1.1664
3	0.2777	---
	0.6230	1.7442
4	0.3291	0.9138
	0.7269	2.8908

Tab. 2. Section coefficients of the optimal all-pass networks.

3.5 Evolutionary Algorithms

Optimization algorithms recently represent a strong instrument for solving many complicated engineering problems. For solving these problems, the group of very powerful algorithms, which apply principles of the Darwin evolution in the nature, was developed within the last several decades. The typical attribute of these algorithms is that the algorithms work with a population of possible solutions. The solutions are called individuals [7]. Fundamental of these algorithms consists in a repeated modifying of the population matrix by applying evolution principles to the best possible solution be found. A general structure of an evolution program can be described as follows:

```
begin
   t ← 0
   initialize P(t)
   evaluate P(t)
   while termination-condition do
   begin
      t ← t+1
      select P(t) from P(t-1)
      alter P(t)
      evaluate P(t)
   end
end
```

where $P(t)=\{x_1^t, ..., x_n^t\}$ is a population matrix generated for iteration t. Each solution of the solved problem x_i^t is evaluated by an objective function; whose value – the so-called fitness value – indicates a measure of quality of each individual. Subsequently, for the iteration $t+1$, the new population matrix is created by a selection of better individuals from the previous population matrix generated for the iteration t; this is the so-called selection mechanism. Some individuals undergo a transformation by genetic operators like mutation and crossover. Evolution algorithm converges to the best possible solution within certain number of iterations. The best solution is represented by an individual with the best fitness value.

The evolution algorithms showed themselves very efficient in the area of the design and optimization of the electrical circuits and systems [4], [14], [15], [23]. The new unconventional methods for the design of the group delay equalizers, which utilize the evolutionary algorithms, were presented in the papers [14], [23]. The main advantage of the evolution based methods is that the methods do not need to determine an initial estimation of the unknown searched variables to ensure convergence of the algorithm. The next advantage is that the evolution-based methods are more robust against convergence to the local extremes in comparison with the design procedures based on classical numerical methods [1], [16], [21].

3.5.1 An Analog All-Pass Filter

As it was mentioned above, the all-pass filters are utilized for the filter group delay frequency response equalization. The group delay equalizer design procedures based on evolutionary algorithms are defined in normalized s*- complex domain, where s^* is the complex variable meaning the normalized frequency defined as:

$$s^* = \Sigma + j \cdot \Omega . \tag{7}$$

The general transfer function of the even order analog all-pass filter can be defined in the form:

$$H_a(s^*) = \frac{\prod_{i=1}^{\frac{n}{2}} \left(s^* - \alpha_i + j \cdot \beta_i\right) \cdot \left(s^* - \alpha_i - j \cdot \beta_i\right)}{\prod_{i=1}^{\frac{n}{2}} \left(s^* + \alpha_i + j \cdot \beta_i\right) \cdot \left(s^* + \alpha_i - j \cdot \beta_i\right)} \tag{8}$$

where n is the order of the all-pass filter. The general transfer function of the odd order analog all-pass filter can be defined in the form:

$$H_a(s^*) = \frac{\left(-s^* + \alpha_0\right) \cdot \prod_{i=1}^{\frac{n}{2}} \left(s^* - \alpha_i + j \cdot \beta_i\right) \cdot \left(s^* - \alpha_i - j \cdot \beta_i\right)}{\left(s^* + \alpha_0\right) \cdot \prod_{i=1}^{\frac{n}{2}} \left(s^* + \alpha_i + j \cdot \beta_i\right) \cdot \left(s^* + \alpha_i - j \cdot \beta_i\right)} \tag{9}$$

where the order of the all-pass filter is $n+1$.

These functions can be simply normalized using the term (10) into the s-complex plane. In this way the resultant all-pass filter modified can be implemented practically.

$$s^* = \frac{s}{\omega_p}, \tag{10}$$

where ω_p is the cut-off frequency of the pass-band filter.

The total group delay frequency response of the cascaded filter and all-pass filter is defined like a sum of individual group delays of each of them, thus:

$$\tau(\Omega) = \tau_a(\Omega) + \tau_f(\Omega) = -\mathrm{Re}\left[\frac{H'_a(s^*)}{H_a(s^*)} + \frac{H'_f(s^*)}{H_f(s^*)}\right]_{s^* = j \cdot \Omega} \tag{11}$$

where $H'_a(s^*)$ is the derivation of the all-pass filter transfer function $H_a(s^*)$ by the variable s^*, $H_f(s^*)$ is the filter transfer function and $H'_f(s^*)$ is the appropriate derivation of $H_f(s^*)$ by s^*.

3.5.2 Flow Diagram of the Equalizer Design Procedure

The flow diagram of the group delay equalizer design procedure described in the paper [14] is shown in Fig. 7.

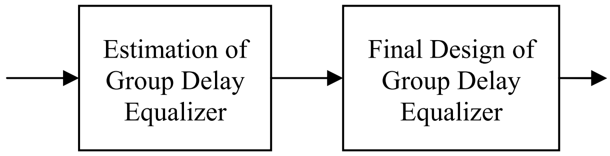

Fig. 7. The flow diagram of the group delay equalizer design procedure.

The main principle of the first block is searching for the all-pass filter transfer function complex poles to decrease the difference $\Delta\tau$ between the maximum and the minimum of the total group delay in the filter pass-band. Using this principle, the objective function (12) can be simply created. Minimization process of the objective function by an evolution algorithm leads to the so-called estimation of the group delay equalizer.

$$F(\underline{x}) = \max[\tau(\Omega)] - \min[\tau(\Omega)] + P_1, \tag{12}$$

where vector $\underline{\mathbf{x}}$ is compounded of real and imaginary parts of the all-pass filter transfer function complex poles. The symbol P_1 is the penalty function defined by (13). The penalty function ensures stability of the searched all-pass filter.

$$P_1 = \sum_{i=1}^{N} \begin{cases} 20000 - 100 \cdot x_i & \text{if} \quad x_i < 0 \\ 0 & \text{otherwise} \end{cases}. \tag{13}$$

The symbol N labels the number of the unknown searched variables and x_i are elements of the vector $\underline{\mathbf{x}}$.

The main principle of the second block is again searching for the group delay equalizer transfer function complex poles. However, in contrast with the first block, the constants ε (ripple of the group delay) and $\tau(0)$ (the group delay constant approximated in the Chebyshev sense), which define an equi-ripple form of the group delay, are searched in addition. The knowledge of the individual group delay extreme positions gained within the first block application is utilized in the second block to obtain equi-ripple form of the total group delay frequency response. Let us have the filter group delay pass-band sampled by N equidistant points. Then, an error function for the individual group delay extremes can be simply defined by:

$$e_k = \begin{cases} \tau(0) + (-1)^k \cdot \varepsilon - \max[\tau(\Omega_k)] & \text{for even } k \\ \tau(0) + (-1)^k \cdot \varepsilon - \min[\tau(\Omega_k)] & \text{for odd } k \end{cases}, \quad (14)$$

$$\Omega_k \in (p_k - N1_k, p_k + N2_k), \quad (15)$$

where k=1,..,n+2, p_k denotes the position of the appropriate k^{th} extreme. The symbol Ω_k denotes the interval in which the appropriate k^{th} extreme is searched. The symbol $N1_k$ is the lower bound of the interval Ω_k and $N2_k$ is the upper bound of the interval Ω_k. For the group delay sampling at 1024 points, the values $N1_k$=100 points and $N2_k$=80 points appear sufficient. Only $N1_1$=0, $N2_1$=0, $N1_{n+2}$=0 and $N2_{n+2}$=0. These values have to be recalculated for sampling by another value of N points. In the case of the odd order all-pass filter design, we have to take into account a modification of the formulae (14) and (15) by parameters k=0,..,n+1 and $N1_0$=0, $N2_0$=0, $N1_{n+1}$=0, $N2_{n+1}$=0.

The second objective function (16) is defined by means of the error function (14):

$$F(\underline{x}) = \left(\sum_{k=1}^{n+2} [e_k]^2 \right) + P_2. \quad (16)$$

The objective function (16) has to be modified by the parameter k=0,..,n+1 in the case of the odd order all-pass filter design. The vector $\underline{\boldsymbol{x}}$ is again compounded of real and imaginary parts of the all-pass filter transfer function complex poles. Moreover, the vector $\underline{\boldsymbol{x}}$ includes the constants $\tau(0)$ and ε. The symbol P_2 means penalty function defined by (17), which ensures both stability of the final all-pass filter and positive values of the constants τ (0) and ε.

$$P_2 = \sum_{i=1}^{M} \begin{cases} 20000 - 100 \cdot x_i & \text{if} \quad x_i < 0 \\ 0 & \text{otherwise} \end{cases}. \quad (17)$$

The symbol M labels the number of the unknown searched variables and x_i are the elements of the vector $\underline{\boldsymbol{x}}$.

The minimization process of the objective function leads to the design of a group delay equalizer satisfying the condition that the total resultant group delay frequency response of the cascaded filter and all-pass filter has the equi-ripple form in the filter pass-band.

The Differential Evolution (DE) algorithm is used for minimization of the objective functions in both optimization blocks. The DE algorithm is a population-based, direct search algorithm designated for solving the global optimization tasks. The original DE algorithm was developed by K.V. Price and Rainer Storn. The genesis of the algorithm comes from a principle of genetic annealing [18]. The first version of the DE algorithm was published by the authors in 1995. The DE algorithm is very simple for implementation (less than 30 lines in C program code [19]). In spite of that, the DE algorithm appeared very efficient for searching the solutions of difficult optimization problems in practical engineering applications. The principles of the algorithm can be found in [4], [14], [15], [18], [19].

In comparison with the design procedure based on the DE algorithm, the method presented in [23] uses a standard genetic algorithm for optimization, which works with floating point number representation of unknown variables [7], [23]. Moreover, this genetic algorithm is combined with a direct search simplex method [6], [11] to increase the convergence rate significantly. However, the disadvantage is a lower resistance of the algorithm against convergence to the local extremes. As to compare the method from the paper [23] with the block diagram in the Fig. 7, the method works only with the first block. Thus, the method does not enable always to achieve such a group delay equalizer to the total group delay has the equi-ripple form.

3.5.3 Testing of the Algorithm

The algorithm was undergone the testing as to optimize the same filter group delay within the same conditions like in the previous case. The results are shown in Fig. 8 to 10.

The design procedure was started with the following initial settings: number of population members NP=150, constants CR=0.9, F=0.9, initial range of unknown variables (the real and imaginary parts of complex poles) was (0 ÷ 1). This initial range is used only for the generation of the initial population matrix. The space of the unknown variables, which is scanned by the DE algorithm, is restricted within the generation of the next population matrices only by the penalization functions. Thus, the DE algorithm in itself does not hold predefined ranges of variables in the searched space. The group delay frequency responses were sampled by 1024 equidistant points.

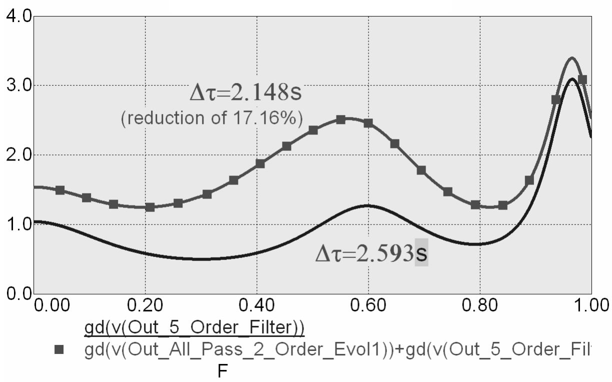

Fig. 8. Filter optimization by using 2nd order all-pass network.

The normalized values of the real and imaginary parts of the complex poles of the 1st and the 2nd order all-pass filter transfer functions are the output of the algorithm, see Tab. 3.

With respect to the above mentioned reasons, the values were recalculated to the values of cut–off frequencies like resonant frequencies and Q-factors, see Tab. 4.

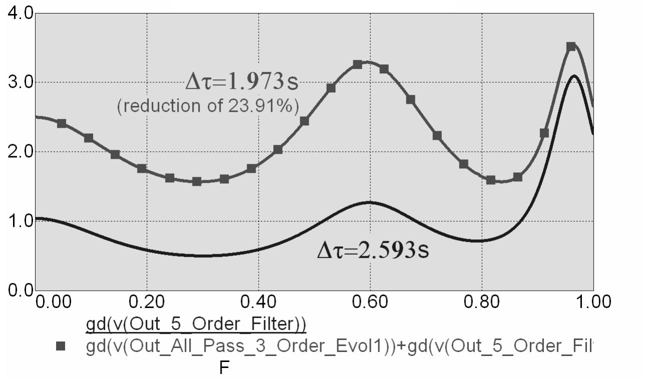

Fig. 9. Filter optimization by using 3rd order all-pass network.

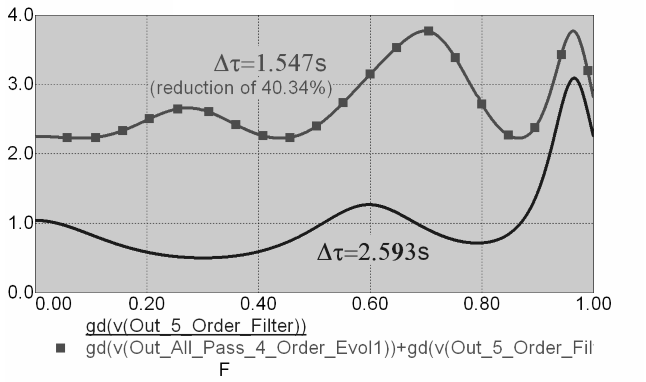

Fig. 10. Filter optimization by using 4th order all-pass network.

2^{nd} order all-pass filter	3^{rd} order all-pass filter	4^{th} order all-pass filter
α_1=0.235109364	α_0=0.273195618	α_1=0.125475236
β_1=0.494789127	α_1=0.178582138	β_1=0.715924523
	β_1=0.597074676	α_2=0.181523558
		β_2=0.275721572

Tab. 3. Normalized parameters of all-pass networks in s*-domain.

All-pass order	$f_0, f_r[Hz]$	$Q[-]$
2	0.5478	1.1650
3	0.2732	---
	0.6232	1.7449
4	0.7268	2.8963
	0.3301	0.9093

Tab. 4. Section coefficients of the optimal all-pass networks.

It is apparent from Fig. 8 to 10 that the group delay frequency responses do not have an equi-ripple form. Therefore, we apply the second part of the presented optimization procedure as to find such parameters of the all-pass filters to the resultant group delay frequency responses have the equi-ripple form in the filter pass-band.

The second part of the described procedure was started with the same initial setting like in the case of the first part design procedure: the number of population members NP=150, the constants CR=0.9, F=0.9, the initial range of unknown variables was again (0 ÷ 1). The group delay frequency responses were sampled by 1024 equidistant points. The resultant values of the all-pass filters normalized parameters are arranged in the Tab.5.

2^{nd} order all-pass filter	3^{rd} order all-pass filter	4^{th} order all-pass filter
α_1=0.136275973	α_0=0.148295218	α_1=0.117548185
β_1=0.486191946	α_1=0.156628801	β_1=0.260769938
	β_1=0.585933796	α_2=0.131741747
		β_2=0.696698362

Tab. 5. Normalized parameters of all-pass networks in s*-domain.

The parameters arranged in the Tab. 6 are achieved by the transformation of the normalized parameters given in the Tab.5 to the *s*-domain. The circuits designed using the parameters were simulated in the Micro-Cap 7 program, see the Fig.11 to 13.

All-pass order	$f_0, f_r[Hz]$	$Q[-]$
2	0.5049	1.8526
3	0.1483	---
	0.6065	1.9361
4	0.2860	1.2167
	0.7090	2.6910

Tab. 6. Section coefficients of the optimal all-pass networks.

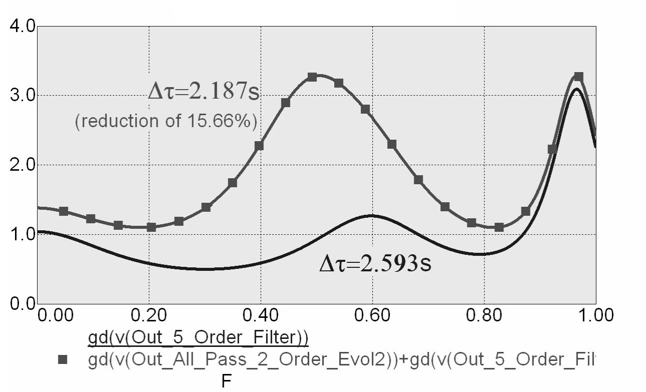

Fig. 11. Filter optimization by using 2nd order all-pass network.

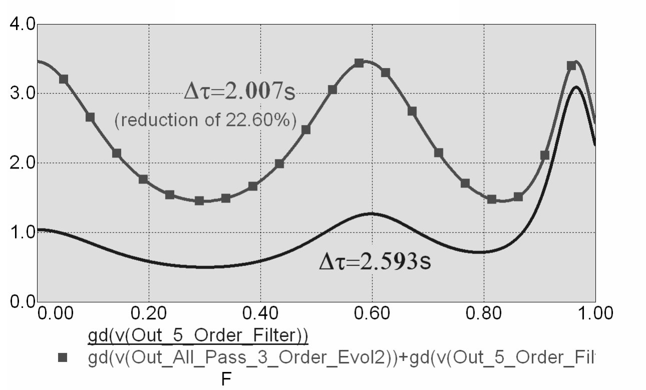

Fig. 12. Filter optimization by using 3rd order all-pass network.

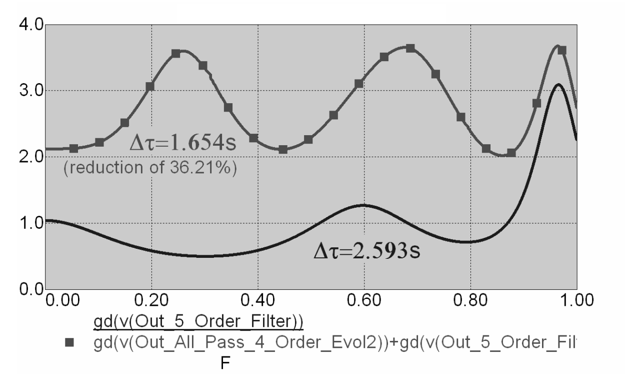

Fig. 13. Filter optimization by using 4th order all-pass network.

From Fig. 11 to 13 it is obvious that we did not succeed to find the exact equi-ripple forms of the resultant group delays. The deformations in the resultant group delay forms are due to the greatly corrugated form of the equalized filter group delay.

3.6 Comparative Optimization by MicroCap7

The outputs of the both described algorithms were compared with an algorithm which is embedded in the Micro Cap 7 commercial simulator [24]. A standard Powell's method is used in the Micro Cap 7 [1], [16]. The method works with a parabola in three points of some interval. The parabola divides the interval into two subintervals. One of subintervals is excluded by evaluation of an objective function. The results of the optimization are shown in Fig. 14, 15, 16 within the same conditions. The resulting optimal coefficients of the all-pass networks transfer function are grouped in Tab. 7.

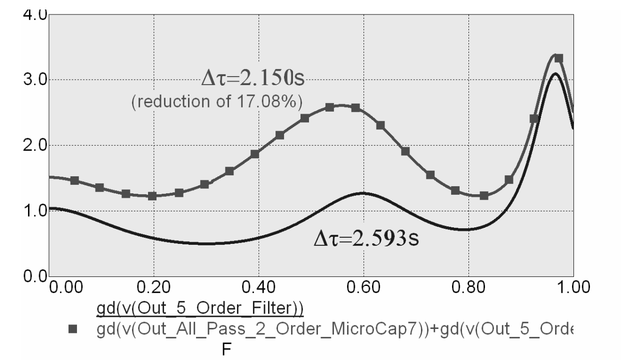

Fig. 14. Filter optimization by using 2nd order all-pass network.

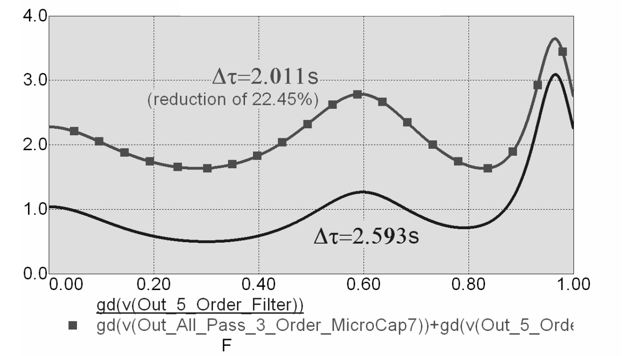

Fig. 15. Filter optimization by using 3rd order all-pass network.

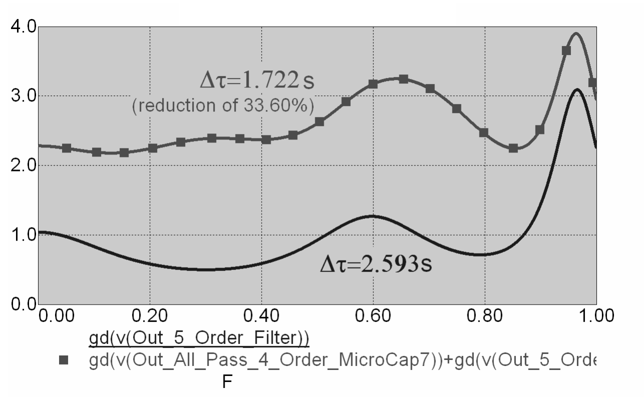

Fig. 16. Filter optimization by using 4th order all-pass network.

All-pass order	$f_0, f_r[Hz]$	$Q[-]$
2	0.5388	1.2510
3	0.3748	---
	0.6529	1.2550
4	0.3800	0.8160
	0.7340	2.0180

Tab. 7. Section coefficients of the optimal all-pass networks.

4. Summary of Optimization Results

The achieved reduction of the analog filter group delay course is well-arranged in Tab. 8. The optimization algorithms mentioned above are described. The second, third and fourth-order all-pass networks were used to optimize the analog filter. The evolutionary algorithm was used twice and these two cases are marked Evol1 and Evol2. The Evol2 algorithm includes an equi-ripple correction. A number in each cell indicates reduction in percents. An ideal case of the reduction is 100 %. It means the original group delay course is optimized into the constant course.

Algorithm	Reduction $\Delta\tau$ in $[\%]$		
	2nd order	3rd order	4th order
Iteration	17.16	23.89	40.34
Evol1	17.16	23.91	40.34
Evol2	15.66	22.60	36.21
Micro-Cap	17.08	22.45	33.60

Tab. 8. Comparison of all optimization outputs.

The algorithms Iteration and Evol1 provide comparable results, as we can see in Tab. 8. The Micro Cap 7 offers the worst results. However, these results depend on current bounds of the optimization parameters. The main disadvantage of the Micro-Cap optimization algorithm is a very long calculation time. It could be possible to compare many other parameters of used algorithms, e.g. setting possibilities, speed etc. This contribution is oriented to direct application and a comparison of ones.

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