Quantum Description of Optical Devices Used in Interferometry

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Abstract. A quantum-mechanical description of the phase shifters, retarders, mirrors and beam splitters is given in the paper. The description is then applied on two types of states. On a coherent state, a classical-like state, and on a number state, hence the strict quantum state.

The quantum description of a beam splitter can be found in the literature. However the description does not treat with the polarization concept.

The paper is aimed to introduce quantum description of an arbitrary oriented retarder and give a description of a beam splitter which treats with the polarization.

Keywords

Phase shifter, retarder, beam splitter, polarization, coherent state, number state, Jones calculus.

1. Introduction

In this section we introduce the two most common states used in the quantum optics. Used relations in this section are taken from [1].

A coherent state is the eigenstate of the annihilation operator

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \tag{1}$$

and can be generated from the vacuum state using the displacement operator

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle = \exp(\alpha \hat{a}^{H} - \alpha^{*} \hat{a})|0\rangle$$
⁽²⁾

where the symbol X^{H} denotes the Hermitian conjugate. A normalized coherent state can be expressed in the number state basis as

$$|\alpha\rangle = \exp\left(-|\alpha|^2 / 2\right) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$
(3)

Note that $|\alpha|$ is related to the amplitude of the field.

Number states are generated from the vacuum according to

$$|n\rangle = \frac{1}{\sqrt{n!}} \left(\hat{a}^{H}\right)^{n} |0\rangle \tag{4}$$

The time evolution of the number states is given by

$$|n(t)\rangle = \exp(-in\,\omega t)|n(0)\rangle \tag{5}$$

where we neglected the global factor $\exp(-i\omega t/2)$, the contribution from the vacuum (does not influence the dynamic of the system).

2. Phase Shifter

A phase shifter \hat{P} acts like time evolution. It adds an extra time delay $\Delta t = (n_p - n_0)\Delta z/c$ which is dependent on refractive index n_p and thickness Δz of the shifter. The symbol n_0 denotes the refractive index of the environment. In the following, the symbol *n* denotes the eigenvalue of energy eigenstate $|n\rangle$. The dependency on the refractive index of the shifter is absorbed in Δt .

Hence the action of the shifter on a number state is

$$\hat{P}|n\rangle = \exp(in\theta)|n\rangle \tag{6}$$

where $\theta = \omega \Delta t$ was introduced and represents an extra phase due to the shifter. Relation (6) can be rewritten in the operator form as (due to the fact that *n* represents the eigenvalue of $|n\rangle$)

$$\hat{P}|n\rangle = \exp(i\hat{n}\theta)|n\rangle \tag{7}$$

where the number operator $\hat{n} = \hat{a}^H \hat{a}$ was introduced. Hence, the shifter is generally described by the following unitary operator

$$\hat{P} = \exp(i\hat{n}\,\theta) \tag{8}$$

When the shifter acts on the coherent state, one obtains

$$\hat{P}|\alpha\rangle = \exp\left(-|\alpha|^2 / 2\right) \sum_n \frac{(\alpha \exp(i\omega\Delta t))^n}{\sqrt{n!}} |n\rangle =$$

$$= |\alpha \exp(i\theta)\rangle.$$
(9)

Note, that the time evolution of a coherent state is given by

$$\left|\alpha(t)\right\rangle = \left|\alpha \exp^{-i\omega t}\right\rangle \tag{10}$$

where the contribution from the vacuum was neglected.

3. Retarder

In this section we describe the action of a retarder by extending the foregoing considerations through the assumption that different phase shifts are experienced for two eigenstates. Thus a retarder is on the contrary from a phase shifter able to change the polarization and one needs to introduce an extra degree of freedom to treat with the polarization.

3.1 Retarder in the Lab Frame

First we consider the situation where the lab frame and frame of the retarder are in the coincidence (so fast axis (FA) of the retarder is parallel to the *y*-axis as is depicted in Fig. 1 on the left).



Fig. 1. Placing of a retarder. On the left, the lab frame coincides with the retarder frame. On the right, the lab frame differs from the retarder frame.

The action of a retarder (slow axis oriented parallel to xaxis) is described by using Jones matrix (expressed in the x-y basis) in the form

$$J_{R_{SAh}} = \begin{bmatrix} 1 & 0\\ 0 & \exp(-i\theta) \end{bmatrix}, \quad \theta > 0$$
(11)

where θ denotes the phase difference between both eigenvalues of the retarder (the relative phase shift). In the Jones formalism matrix (11) is applied on the input vector where each entry of the vector describes the orthogonal (respect with the polarization) electric field component

$$\begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \exp(-i\theta) \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$
(12)

The usual way for obtaining a quantum description is replacing the classical complex field amplitudes by a set of annihilation operators as

$$\begin{pmatrix} \hat{a}'_x \\ \hat{a}'_y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \exp(-i\theta) \end{pmatrix} \begin{pmatrix} \hat{a}_x \\ \hat{a}_y \end{pmatrix}.$$
(13)

Now we can introduce new basis states as

$$|n_{x}, n_{y}\rangle = (n_{x}!n_{y}!)^{-1/2} (\hat{a}_{x}^{H})^{n_{x}} (\hat{a}_{y}^{H})^{n_{y}} |0,0\rangle$$
(14)

where an extra degree of freedom was introduced in order to include polarization. A basis state $|n_x, n_y\rangle$ can be interpreted as the state containing exactly n_x x-polarized photons and n_y y-polarized photons.

The input and output modes of a retarder are related according to (using (13))

$$\hat{a}'_{x} = \hat{a}_{x}$$

$$\hat{a}'_{y} = \exp(-i\theta)\hat{a}_{y}.$$
(15)

Note that the familiar commutations relations

$$[\hat{a}_{i}, \hat{a}_{j}^{H}] = \delta_{ij}, [\hat{a}_{i}, \hat{a}_{j}] = 0 = [\hat{a}_{i}^{H}, \hat{a}_{j}^{H}]$$
(16)

are still satisfied by transformation (15) (indexes *i*, *j* represent input and output modes for each polarization).

Formally relation (15) may be written as

$$\begin{pmatrix} \hat{a}'_x \\ \hat{a}'_y \end{pmatrix} = \hat{R}^H \begin{pmatrix} \hat{a}_x \\ \hat{a}_y \end{pmatrix} \hat{R}$$
 (17)

where \hat{R} is a unitary operator representing a retarder. The operator satisfying (17) and (15) was found in the form

$$\hat{R} = \exp\left(-i\theta \hat{a}_{y}^{H} \hat{a}_{y}\right).$$
(18)

As an example let us act a retarder on the input state which consists of one photon polarized diagonally

$$\frac{1}{\sqrt{2}} \left(\hat{a}_x^H + \hat{a}_y^H \right) \left| 0_x, 0_y \right\rangle = \frac{1}{\sqrt{2}} \left(\left| 1_x, 0_y \right\rangle + \left| 0_x, 1_y \right\rangle \right)$$

$$\xrightarrow{R} \frac{1}{\sqrt{2}} \left(\left| 1_x, 0_y \right\rangle + \exp(-i\theta) \left| 0_x, 1_y \right\rangle \right)$$
(19)

where we used inverse relation (from (15))

$$\hat{a}_x^H = (\hat{a}_x')^H$$

$$\hat{a}_y^H = \exp(-i\theta)(\hat{a}_y')^H.$$
(20)

Investigating (19) one can see that in the case of quarterwave plate ($\theta = \pi/2$), from a photon initially diagonally polarized one obtains a photon which is right-handed polarized.

3.2 Retarder in the Device Frame

Generally the retarder is placed as shown on the right in Fig. 1. Then the Jones matrix $J_{R_{Sdh}}$ is undergone the active transformation

$$U(-\delta)J_{R_{S,th}}U(\delta) \tag{21}$$

where

$$U = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix}$$
(22)

Note that $U^{H}(\delta) = U(-\delta)$.

Working out the multiplication in (21) one again can associate the fields with annihilation operators as

$$\hat{a}'_{x} = \cos^{2} \delta(\hat{a}_{x}) + \cos \delta \sin \delta(\hat{a}_{y}) + + \exp(-i\theta) \sin^{2} \delta(\hat{a}_{x}) - \exp(-i\theta) \sin \delta \cos \delta(\hat{a}_{y})$$
(23)

and

$$\hat{a}'_{y} = \exp(-i\theta)\cos^{2}\delta(\hat{a}_{y}) + \cos\delta\sin\delta(\hat{a}_{x}) + \\ +\sin^{2}\delta(\hat{a}_{y}) - \exp(-i\theta)\sin\delta\cos\delta(\hat{a}_{x}).$$
(24)

Note that commutation relations (16) are again satisfied for the operators (23) and (24). Relations (23) and (24) can be formally written as

$$\begin{pmatrix} \hat{a}'_x \\ \hat{a}'_y \end{pmatrix} = \hat{R}^H \hat{U}^H \begin{pmatrix} \hat{a}_x \\ \hat{a}_y \end{pmatrix} \hat{U} \hat{R}$$
(25)

where \hat{R} is given by (18) and

$$\hat{U} = \exp\left(\delta\left(\hat{a}_x^H \hat{a}_y - \hat{a}_x \hat{a}_y^H\right)\right) \tag{26}$$

Eqn.(26) is the quantum analogy of (22). Hence the unitary operator describing an arbitrary placed retarder is \hat{UR} .

As an example let us suppose $\delta = \pi/4$, $\theta = \pi/2$ and the input state $|2_x, 0_y\rangle$. Using (23) and (24) one obtains

$$\hat{a}_{x}^{H} = \frac{1}{\sqrt{2}} \Big((\hat{a}_{x}')^{H} + i (\hat{a}_{y}')^{H} \Big)$$
(27)

Then

$$|2,0\rangle \xrightarrow{R} \left(\left(\hat{a}'_{x} \right)^{H} + i \left(\hat{a}'_{y} \right)^{H} \right)^{2} |0,0\rangle =$$

$$= |2,0\rangle + |0,2\rangle + 2i|1,1\rangle =$$

$$= \left(|1,0\rangle + i|0,1\rangle \right) \otimes \left(|1,0\rangle + i|0,1\rangle \right) \equiv |0_{R},2_{L}\rangle.$$
(28)

And generally $|n_x, 0_y\rangle \xrightarrow{R} |0, n_L\rangle$. So we obtained left-handed polarized photons.

As the last example let us suppose input *x*-polarized coherent state and the properties of the retarder from the previous example. Then

$$\exp\left(\alpha \hat{a}_{x}^{H} - \alpha^{*} \hat{a}_{x}\right) |0_{x}, 0_{y}\rangle \xrightarrow{R} \\ \xrightarrow{R} \exp\left(\frac{1}{\sqrt{2}} \alpha (\hat{a}_{x}')^{H} - \frac{1}{\sqrt{2}} \alpha^{*} \hat{a}_{x}'\right) \cdot \\ \cdot \exp\left(\frac{i}{\sqrt{2}} \alpha (\hat{a}_{y}')^{H} - \frac{-i}{\sqrt{2}} \alpha^{*} \hat{a}_{y}'\right) |0_{x}, 0_{y}\rangle = \\ = \left|\frac{\alpha}{\sqrt{2}} \sum_{x} \frac{i\alpha}{\sqrt{2}} \right|_{y}.$$

$$(29)$$

So we obtained a left-handed polarized beam.

4. Mirror

For normal incidence a common mirror has Jones matrix in the form

$$J_{Mirror} = \begin{bmatrix} -1 & 0\\ 0 & 1 \end{bmatrix}$$
(30)

in the accordance with the Frenel's equations (see Fig. 2).



Fig. 2. Used notation for the components of the incident field and Frenel's equations.

Hence the action of a mirror is the same as half-wave plate (HWP) when the fast axis is parallel to the x-axis. The mirror is then described by the unitary operator satisfying

$$\hat{R} = \exp\left(i\pi\hat{a}_x^H\hat{a}_x\right) \tag{31}$$

where we neglected the global phase factor $\pi/2$ (is same for both field components) due to the reflection.

Now we apply the mirror on a coherent left-handed polarized light

$$\left|\frac{\alpha}{\sqrt{2}_{x}},\frac{i\alpha}{\sqrt{2}_{y}}\right\rangle \xrightarrow{M} \exp\left(\frac{-1}{\sqrt{2}}\alpha(\hat{a}'_{x})^{H}-\frac{-1}{\sqrt{2}}\alpha^{*}\hat{a}'_{x}\right) \cdot \exp\left(\frac{i}{\sqrt{2}}\alpha(\hat{a}'_{y})^{H}-\frac{-i}{\sqrt{2}}\alpha^{*}\hat{a}'_{y}\right) |0_{x},0_{y}\rangle =$$
(32)
$$=\left|\frac{-\alpha}{\sqrt{2}_{x}},\frac{i\alpha}{\sqrt{2}_{y}}\right\rangle$$

where we used (15) with $\theta = \pi$. The left-handed beam is converted to the right-handed beam due to the reflection.

This is obvious because the Jones matrix (30) in the left-right handed (helicity) basis is expressed as

$$J'_{Mirror} = \begin{bmatrix} 0 & 1\\ 1 & 0 \end{bmatrix}.$$
 (33)

5. Non-Polarizing Beam Splitter

5.1 Classical Scalar Description of a Non-Polarizing Beam Splitter

The classical scalar (input and output beams are assumed to have a common linear polarization) description of a non-polarizing beam splitter (NBS) can be found in [3]. The output fields are related to the input fields by relation

$$\begin{bmatrix} E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} t_0 & r_1 \\ r_0 & t_1 \end{bmatrix} \begin{bmatrix} E_0 \\ E_1 \end{bmatrix}$$
(34)

where the meaning of reflection and transmission coefficients is illustrated in Fig. 3.



Fig. 3. Classical scalar (a) and vector (b) description of a NBS.

The phases of reflection and transmission coefficients are related via (the formula derived by considering the energy conversation [3])

$$\arg(r_0) + \arg(r_1) - \arg(t_0) - \arg(t_1) = \pm \pi$$
. (35)

Note that there are two choices of phases which have different observable effects. These choices depend on the construction of the beam splitter.

For a beam splitter cube the conventional choice is

$$\arg(r_0) = \arg(r_1) = \arg(t_0) = 0$$
 and $\arg(t_1) = \pi$ (36)

whereas for a single dielectric layer beam splitter the conventional choice is

$$\arg(t_0) = \arg(t_1) = 0$$
 and $\arg(r_0) = \arg(r_1) = \pi/2$. (37)

Of course, the phase choice in (36) and (37) is not unique.

Hence for a 50:50 beam splitter cube one can write

$$E_2 = \frac{E_0 + E_1}{\sqrt{2}} \text{ and } E_3 = \frac{E_0 - E_1}{\sqrt{2}}.$$
 (38)

5.2 Classical Vector Description of a Non-Polarizing Beam Splitter

In section 5.1 we assumed that all beams have the common linear polarization. The reason is following. In general, an optical device divides an incident field into two parts, the eigenstates (eigenvectors) of the optical device. And these eigenstates are treated independently. As an example we measured Jones matrix of NBS 10701A

$$NBS_{t} = \begin{bmatrix} 0.69 & 0\\ 0 & 0.73 \exp(-3.5^{\circ} i) \end{bmatrix}$$
(39)

for the transmitted beam and

$$NBS_r = \begin{bmatrix} 0.67 & 0\\ 0 & 0.62 \exp(172.9^\circ i) \end{bmatrix}$$
(40)

for the deflected (reflected) beam. Matrices (39) and (40) are expressed in the linear basis (Fig. 2). Hence matrix component NBS_{11} tells us how *x*-component of the field evolves. From (39) and (40) one can guess an ideal 50:50 NBS as

$$NBS_t = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0\\ 0 & 1 \end{bmatrix},\tag{41}$$

$$NBS_r = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0\\ 0 & -1 \end{bmatrix}.$$
 (42)

However in general, the input beams are in the superposition of eigenstates of NBS. Hence the reflection and transmission coefficients are different for *x* and *y*-polarization. Now we suppose a symmetric beam splitter cube $(|r_0|=|r_1| \text{ and } |t_0|=|t_1|)$. Then the vector description of a beam splitter cube can be expressed as

$$\vec{E}_2 = NBS_t \cdot \vec{E}_0 + NBS_r \cdot \vec{E}_1$$

$$\vec{E}_3 = NBS_r \cdot \vec{E}_0 - NBS_t \cdot \vec{E}_1$$
(43)

Note, that for a non-symmetric beam splitter cube one needs to measure Jones matrices for both input beams (hence the second formula in (43) will obtain different Jones matrices from the first formula).

If we now assume that input beams are both x-polarized

$$\vec{E}_0 = \begin{pmatrix} E_0 \\ 0 \end{pmatrix}$$
 and $\vec{E}_1 = \begin{pmatrix} E_1 \\ 0 \end{pmatrix}$ (44)

and next we suppose an ideal 50:50 NBS, using (41), (42) and (43) we arrive to (38).

5.3 Quantum Non-Polarization Description of NBS

The quantum scalar description of a beam splitter is given in [1], [3]. The electric field vectors in (35) are replaced by annihilation operators (Fig. 4(a))

$$\begin{bmatrix} \hat{a}_2\\ \hat{a}_3 \end{bmatrix} = \begin{bmatrix} t_0 & r_1\\ r_0 & t_1 \end{bmatrix} \begin{bmatrix} \hat{a}_0\\ \hat{a}_1 \end{bmatrix}.$$
(45)

Formula (45) can be formally written as

$$\begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = \hat{B}^H \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix} \hat{B}$$
 (46)

where the unitary operator of a 50:50 beam splitter has a form [1]

$$\hat{B} = \exp\left(i\frac{\pi}{4} \left(\hat{a}_0^H \hat{a}_1 + \hat{a}_0 \hat{a}_1^H\right)\right).$$
(47)



Fig. 4. Quantum "scalar" (a) and "vector" (b) description of a beam splitter.

If a beam splitter is acted on the state $|1\rangle_0|1\rangle_1$ (in both inputs is exactly one photon) one obtains

$$|1\rangle_{0}|1\rangle_{1} \xrightarrow{B} \frac{i}{\sqrt{2}} \left(|2\rangle_{2}|0\rangle_{3} + |2\rangle_{3}|0\rangle_{2}\right).$$

$$(48)$$

The possibilities $|1\rangle_0|0\rangle_1$ and $|0\rangle_0|1\rangle_1$ do not occur due to the fact that the processes shown in Fig. 5 are indistinguishable and interfere destructively.



Fig. 5. Two indistinguishable processes

For example for input coherent states and a single dielectric layer beam splitter one obtains

$$|\alpha\rangle_{0}|\beta\rangle_{1} \xrightarrow{B_{1}} \left|\frac{\alpha + i\beta}{\sqrt{2}}\right\rangle_{2} \left|\frac{i\alpha + \beta}{\sqrt{2}}\right\rangle_{3}$$

$$\tag{49}$$

and for a beam splitter cube, made of a right angle prisms, one obtains

$$|\alpha\rangle_{0}|\beta\rangle_{1} \xrightarrow{B_{1}} \left|\frac{\alpha+\beta}{\sqrt{2}}\right\rangle_{2} \left|\frac{-\alpha+\beta}{\sqrt{2}}\right\rangle_{3}.$$
 (50)

5.4 Quantum Polarization Description of NBS

From (41) and (42) one can see that for the ideal beam splitter there is the change of the polarization for the reflected beam. Note relation (42) differs from (30) only in the global factor which is unimportant for us now (it has no observable effects).

Thus if we assume that the photon polarization is swap under a reflection from left-handed to right-handed and vice versa (see Fig. 6) then the processes in Fig. 5 are truly indistinguishable only if the inputs of the beam splitter are both x or y-linearly polarized.



Fig. 6. Two distinguishable processes.

Thus for the complete description one needs to introduce an extra degree of freedom to treat with the polarization. For this purpose we use the following definition (applicable only for a lossless beam splitter) for the reflection and transmission coefficients [2]

$$\cos\theta = |r|$$
 and $\sin\theta = |t|$ (51)

where we set $|r_0|=|r_1|=|r|$ and $|t_0|=|t_1|=|t|$ (assuming a symmetric beam splitter). Note that the angle θ has no geometrical interpretation. For instance $\theta=\pi/4$ represents a 50:50 beam splitter.

In Fig. 4(b) the new set of operators for the input and output modes is shown. In this section we use right-handed and left-handed basis. In this basis the transmitted photon is left untouched and the reflected photon is swap from $R \rightarrow L$ and vice versa.

Then the action of a beam splitter cube can be expressed as

$$Ba_{0R}^{H}B^{H} = a_{2L}^{H}\cos\theta + a_{3R}^{H}\sin\theta$$

$$Ba_{0L}^{H}B^{H} = a_{2R}^{H}\cos\theta + a_{3L}^{H}\sin\theta$$

$$Ba_{1R}^{H}B^{H} = -a_{2R}^{H}\sin\theta + a_{3L}^{H}\cos\theta$$

$$Ba_{1L}^{H}B^{H} = -a_{2L}^{H}\sin\theta + a_{3R}^{H}\cos\theta.$$
(52)

As an example we suppose the following input states of a beam splitter

$$\begin{aligned} &|1_{R},0_{L}\rangle_{0}|1_{R},0_{L}\rangle_{1} \xrightarrow{B} B|1_{R},0_{L}\rangle_{0}|1_{R},0_{L}\rangle_{1} = \\ &= B\left(a_{0R}^{H}\right)B^{H}B\left(a_{1R}^{H}\right)B^{H}B|0_{R},0_{L}\rangle_{0}|0_{R},0_{L}\rangle_{1} = \\ &= -\cos\theta\sin\theta|1_{R},1_{L}\rangle_{2}|0_{R},0_{L}\rangle_{3} + \\ &+ \cos^{2}\theta|0_{R},1_{L}\rangle_{2}|0_{R},1_{L}\rangle_{3} - \\ &- \sin^{2}\theta|1_{R},0_{L}\rangle_{2}|1_{R},0_{L}\rangle_{3} + \\ &+ \cos\theta\sin\theta|0_{R},0_{L}\rangle_{2}|1_{R},1_{L}\rangle_{3} \end{aligned}$$
(53)

where we used the unitary property $B^H B=1$ and $B|0_R,0_L\rangle_0|0_R,0_L\rangle_1=|0_R,0_L\rangle_2|0_R,0_L\rangle_3$. From (54) it is clear that if we do not treat with the polarization and assume a 50:50 beam splitter ($\theta=\pi/4$) the two middle terms in (53) cancel and we obtain a relation similar to (48) (the different phases are due to the fact that in (48) we used a single dielectric layer as a beam splitter and in (43) a beam splitter cube).

6. Summary

The main goal of the article was to give quantum description of an arbitrary placed retarder and a vector description (description witch treats with the polarization) of a beam splitter.

The derivation was based on the analogy with Jones matrix calculus where electric fields vectors were replaced by annihilation operators.

In quantum case we supposed lossless components (no interaction with the environment) to represent them by unitary operators. This is the fundamental difference in comparison with Jones matrix concept where one can represent the loss optical device by Jones matrix. This difference is due to the fact that the Jones concept arises from Maxwell's equations hence analogy between Jones calculus and quantum description can't be taken too seriously. The description of loss systems in quantum domain needs to introduce an environment witch interacts with the principal system [2].

Jones calculus can't be used when the experiments are treated in the quantum domain. Hence in the single photon interferometry or interferometry which uses typical quantum states (for example number states). The quantum description of retarders, phase shifters and beam splitters is needed for a single photon transmission and quantum computation where mentioned devices are used for manipulation with single photons.

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Petr KUČERA received his master degree in electrical engineering and communication from the University of Technology, Brno, in 2004. From 2004 to 2007 he was a PhD student at the Dept. of Radio Engineering, University of Technology, Brno. In 2007 he joined the Pforzheim University as a project co-worker. His research interests include the light polarization measurements and quantum communication.

Correction

Switched-Capacitor Filter Optimization with Respect to Switch On-State Resistance and Features of Real Operational Amplifiers

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In the previous issue of the Radioengineering journal (June 2007, vol. 16, no. 2, p. 34–39), one equation was deleted by mistake from the manuscript of the paper during its editing in the publishing department. This equation shall follow the equation (15), and expresses the value of F_{Ui} :

$$F_{\mathrm{U}i}(C_1,...,C_7) = \begin{cases} \frac{M_{2\mathrm{N}}(f_i,C_1,...,C_7) - B_{\mathrm{U}}(f_i)}{B_{\mathrm{U}}(f_i)} \\ & \text{if } M_{2\mathrm{N}}(f_i,C_1,...,C_7) > B_{\mathrm{U}}(f_i), \\ 0 & \text{else.} \end{cases}$$
(16)

I apologize both to the authors and to readers.

Hymel daida