Simplified Frame and Symbol Synchronization for 4–CPFSK with *h*=0.25

Marek BOBULA [1], *Karel DANĚK* [2], *Aleš PROKEŠ* [1]

[1] Dept. of Radio Electronics, Brno University of Technology, Purkyňova 118, 612 00 Brno, Czech Republic
[2] Racom Ltd, Mírova 1283, 592 31 Nové Město na Moravě, Czech Republic

xbobul00@feec.vutbr.cz, karel.danek@racom.eu, prokes@feec.vutbr.cz

Abstract. *This paper examines the problem of rapid frame and symbol synchronization techniques intended particularly for constant envelope modulation formats M–CPFSK with modulation index h=1/M which are used in strictly bandwidth limited narrowband industrial applications. The data aided and non data aided versions of the algorithm based on digital frequency discrimination are discussed and compared against the synchronization techniques found in literature. Sample wise pattern correlation technique for joint frame and symbol synchronization is also studied. With the focus on a practical digital implementation the advantages and disadvantages of the described approaches are discussed.*

Keywords

M–CPFSK, symbol timing recovery, frame based communication, synchronization.

1. Introduction

Besides an overwhelming public interest in evolution of commercial wireless communication, systems for telemetry, SCADA, maritime radio and other industrial applications also evolve. As such, communication systems specified mostly by *EN 300 113* [1] use – in most cases – constant envelope modulation M-CPFSK at least due to the strict adjacent channel power requirements and the need for good power and spectral efficiency. Since a stochastic multiple access and frame based communication are used in such applications, rapid frame and symbol synchronization is essential to use the radio channel as effectively as possible. With today's digital signal processing enrichments these tasks can be enhanced by developing more and more sophisticated algorithms, but care must be taken in trading off the performance gained by such algorithms and their complexity.

Some synchronization algorithms can be found in literature [2]–[9] but only few of them, e.g. [7], [8] focus on rapid synchronization techniques based on a specific preamble as is often the case in practical systems. In reference [2], the synchronization method is proposed for orthogonal M-CPFSK signaling and as such cannot be adopted to systems with h=1/M and non–orthogonal signaling. Synchronization techniques described in [3]–[5] are non-data aided and to suppress significant pattern jitter introduced in a recovered timing estimate, long integration periods starting from 128 symbols are needed. Significant processing power is also required for their practical implementation. The synchronization method described in reference [6], [7] based on specific preamble pattern and symbol correlation is declared as a well suitable for burst mode communication systems with only 16 symbols required to synchronize from the receiving frame. Although this method can provide system with information about the symbol timing estimation, the exact moment when the value of timing estimate is valid becomes unknown to the receiver in a case when it tries to synchronize from received noise. Since this is always the case, the problem of rapid symbol timing recovery is very closely related to frame synchronization and the preamble structure should provide the system with all the information needed to synchronize for incoming frame. In this case the sample wise pattern correlation technique described in this article seems to be providing good results.

In this paper we derivate a method for symbol timing recovery based on digital frequency discrimination as an alternative technique to the one introduced in [6], [7]. We compare the performance of its *Data Aided – DA* and *Non Data Aided – NDA* modes of operation. It is shown, that the method, however straightforward it is, gives comparable results but the computational burden is reduced. We consider this method to be suitable for communication systems where long frames are being used as a secondary loop to adjust timing along the receiving frames. For systems where the computational power is limited or the frame and symbol timing synchronization is to be combined in rapid synchronization algorithm, we suggest sample wise pattern correlation technique as an option.

2. M–CPFSK Signal Model

Following the signal notations from [4] let start with an information – modulation – signal to be transmitted

$$m(t,\alpha_k) = \sum_{k=-\infty}^{\infty} \alpha_k . g(t-kT) \tag{1}$$

where $\{\alpha_k\}$ are the data symbols taking on the values from the symbol alphabet $\{\pm 1, \pm 3, ..., \pm M\}$, T is the symbol period and $g(t)$ is a frequency pulse or an impulse response of the pulse shaping filter, usually the root raised cosine one [10] to reduce ISI. We see, that the modulation signal $m(t,\alpha_k)$ is a superposition of the modulation impulses weighted by the information symbols. If we were dealing with a linear modulation we would be done at this stage. For nonlinear modulation as M–CPFSK we need to continue. Due to the requirement for continuous phase change the relation between the frequency pulse $g(t)$ and phase pulse $q(t)$ in all CPM modulations is defined as

$$q(t) = \int_{-\infty}^{t} g(\tau) d\tau \,, \tag{2}$$

thus the resulting continuous phase $\psi(t,\alpha)$ of the modulated signal changes in time according to

$$\psi(t,\alpha) = 2\pi h \sum_{k=-\infty}^{\infty} \alpha_k q(t-kT) \tag{3}$$

where h is the modulation index defining the relation between the *maximum* frequency deviation Δf and a symbol rate R according to

$$h = \frac{2\Delta f}{R(M-1)} \,. \tag{4}$$

It also tells us how much the phase changes during the symbol period respecting the symbol value of α_k .

The resulting complex envelope $s(t)$ of the M–CPFSK modulated signal can be then written as

$$s(t) = \exp\left\{ j\, 2\pi\, h \left(\sum_{k=-\infty}^{\infty} \alpha_k q(t-kT) \right) \right\} \tag{5}$$

which can be further modified into more FM like form

$$s(t) = \exp\left\{ j\, 2\pi\, k_{FM} \int_{-\infty}^{t} \left(\sum_{k=-\infty}^{\infty} \alpha_k g(\tau-kT) \right) dt \right\} \tag{6}$$

where k_{FM} is a frequency modulator sensitivity per modulation state defined as

$$k_{FM} = \frac{\Delta f}{(M-1)} \,. \tag{7}$$

If only an AWGN noise is assumed to be corrupting the radio channel, the complex envelope of the received signal can be described as

$$r(t) = \exp\{ j 2\pi \nu t + \theta \} s(t-\tau) + w(t) \tag{8}$$

where ν, θ and τ are unknown values of the frequency offset, carrier phase and symbol timing epoch respectively. The $w(t)$ is the AWGN noise modeled as a complex random signal with independent real and imaginary parts having a two-sided power spectral density $N_0/2$.

From all the operations performed by the receiver, filtering – either anti-aliasing or main selectivity one – and sampling of the received signal have the biggest impact on a system performance. So first the complex envelope of the received signal after filtering is

$$x(t) = \exp\{ j 2\pi \nu t + \theta \} s_r(t-\tau) + n(t) \,. \tag{9}$$

Here $s_r(t)$ is the transmitted signal shaped by all of the receiver filters and $n(t)$ is a band limited noise with a noise bandwidth B_N. Thus the SNR of the studied signal with energy per symbol E_s can be expressed as

$$SNR = \frac{E_S}{N_0} \left(\frac{R}{B_N} \right) . \tag{10}$$

After the equidistant sampling process with the sampling rate of $N.R$, the samples $x_k(i)$ of the $x(t)$ taken at $t=(kT+iT_s)$ are finally the one seen by the synchronizer

$$\begin{aligned} x_k(i) = \exp\{ j\psi(kT + iT_s - \tau; \alpha_k) \} \\ .\exp\{ j2\pi\nu(kT + iT_s) + \theta \} + n_k(i) \\ -\infty \le k \le \infty;\; 0 \le i \le (N-1) \end{aligned} \tag{11}$$

where k is a symbol index and i is the sample index.

3. Symbol Timing Recovery Scheme

According to [6], [7], the timing estimation can be calculated by using 4–th order m–lag autocorrelation over the L_0 symbols α_k of the specific preamble α_k= {...0 0 1 1 0 0 1 1 ....}

$$\tilde{\xi}(i) = \frac{1}{L_0} \sum_{k=0}^{L_0-1} [x_k(i).x_k^*(i)]^4 \tag{12}$$

where $x^*(i)$ denotes a complex conjugate. The $\xi(i)$ contains the information about the timing epoch which can be extracted by calculating the Fourier transform of its absolute value

$$\tilde{\tau} = \frac{T}{2} - \frac{T}{2\pi} \arg\left\{ \sum_{i=0}^{N-1} \left| \tilde{\xi}(i) \right| e^{\frac{-j2\pi}{N} i} \right\} . \tag{13}$$

For L_0=16 and m=1 the algorithm is considered to recover symbol timing information which is supposed to be constant over the entire incoming frame. The practical implementation obstacles of this equation can be seen in significant computational burden required to calculate the forth power of the correlation function as well as the non-symmetrical preamble structure which does not fulfill the symbol alphabet.

In this section we show that the comparable algorithm can be derived using frequency discrimination method and with reduced complexity it provides sufficiently small MSE *(Mean Square Error)* [4], [11].

Following the vector notations shown in Fig. 1 and equation (11) for the time instant with a simplified notation $t=(kT+iT_s)$=>$[i]$ we can calculate the instantaneous phase of the received signal $x[i]=x_I[i]+jx_Q[i]$ as

$$arctg\left\{\frac{x_Q[i]}{x_I[i]}\right\}=\psi\big([i-\tau];\alpha_k\big)+2\pi v[i]+\theta\cdot \tag{14}$$

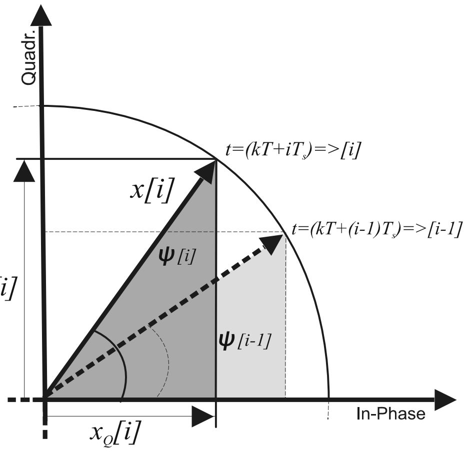

Fig.1. Vector representation of the received CPFSK signal.

In equation (14) we only calculate the argument of the function and ignore for the moment the noise contribution altogether. The instantaneous phase should contain all the information about the parameters v, θ and τ. Although we would be able to calculate the timing information right from (14), let proceed with frequency discrimination to get rid of the frequency and phase offsets and to extract the timing information by using an equation which is more suitable for hardware realization. We differentiate the instantaneous phase by discrete time approximation of the derivation – inverse operation to integration shown in (2) and (6)

$$\frac{df(t)}{dt}\approx\frac{f[i]-f[i-1]}{T_s\big([i]-[i-1]\big)}+\varepsilon[i] \tag{15}$$

where $\varepsilon[i]$ can be seen as an amplitude imperfection caused by the approximation. As we show later in the article if N is chosen reasonably high, e.g. 8, the imperfection has a negligible impact on a system performance.

By using the term for derivation of the arctg[$f(t)$] function we can rewrite (14) into equation (16) which can be used for the instantaneous frequency discrimination:

$$m'[i-\tau]=\frac{x_I[i-1]x_Q[i]-x_I[i]x_Q[i-1]}{2\pi k_{FM}T_s\left[(x_I[i])^2+(x_Q[i])^2\right]}\cdot \tag{16}$$

Respecting the constant magnitude character of the M–CPFSK modulated signal we can see that the whole denominator of (16) is a constant. Although it is definitely true for the transmitted signal – if no additional channel filtering is performed by the transmitter – the received signal has a significant magnitude variance caused mostly by the main channel selectivity filter. This amplitude variance cannot be omitted if equation (16) is used for frequency demodulation in a digital M–CPFSK baseband receiver. However, for the symbol timing estimation the information about the amplitude of the demodulated signal is not crucial and equation (16) can be considerably reduced to

$$m''[i-\tau]=x_I[i-1]x_Q[i]-x_I[i]x_Q[i-1] \tag{17}$$

where $m''[i-\tau]$ is a demodulated replica of the modulation signal defined by (1). By discrete differentiation the frequency offset v transforms into a small constant with negligible impact on timing estimate and the initial phase θ vanishes altogether.

From now on, extracting of the timing estimate is similar to the procedure described in [6], [7]. We need to calculate the expectation function of the signal which carries the timing information. In our case however, it is the expectation function of an absolute value of the demodulated signal (18) over the specified signal interval of L_0 symbols

$$\tilde{\xi}(i)=\frac{1}{L_0}\sum_{k=0}^{L_0-1}\left|x_{I,k}(i-1)x_{Q,k}(i)-x_{I,k}(i)x_{Q,k}(i-1)\right| \tag{18}$$

$$0\le k\le L_0-1 \quad 0\le i\le N-1$$

and finally we extract the timing estimate by calculating the Fourier transform to extract the timing information

$$\tilde{\tau}=\frac{T}{2\pi}\arg\left\{\sum_{i=0}^{N-1}\left|\tilde{\xi}(i)\right|e^{\frac{-j2\pi}{N}i}\right\}. \tag{19}$$

By comparing equation (18) to (12) we can readily see that the forth power has reduced to a single complex multiplication which can be effectively implemented especially in modern FPGA circuits.

For NDA operation the length of the observation interval L_0 needs to be chosen large enough to suppress the significant amount of pattern jitter. However, if the rapid synchronization is necessity as is always the case for burst mode communication, the preamble based estimation – DA operation – can be used and thus significantly shorten the observation interval L_0. We will show that when simple harmonic preamble pattern in form of α_k={-(M-1),(M-1)... -(M-1),(M-1)} is chosen the length of the observation interval can be reduced to L_0=16 or even 8 symbols and still reach acceptable *MSE* of the timing estimate for practical purposes.

3.1 Simulation Results

In this section, the performance of the proposed synchronization scheme is analyzed. Since the algorithm is based on the discrete time approximation of the derivation operation (15), let begin the analysis by studying the approximation imperfection versus the number of signal samples used per one modulation symbol. As it is shown in Fig. 2 for N≥8 the maximum approximation error max($\varepsilon[i]$) defined by

$$\max\left(\varepsilon[i]\right) = \frac{\max\left|m[i] - m'[i]\right|}{\max\left|m[i]\right|} \tag{20}$$

drops to one percent of the signal amplitude and is considered acceptable for the real system. It is worth stressing at this point that by increasing the number of samples per symbol the performance of most digital algorithms is enhanced, but the cost is an implementation complexity and an increase in a computational burden. Therefore, in the following simulation N=8 samples per symbol have been selected. We use 4–CPFSK with h=0.25 as a modulation technique. The studied signal and white noise at the receiver have been band limited by a 27th order digital main channel selectivity FIR filter with a low pass characteristic having 3 dB attenuation at 0.75R frequency offset. As a pre modulation and post modulation filter the root raised cosine with roll–off 0.5 has been used.

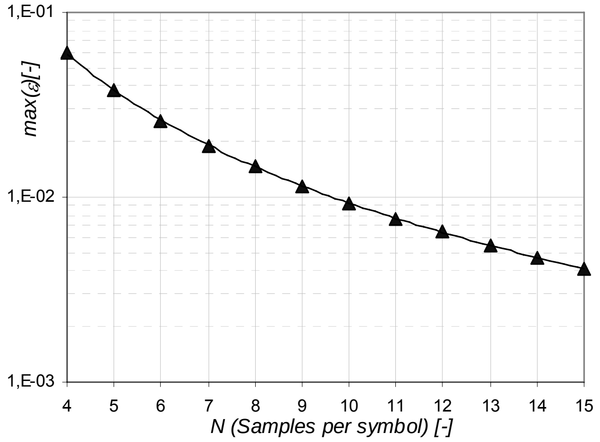

Fig.2. Maximum differentiation imperfection max($\varepsilon[i]$) versus the number of samples per symbol.

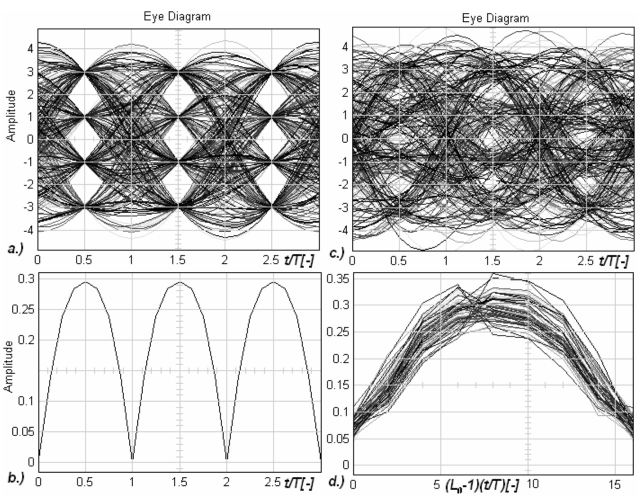

Fig.3. **(a)** Eye diagram of the demodulated 4–CPFSK signal $m'[i-\tau]$ (Tx and Rx Root raised cosine filters employed with RollOff=0.5 **(b)** Demodulated preamble signal $m''[i-\tau]$ containing exact timing information. **(c)** Eye diagram of the demodulated 4–CPFSK signal with AWGN: E_s/N_0=15 dB (-3dB Low pass Noise Bandwidth B_N=0.75R). **(d)** Instances of the expectation function of $\xi(i)$ calculated according to (18).

In Fig. 3 the result of the equation (16) is shown as an eye diagram of the frequency demodulated 4–CPFSK signal. We see that the distortion caused by the selectivity filter and the approximation imperfection is insignificant. Fig. 3b shows the relation between the absolute value of the demodulated preamble signal $m''[i-\tau]$ and the corresponding optimum sampling instant – maximum eye opening. In a presence of AWGN noise (Fig. 3c) the additional filtering or averaging of the extracted preamble is needed in form of calculating (18). The result is an averaged instance of $\xi(i)$ containing timing information. Several instances of $\xi(i)$ are shown in Fig. 3d. Since the maximum of $\xi(i)$ varies in each trace we can only calculate an estimate of the timing information if the AWGN noise is present.

Fig. 4 illustrates the performance of the DA operation of the suggested algorithm by showing the normalized timing mean square error MSE_τ

$$MSE_\tau = E\left\{\left[\frac{(\tilde{\tau} - \tau)}{T}\right]^2\right\} \tag{21}$$

as a function of E_b/N_0 in above mentioned channel bandwidth. From Fig. 4 we can see that the suggested preamble structure enables us to decrease its length to 16 or even 8 and still reach sufficiently small timing MSE. By taking the longer preamble structure, the number of traces to average is increased and naturally the noise contribution is reduced.

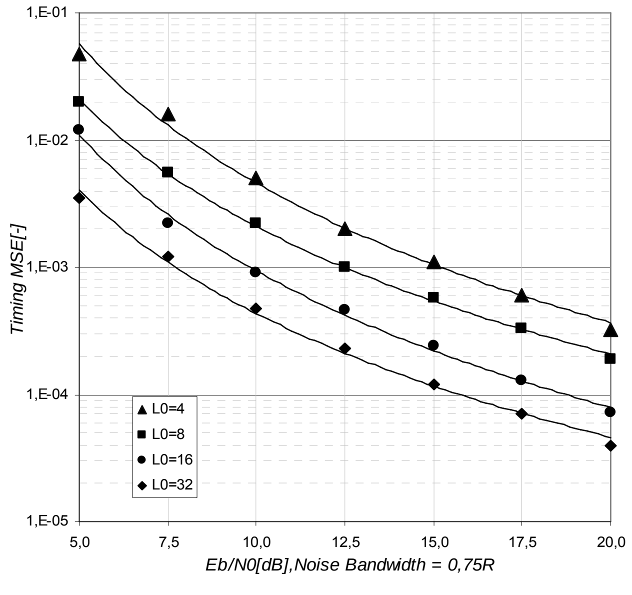

Fig. 4. Timing estimation MSE for different length of the preamble L_0=4,8,16,32.

For NDA operation of the timing synchronizer much longer – Fig.5 – observation interval, at least 128 symbols, is needed, what is in accordance with the results available in the literature [3]–[7]. In our case however, the calculation of both types has a simplified form of (18) and (19) and is considered better suitable for digital implementation, especially in situations where the digital frequency discrimination is already employed. In this situation the timing estimation only needs to implement the averaging filter and the Fourier transform function which is needed for almost every available synchronizer. We suggest to use the DA version for rapid synchronization and to generate the first timing estimate for the system and the NDA version to slowly trace the timing drift over the entire frame being received.

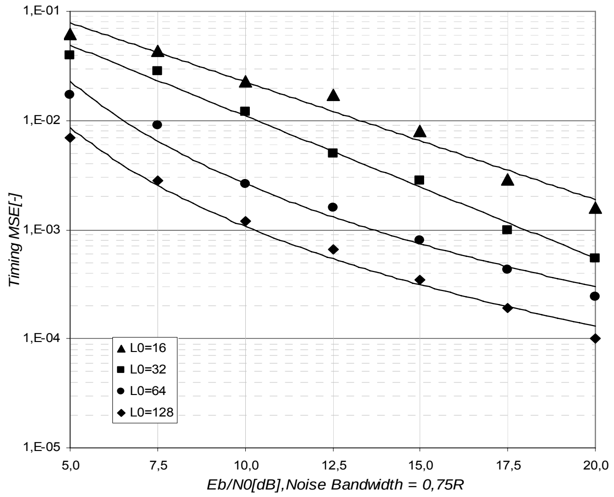

Fig. 5. Timing estimation MSE for different length of the NDA operation L_0=16,32,64,128.

3.2 Drawbacks of the Algorithm

Despite the encouraging results presented in the previous section some drawbacks of such approach can be seen mostly in:

- Need for Fourier transform calculation and fractional Farrow interpolation [13], [14] to implement the complete synchronization system.
- Relatively high number – e.g. 8 to 10 – of samples per symbol is needed to approximate the derivation operation, Fig. 2.
- Significant variance of the timing estimate $\xi(i)$ in situations when E_b/N_0 is low and interval L_0 is not long enough to average the noise contribution. This situation is depicted in Fig. 6.

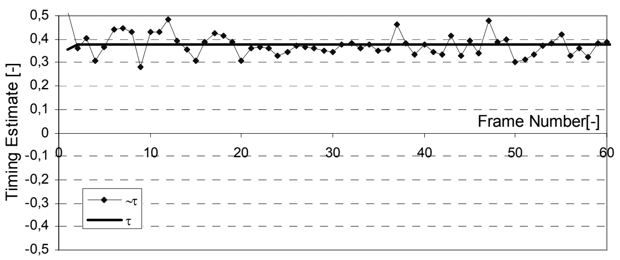

Fig. 6. Timing estimation variance over the received frames. E_b/N_0=10 dB, preamble length L_0=16, 4–CPFSK, Tx root raised cosine filter employed with RollOff=0.5.

- For the frame based communication the exact moment when the timing estimate has the lowest MSE with respect to its optimum value remains unknown to the system synchronizing from noise, and needs to be further calculated, Fig.7.

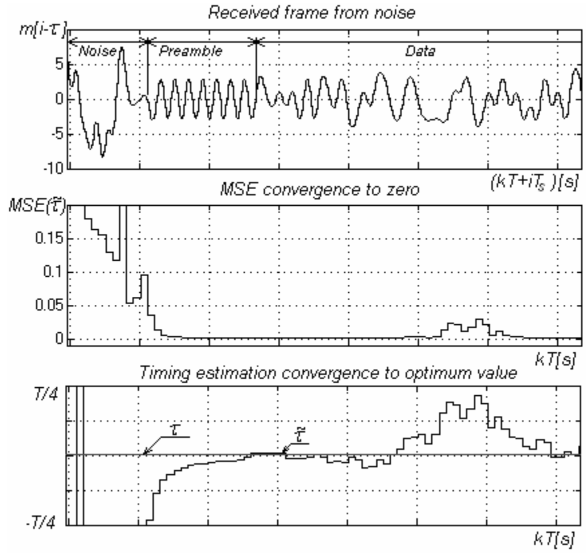

Fig. 7. Example of the timing estimate calculation over the receiving frame from band limited noise. E_b/N_0=25 dB, L_0=16, 4–CPFSK, Tx and Rx root raised cosine filter employed with RollOff=0.5.

4. Sample wise Pattern Correlation

The last problem mentioned in the previous section is critical especially in frame based communication systems where the only source of the timing information is the receiving frame itself. It is worth stressing that such assumption needs to be made with almost all similar algorithms described in literature [3]–[7], regardless the fact, how accurately they estimate the timing information. Some pattern recognition technique would ease the problem but at this point we have not yet recovered the symbol timing and thus the situation reminds a “chicken or the egg” problem. We believe that in such systems the symbol timing and frame synchronization should be combined.

Let suppose that the frequency discrimination according to (16) or (17) is already employed in the system as well as the post modulation filtering by root raised cosine filter. The result would look similarly to the one shown on the top in Fig. 7. Since we cannot distinguish symbols at this point, only the information about the signal polarity is used to calculate the correlation function with a priori bipolar preamble pattern α_k:

$$y[i]=\sum_{k=0}^{L_0-1} bip\left(m'[i-kN]\right).bip\left(\alpha_k\right) \quad -\infty \le i \le \infty \quad (22)$$

where i is the sample index, N is the number of samples per symbol, α_k is the preamble of length L_0 and bip denotes that

$$bip(x)=\begin{cases} 1 & x\ge 0 \\ -1 & otherwise \end{cases}. \quad (23)$$

Correlation function $y[i]$ achieves its maximum each time it finds out the corresponding sequence α_k. Ideally, for the rectangular modulation pulse the maximum would spread for N samples. In real systems however, it will be less than N due to the ISI *(inter symbol interference)* introduced by the frequency characteristic of the communication channel including all filters. This situation is depicted in Fig. 8, where the correlation extreme spans for 4 samples even though N has been set to 8.

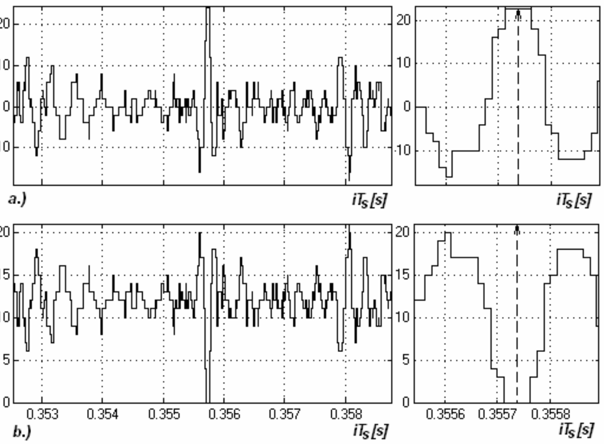

Fig. 8. **(a)** The result of the sample wise pattern correlation calculation (22), L_0=24, E_b/N_0>25 dB. Detailed scan of the correlation maximum. Its center is taken for symbol synchronization. **(b)** The result of equation (24) with the detailed scan of the correlation minimum.

By analyzing the correlation peak of $y[i]$ we realized that it is in direct relation to the maximum eye opening shown in Fig. 3a. Thus it can be used not only for frame synchronization but also for symbol timing estimation. As it is shown in Fig. 8, we take the center of the correlation peak as the initial time instant for symbol timing recovery. Before we analyze how accurate such straightforward approach is, let simplify it even further. In (22) we change the bipolar signaling into unipolar and the multiplication into *XOR* operation:

$$y[i] = \sum_{k=0}^{L_0-1} sign(m'[i-kN])\, xor\, sign(\alpha_k) \quad -\infty \le i \le \infty \qquad (24)$$

where $sign(x) = \begin{Bmatrix} 1 & x \ge 0 \\ 0 & otherwise \end{Bmatrix}$. (25)

Now the required information about the frame and symbol synchronization is carried in a minimum of the correlation function $y[i]$. The main advantages of this modification can be seen in a fact that the algorithm is no more dependent on the preamble length L_0 and the computational requirements are substantially reduced.

4.1 Simulation Results

Since the sample wise pattern correlation technique works on a discrete sample basis there is not much sense in calculating the MSE of the symbol timing estimation as a continuous function of E_b/N_0 as in the previous section. However, it is worth a note at this point that the MSE of such symbol timing estimation cannot be considered better that the lowest limit defined by

$$\min\{MSE(\tilde{\tau})\} \ge \left(\frac{1}{2N}\right)^2. \qquad (26)$$

We set up a simulation instead to evaluate *SER (symbol error ratio)* as a function of E_b/N_0 calculated over a high number of received frames, each of which has been self synchronized from noise by using only the sample wise pattern correlation approach. An example of the preamble structure has been set according to [12] to have a pseudo random form of α_k={ -1 1 -1 1 -1 3 -3 3 -3 -1 1 -3 3 3 -1 1 -3 -3 1 3 -1 -3 1 3} with length of L_0=24. Please note that other preamble patterns – having good autocorrelation properties such as PRBS sequences – can also be selected. The length of the preamble has been optimized regarding its false synchronization probability and noise immunity. All other parameters of the simulations were set correspondingly to simulations described in the previous sections. It can be readily seen from Fig. 9 that for the selected preamble structure and 4–CPFSK modulation with h=0.25 the degradation in power efficiency – in a whole studied SER interval – is not exceeding 2 dB with respect to ideally synchronized hard decoder receiver. It is an interesting remark that the performance of the simulated synchronizer does not tend to change with different number of samples per symbol. Having the simplicity of the algorithm in mind we can judge such performance acceptable for several practical applications. If more precise timing estimate is required in the system, the synchronization algorithm described in Sec. 3 based on frequency discrimination can be employed. Even though it has a simplified form of (18) – comparing for instance to (12) – its precision is achieved with expanse in overall implementation complexity.

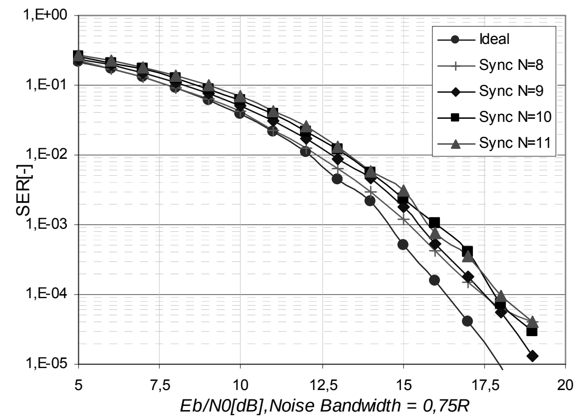

Fig. 9. Simulation results of the 4-CPFSK power efficiency with sample wise pattern correlation algorithm implemented.

5. Conclusion

Symbol synchronization method based on digital frequency discrimination described in this article is found to provide satisfactory results with reduced computational complexity. Its DA operation recovers the synchronization information over 8 to 16 symbols of the harmonic preamble structure. The NDA version requires at least 128 randomly distributed symbols of the receiving frame to calculate the timing estimate accurately. Since both versions have the same structure, we suggest to use the DA version for rapid synchronization and to generate the first timing estimate for the receiver and to use its NDA version to slowly trace the timing variance over the entire receiving packet. Such approach can be adopted even for the system where long frames e.g. more than 1500 bytes are used. Despite its simplified form the calculation of the Fourier transform and fractional Farrow interpolation is needed to implement the complete synchronization algorithm.

For systems where the computational power is limited and the frame and symbol timing synchronization should be combined in a simple synchronization algorithm, the sample wise pattern correlation technique described in this article is shown to provide satisfactory results. We see the advantages of the described synchronization technique mostly in its:

- Simplicity, the algorithm does not require any additional multiplications, Fourier transform calculation, CORDIC or fractional Farrow interpolator to be implemented in the system.
- Accuracy, the power efficiency of the communication system employing sample wise pattern correlation should not degrade for more than 2 dB – with respect to ideally synchronized receiver – caused by imperfect symbol timing estimation.

Due to its simplified form, the sample wise pattern correlation approach was tested in a software defined digital radio modem prototype and found robust in real communication environment. In this case however, the rigorous evaluation of the synchronization algorithm performance is a difficult task mainly because of the intricacy of a number of different negative influences such as nonlinearity of the radio channel, precise E_b/N_0 calculation, accuracy of the frequency discrimination, etc.

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About Authors...

Marek BOBULA (*1983 in Svit, Slovakia) received MSc degree from the Brno University of Technology (BUT) in 2006. Since 2006 he has been engaged in research and development in the field of digital radio signal processing in Racom Ltd. with a main emphasis on FPGA technology. He is currently a postgraduate student at the Dept. of Radio Electronics, Brno University of Technology.

Karel DANĚK (*1961 in Nové Město na Moravě, Czech Rep.) received his MSc and PhD degrees from the BUT, Dept. of Telecommunication in 1984 and 2001, respectively. He is author of a number of articles and a book *"Moderní rádiový přijímač, Kniha o jeho návrhu",* published in Prague, 2005. Since 1991 he has been a chief hardware designer in Racom Company. His present professional interests are in the land mobile radio communications, RF and software digital radio circuit design.

Aleš PROKEŠ (*1963 in Znojmo, Czech Rep.) received the MSc and PhD degrees in electrical engineering from the BUT, in 1988 and 2000, respectively. He is currently an Assistant Professor at the Dept. of Radio Electronics. His research interests include nonuniform sampling, velocity measurement based on spatial filtering and free-space optical communication with emphasis on optical receiver performance analysis and optimization.