Abstract. This paper deals with mathematical simulations of EMI filters’ performance. These filters are commonly used for the suppressing of electromagnetic interference which penetrates through the power supply networks. The performance of these filters depends on terminating impedances which are plugged to the inputs and outputs clamps of the EMI filters. This paper describes the method by which it is possible to calculate the insertion loss of the filters. The method is based on the modified nodal voltage method. The circuitry of the EMI filters is used for their description. The effect of spurious components is not taken into account. The calculated results were compared with measured data of several filters for several impedance combinations. Different test setups, like asymmetrical, symmetrical, etc. were taken into account. The simplicity and accuracy of the presented method is discussed in the conclusion. The achieved accuracy is on high level. The described method is universal, but for filters with more than one current compensated inductor, the mentioned method is complicated. The size of the final equation for calculating the insertion loss rapidly increases with the number of current compensated inductors.

Keywords
Electromagnetic compatibility EMC, EMI mains filter, insertion loss, impedance termination, filter model, modified nodal voltage method, current compensated inductors.

1. Introduction
The EMI filters are one of most often used tools for suppression of electromagnetic interference (EMI) which penetrates through the power network. The foremost task of the EMI filters is usually to increase the immunity of an electronic equipment operated on power line inputs and simultaneously to decrease the level of high frequency emissions supplied by the equipment into outer power network. The most important characteristics of the EMI filters are values of insertion loss or RF attenuation. The insertion loss of the EMI filters depends on the current frequency of the interference signal. The performance of the EMI filter depends on the current impedance terminating the filter terminals, current load, etc. In general, there are lot of parameters, conditions and influences which degrade the insertion loss of the EMI filters. Covering all these possibilities in measuring setup and in the data sheets in catalogues is nearly impossible.

The EMI filters usually have more then two clamps at the input and output side, e.g. the single-phase EMI filter has minimally these three clamps (live, neutral and earth) at the input and also at the output side. This situation complicates the correct presentation of the EMI filter performance data. Thus, it is possible to determine several configurations by which interfering signals should penetrate through the power network. This situation is depicted in Fig. 1. Three different measurements could be carried out on a single-phase EMI filter. The most common presented data have been measured in asymmetrical mode (a) in Fig. 1). Several producers give the data for the system b) (Fig. 1), which is called symmetrical. It is also possible to determine the system, which is depicted in Fig. 1 c). For the purpose of this paper, this system will be denoted as a "non-symmetrical system". The measurements itself were done according to the ČSN CISPR 17 and as signal receivers the HP E7404A and HP 35665A spectral analysers were used. The R&S SML03 and Agilent 33220A were used for signal generating.

The producers of EMI filters usually give the insertion loss characteristics for the asymmetrical system and for the impedance system 50 Ω/50 Ω [1], [2]. The first number means the value of the terminating impedance on the input terminals of the EMI filter. The second number refers to the output terminating impedance. Several producers sometimes give the data also for the symmetrical system for several terminating impedances, e.g.: 50 Ω/50 Ω; 0.1 Ω/100 Ω and vice versa. The insertion loss measuring with the terminating impedances 0.1 Ω/100 Ω and vice versa is defined by the international standard CISPR 17 Methods of measurement of the suppression characteristics of passive radio interference filters and suppression components [3]. Schaffner company, which is the widely known producer of EMI fil-
systems are very specialized for specific tasks. The EMI filters performance analysis should be very universal and variable because the configuration of each filter is very variable. The circuitry knowledge of the certain EMI filter is other precondition, which should be fulfilled. For that reasons, the Matlab® was chosen for this analysis. Using of the Matlab® brings universality because the determination of the insertion loss relation uses only general Y parameters. These parameters could be effectively changed in relation with the circuitry of the concrete EMI filter. The basic single-phase EMI filter, which diagram of connections is depicted in Fig. 2, could be described by the following equations

\[ I_{L1} = Y_{11}U_{L1} + Y_{12}U_{N1} + Y_{13}U_{L2} + Y_{14}U_{N2}, \]
\[ I_{N1} = Y_{21}U_{L1} + Y_{22}U_{N1} + Y_{23}U_{L2} + Y_{24}U_{N2}, \]
\[ I_{L2} = Y_{31}U_{L1} + Y_{32}U_{N1} + Y_{33}U_{L2} + Y_{34}U_{N2}, \]
\[ I_{N2} = Y_{41}U_{L1} + Y_{42}U_{N1} + Y_{43}U_{L2} + Y_{44}U_{N2}, \]

where \( I_{L1} \) is the input current for clamps L or N, \( I_{L2} \) is one of the two output currents. In the same manner the input and output voltages are determined as it is shown in Fig. 2. The \( Y_{xy} \) is the admittance parameter of the tested EMI filter. The single admittance parameters in admittance matrix \( Y \) could be easily calculated by the modified nodal voltage method. By these admittance parameters it is possible to construct the admittance matrix \( Y \). Equations (1) to (4) could be rewrite in to the matrix form

\[ \mathbf{I} = \mathbf{Y} \cdot \mathbf{U} \quad (5) \]

where \( \mathbf{I} \) is the vector of the unknown currents, and \( \mathbf{U} \) is the vector of the variable voltages. The equations (1) to (4) exactly describe the properties of an arbitrary EMI filter, but for correct calculations, it is necessary to add more equations which will refer to configurations of the impedance network (Fig. 1) and to the location of the source of the interference signal. The insertion loss data are obtained after calculations of these several equations. The real frequency on which the insertion loss of the tested filter is calculated is also included in each element of matrix \( Y \). By this method it is possible to determine the insertion loss data, e.g. (\( L \) in dB).

2. Simple Models of EMI Filters

The comparison of several different filters from several companies could not be easy, as it is written in the introduction. The biggest problem is to obtain data for different filters and in the same time for several combinations of impedance terminations. One possible way could be through the analysis of filters’ models. This approach has several advantages. The method based on the models is not so time consuming as a lot of measurements with different impedance terminations. For making the models, plenty of commercial software could be used (PSpice, Micro-Cap, Ansoft Designer®, etc). These all software

![Fig. 1. Possible insertion loss measuring systems: a) asymmetrical; b) symmetrical; c) "non-symmetrical".](image)

![Fig. 2. Tested filter and distribution of currents and voltages.](image)
nodal voltage method. This method fits well for description of simple and linear electronic circuits. The above shown method has to be extended for correct determination of admittance matrix $Y$ of the EMI filters. The admittance matrix $Y$ has to be enlarged by two columns and two lines. The influence of the current compensated inductors is written into the added cells. By this step the equations (1) to (4) will be added up by the following two equations

$$U_{ab} = j\omega L_1 I_1 + j\omega MI_2,$$

$$U_{cd} = j\omega MI_1 + j\omega L_2 I_2$$

where the meaning of variables $U_{ab}$, $U_{cd}$, $I_a$ to $I_d$ is obvious from Fig. 3. The constants $L$ and $M$ represent own and mutual coefficients of induction of the current compensated inductor. The relationship between these two quantities is given by

$$M = k\sqrt{L_1L_2},$$

where $k$ is the coupling coefficient. The values of the own coefficients of induction $L_1$ and $L_2$ are commonly the same for most of EMI filters ($L = L_1L_2$).

The electromagnetic circuit given in Fig. 3 could be described by the following equation [6]

$$\begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} U_a \\ U_b \\ U_c \\ U_d \\ 0 \\ 0 \end{bmatrix}.$$  \hspace{1cm} (9)

The admittance parameters have to be added into the equation (9). The influence of current compensated inductors is not taken into account. The final obtained matrix of the filter could be written as following

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & 1 & 0 \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} & 0 & 1 \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} & -1 & 0 \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} & 0 & -1 \\ -1 & 0 & 1 & 0 & j\omega L & j\omega M \\ 0 & -1 & 0 & 1 & j\omega M & j\omega L \end{bmatrix}.$$  \hspace{1cm} (10)

This presented matrix is deduced for single-phase EMI filter which contains only one current compensated inductor. For the description of filters with more inductors it is necessary to create a bigger matrix. This fact rapidly reduces an efficiency and degrades universality of this analysis. More universal method could be made by using a firmly set of the matrix dimension. The smallest dimension of the matrix could be $4 \times 4$ because single-phase filters have usually 2 input and 2 output clamps. Thus, it is possible to produce a universal relation for insertion loss calculation, which depends only on the admittance parameters of the filters ($Y_{11}$ to $Y_{44}$). These parameters are defined for input and output nodes (clamps) of the EMI filter. The rest of the nodes has to be reduced onto the dimension $4 \times 4$. For this reduction it is possible to use the pivot condensation [7]. The principle of the reduction is possible to write down in this mathematical form

$$M_R = M_E - M_{EI} \cdot (M_I)^{-1} \cdot M_{IE}.$$  \hspace{1cm} (11)

Matrices $M_E$, $M_{EI}$, $M_I$ and $M_{IE}$ were created from the admittance matrix $Y$ of the EMI filter by the following way

$$\begin{bmatrix} M_E & M_{IE} \\ M_{EI} & M_I \end{bmatrix} \cdot \begin{bmatrix} X_I \\ X_E \end{bmatrix} = \begin{bmatrix} L_I \\ L_E \end{bmatrix}.$$  \hspace{1cm} (12)

where $X_I$ and $X_E$ represent the internal and external unknowns. The $L_I$ and $L_E$ represent external sources. The matrix $M_R$ is the final reduced matrix after pivot condensation. This matrix has the desired dimension of $4 \times 4$. Each matrix element is afterwards established into the relation for calculating the insertion loss data. This element depends on the frequency.

By the described method it is possible to calculate insertion loss of single-phase EMI filters. The method could be modified for multi-phase filters. This setup calculates only with the data which are written in the data sheet. From this condition it follows that calculations of insertion loss are not possible on higher frequencies, because in this setup, spurious properties of real electronic parts and devices are not covered. The value of coupling coefficient $k$ should be set by measuring or by optimization. The measured insertion loss data, e.g. in 50 $\Omega$, 50 $\Omega$, which should be given in data sheets could be used for this optimization. The equation for the calculation of the insertion loss can be determined for each system shown in Fig. 1.

The similar setup could be used for the estimating of the insertion loss with different impedance terminations [8].
3. Results

The method mentioned above was tested on seven filters: Schurter 5110.1033.1, Schaffner FN 321 1/05, FN 2020-16-06, FN 2070-10-06, Elfis 1ELF16V, 1ELF16VY-4, and Filtana TS 800 1006. The calculated insertion loss data were compared with measured ones. Several results and examples will be discussed in the following paragraphs.

Firstly, the insertion loss of the Schurter 5110.1033.1 EMI filter, whose inner circuitry is depicted in Fig. 4, was tested. The data sheet of the filter could be found in [9], for comparing the results. The insertion loss performances of this filter are shown in Fig. 5, 6 and 7. In these figures, the measured data are compared with the calculated insertion loss of this filter. The filter was tested in several different systems and also with different impedance terminations. The usual impedance terminations like 50 Ω/50 Ω, 0.1 Ω/100 Ω and 100/Ω/0.1 Ω were tested, and several impedance systems, which are not so common like 0.1 Ω/0.1 Ω and 100/Ω/100 Ω were thought too. From the presented data depicted in Fig. 5, 6 and 7, it is possible to say that the performance of the described method is really high.

![Fig. 4. Typical single-phase filter’s circuitry (Schurter 5110.1033.1).](image)

The measured and calculated performance data are shown in Fig. 9, 10, and 11. If we compare the measured and calculated data, the great accuracy in the domain of low frequencies can be seen.

![Fig. 5. The insertion loss of the Schurter 5110.1033.1 in asymmetrical systems.](image)

![Fig. 6. The insertion loss of the Schurter 5110.1033.1 in symmetrical systems.](image)

![Fig. 7. The insertion loss of the Schurter 5110.1033.1 in "non-symmetrical" systems.](image)

For the second example, the Elfis 1ELF16V filter was chosen. The data sheet with the technical data could be found in [10]. The basic circuitry of this filter is depicted in Fig. 8. This schema is in fact the mirrored schema of the Schurter EMI filter. The discharging resistors are added up.

![Fig. 8. The single-phase filter’s circuitry of Elfis 1ELF16V.](image)

The last example is given for the Schaffner FN 2070-10-06 EMI filter. This filter contains two current compensated inductors. These two inductors improve insertion loss of the filter, but the mathematical description of this filter is more complicated. In practise, the equation for calculat-
measured by the producer could be also found there. The circuitry of this filter is given in Fig. 12. The measured and calculated data are depicted in Fig. 13, 14, and 15. From Fig. 13, 14, and 15 is obvious that the performance of mathematical models is very good in low frequency range. This is due to the neglecting of the spurious components of each electrical components of the EMI filter.

Fig. 12. Typical single-phase filter’s circuitry schema with two current compensated inductors (Schaffner FN 2070-10-06).

Fig. 13. The insertion loss of the Schaffner FN 2070-10-06 in asymmetrical systems.

4. Conclusions

This paper deals with the simple mathematical models of single-phase EMI filters. By these models it is possible to describe whichever EMI filter, but the inner circuitry of the filter has to be known including the quantity of each electronic component. The filter itself is described by admittance parameters. The spurious components have not been taken into account. The insertion loss of the EMI filter was deduced by using the modified nodal voltage method. The biggest problem is with the universality of the method because the EMI filters have not the same number of nodes and the current compensated inductors added up several rows and columns into the admittance matrix Y. For that reasons, the final admittance matrix should be reduced by the pivot.

Fig. 9. The insertion loss of the Elfis 1ELF16V in asymmetrical systems.

Fig. 10. The insertion loss of the Elfis 1ELF16V in symmetrical systems.

Fig. 11. The insertion loss of the Elfis 1ELF16V in "non-symmetrical" systems.
condensation. The user has to be careful during the condensation and does not condensate the outer nodes where the filter is connected into the outer system: power supply network at the input of the EMI filter and outputs for connecting the protected device. After this step the reduced admittance matrix $\mathbf{Y}$ is obtained. This matrix has the dimension of $4 \times 4$. The dimension corresponds with the outer clamps of the single-phase EMI filters.

The performance of the described method was tested on seven single-phase EMI filters from several producers with a different circuits. There were: Schurter 5110.1033.1, Schaffner FN 321 1/05, FN 2070-10-06, Elfis 1ELF16V, 1ELF16VY-4, and Filtana TS 800 1006. The testing was done in asymmetrical, symmetrical and “non-symmetrical” configurations. The following combinations of impedance termination were used: $50 \, \Omega/50 \, \Omega$, $0.1 \, \Omega/100 \, \Omega$, $100 \, \Omega/0.1 \, \Omega$, $0.1 \, \Omega/2 \, \Omega$ and $100 \, \Omega/100 \, \Omega$ were used. The odd clamps in “non-symmetrical” mode were terminated by the $50 \, \Omega$ in all cases. The insertion loss data were measured and calculated for these impedance systems. These data for three filters are given in Fig. 5, 6, 7, 9, 10, 11, 13, 14, and 15.

After comparison of the calculated data with the measured ones, it is possible to conclude. The simple models work quite well in the range of low frequencies. This fact is caused by ignoring the spurious properties of real electronic parts. But on the other hand, the accuracy in this low frequency range is really good; under $3 \, \text{dB}$ in the most cases. The advantage of the described method is universality caused by using the modified nodal voltage method and by using the admittance matrix for describing the EMI filters properties. The method also respects the real filter’s circuitry and also works with the current compensated inductors. The simple models are also applicable for different testing configurations, like asymmetrical or symmetrical test setups. Disadvantages are usually caused by inaccuracy of electronic components used in the tested EMI filters. In general, the simple models and also the described method could be used for quick analysing and comparing several EMI filters in different impedance systems. For obtaining the final equation for calculating insertion loss the Maple software was used. Final calculations in the frequency domain were done in the Matlab®. The final equations for EMI filter’s with two current compensated inductors reached hundreds of pages in the Microsoft® Word editor unformatted text. The measurements has been commanded by VEE Pro from Agilent Technologies®.

The presented method could be modified for quick estimation of filter performance or estimation of EMI filter’s “worst-case”. More details could be found in [8]. These techniques are available only for asymmetrical and symmetrical systems and reliably identify the mentioned “worst-case” performance of the EMI filter.

Further work will be focused on the improvement of the performance and the accuracy of described method mainly in high frequency range. The accuracy could by improved by implementing the effects of spurious parts of electronics components which are included into the EMI filters. The quantities of the spurious components will be estimated from the measured insertion loss data in $50 \, \Omega/50 \, \Omega$ systems. For estimating of spurious parameters of individual components, the Particle swarm optimization could be used.

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References


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