



primary feed radiates the spherical waves. The program offers several variants such as vertical or horizontal polarization calculations for horn antennas (pyramidal or conical) and radiation field calculations using aperture dimensions or approximations based on measured radiation patterns in the main planes using the least squares method with polynomials of order 6. Moreover, the primary feed could be formed by one feed or an array of two or four equally spaced radiators.



Fig. 2. The RL-41 antenna - doubly curved reflector with two independent pyramidal horns and SSR antenna above.

The far field  $\mathbf{E}$  in terms of the reflector complex current density using physical optics [3], [7] is

$$\mathbf{E} = -j \frac{e^{-jkR}}{\lambda R} \int_S [\mathbf{n} \times (\mathbf{i}_r \times \mathbf{E}_i)] e^{jk \mathbf{i}_r \cdot \mathbf{i}_r r} dS \quad (1)$$

where  $R$  is the distance from the origin to the observation position  $P$  (Fig. 1),  $\mathbf{n}$  is the normal unit vector at the reflector surface,  $\mathbf{i}_r$  is the position vector from the origin to the reflector,  $\mathbf{E}_i$  is the incident electric field vector at the reflector surface, determined by primary feed field calculations considering the relevant phases and the primary feed gain,  $\mathbf{i}_R$  is the far-field observation position unit vector,  $r$  is the distance from the origin to the reflector and  $S$  is the reflector surface. The surface integral (1) can be evaluated as a double integral of a function  $f(Y, \Phi)$ , where  $Y$  axis is perpendicular to the symmetry plane and  $\Phi$  is the angle measured in the symmetry plane. Inner integrals  $F(Y)$  are firstly evaluated and then outer integrals are calculated over the variable  $Y$  (details can be found in [7]). If the gain of primary feed is known, the gain of the whole antenna could be calculated [7].

The reflector surface  $S$  and its contour cannot be determined analytically but the reflector central section is calculated using geometrical optics and the surface  $S$  is formed by parabolic ribs [3], [5] - [8]. Therefore, the input data for surface  $S$  calculations are vectors of reflector central section and reflector contour. Moreover,  $\mathbf{E}_i$  calculations are very complicated. That means the far field (1) should be calculated numerically. The crucial problem for physical

optics is the numerical integration as the integrand of the radiation integral (1) oscillates rapidly for large reflectors. It is therefore impossible to evaluate (1) analytically. Moreover, integral (1) should be evaluated each time, when the observation angle changes. Various methods for physical optics have been suggested to increase the speed. The very simple, accurate and powerful integration methods have been proposed (details including the accuracy analyses can be found in [7], [8]), which would require substantially less RAM and computer time. The  $F(Y)$  inner integrals over a variable  $\Phi$  could be evaluated in advance using the generalized trapezoidal method with 80 points. Then the Gaussian integration with respect to  $Y$  axis is chosen. That allows numerical integrations for reflector diameters  $D_H$  greater than 100 wavelengths using just 8  $F(Y)$  values of inner integrals. Considering the first two or three sidelobes 8  $F(Y)$  values could be only used for the other observation positions with sufficient accuracy [7]. If the primary feed movements and tolerances of reflector surfaces are small, changes in amplitude can be neglected and the phase changes could be only considered. That means that computation (1) allows the determination of far fields considering reflector surface deviations as well as the feed movements. A slightly more complicated (more time consuming) procedure should be used for bigger feed movements.

Using physical optics the radiation patterns can be only computed for angles near by the main beam (usually the first two or three sidelobes), as the other sidelobes are strongly affected by antenna tolerances [5], [8] and neglecting of current at the shadow regions (that can be calculated using another approximate methods such as geometric theory of diffraction).

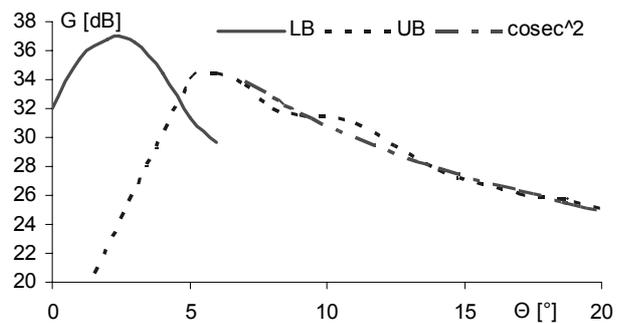


Fig. 3. The RL-41 antenna - the lower and upper beams and cosecant squared ( $\text{cosec}^2$ ).

### 3. Experimental Results

The beam widths and directivity of a relatively large planar array or aperture antennas are related by the well-known simple approximate equation [1]:

$$D = \frac{d_b}{\vartheta_1 \vartheta_2} \quad (2)$$

where  $\vartheta_1$  and  $\vartheta_2$  are half-power beam widths in the principal planes of the elliptical beam at any scan angle,  $d_b$  is constant (directivity-beam width product). The  $d_b$  values of 26 000 up to 52 525 could be considered for various cases. The directivity of planar antennas has been calculated with  $d_b = 32\,400$  and differences for various distributions have been determined [1]. Therefore, the value of 32 400 could be used. According to [1], the value of  $d_b = 26\,000$  could be used for practical antennas (i.e. 1 dB difference against 32 400).

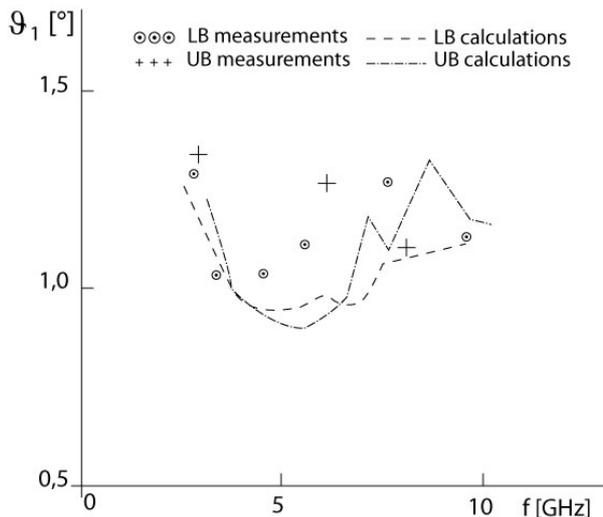


Fig. 4. Measurements and calculations (using physical optics) of  $\vartheta_1$  half-power beam widths.

Fig. 4 and 5 show the measurements and calculations (using physical optics) of  $\vartheta_1$  and  $\vartheta_2$  half-power beam widths for the lower beam (LB) and upper beam (UB) of RL-41 antenna. The measurements and calculations (using physical optics) of gains for RL-41 antenna are shown in Fig. 6 and 7. That are compared with gain approximation according to (2) for  $d_b = 32\,400$  (D) and  $d_b = 26\,000$  (D-1).

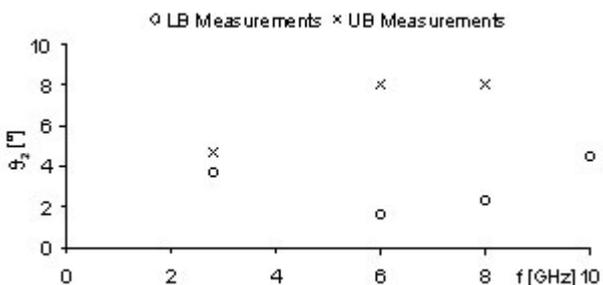


Fig. 5. Measurements of  $\vartheta_2$  half-power beam widths.

The differences between measurements and calculations (using physical optics) for higher frequencies in Fig. 4 to 7 are given by simplifications. In this case, the rectangular waveguide was considered instead of real feed, i.e. the rectangular aperture without any phase errors was used for calculations. That creates higher differences for horn with higher phase errors and tolerance problems. It is well

known that the horn quadratic phase error is directly proportional to frequency, and therefore for higher frequencies the real feed beams are broader, the nulls between the main lobe and the first sidelobe disappear and sidelobe levels are raised [3] - [5]. Moreover, the horn phase characteristics are changed. A possible radiation of higher modes creates substantial changes of primary feed radiation patterns such as maximum oscillations and sharper radiation patterns [11]. Manufacturing tolerances (the surface imperfections) change the optical path length from the feed to the reflector aperture plane, and therefore phase errors are higher for higher frequencies. Moreover the utilization of two horns, which are not in focus, causes that the antenna is even more tolerance sensitive.

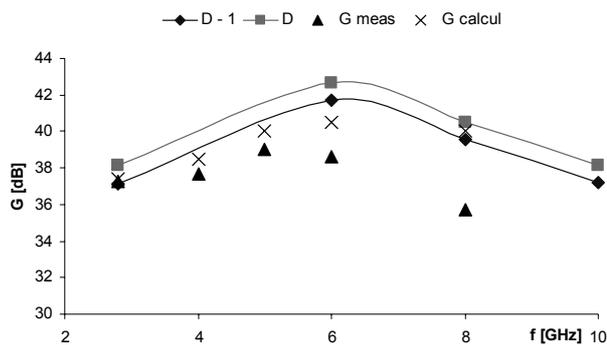


Fig. 6. Measurements and calculations (using physical optics) of gains and directivities for  $d_b = 32\,400$  (D) and  $d_b = 26\,000$  (D - 1) for lower beam.

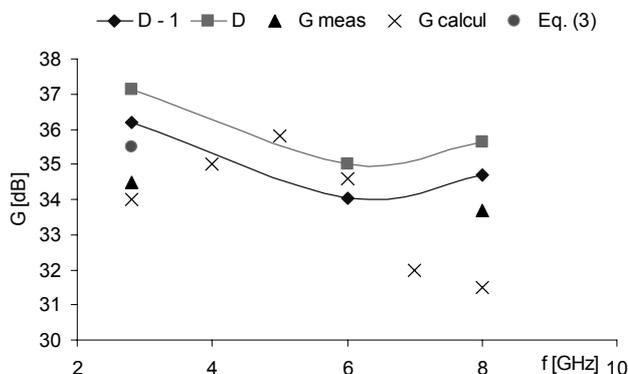


Fig. 7. Measurements and calculations (using physical optics) of gains and directivities for  $d_b = 32\,400$  (D),  $d_b = 26\,000$  (D - 1) for upper beam and  $D_s$  according to (3).

For the shaped beam (upper beam), it could be possible to modify the simple approximation (2). Directivity can be evaluated using the normalized radiation pattern  $F(\theta, \varphi)$

$$D_s = \frac{4\pi}{\int_0^{2\pi} \int_{-\pi/2}^{\pi/2} |F(\theta, \varphi)|^2 \cos \theta \, d\theta \, d\varphi} \quad (3)$$

$$\approx \frac{4\pi}{\varphi_e \int_{-\pi/2}^{\pi/2} |F(\theta)|^2 \cos \theta \, d\theta} \approx \frac{4\pi}{\varphi_e \theta_e}$$

where  $\theta_e, \varphi_e$  are the effective half-power beam widths of the shaped beam. To estimate  $\theta_e$  the integral could be evaluated numerically considering the cosecant squared vertical pattern of the upper beam. That approach could offer slightly better estimation (see Fig. 7). However, that is rather more complicated and cannot be generally used (the radiation pattern is not usually known in advance). Therefore, it is not generally possible to use (3) for simple gain approximation.

## 4. Conclusion

The paper presents the well-known simple formula (2) of gain estimation. It can be seen that for the lower beam, which can be roughly considered as a pencil beam, the experimental results and calculations using physical optics correspond with the simple formula of gain estimation according to (2) for the operation (2.7 to 2.9 GHz) frequencies, when the limits of  $d_b = 32\,400$  and  $d_b = 26\,000$  are taken into account (i.e. 1 dB difference). For out of operation (3 to 10 GHz) frequency bands, the differences between (2) and calculations (using physical optics) are nearly acceptable. The greater differences between experiments and calculations are explained above.

For the upper beam, which can be considered as a shaped beam, the greater differences between experimental results, calculations using physical optics and the simple formula of gain estimation according to approximation (2) exist. The approach (3) could offer slightly better estimation but it is rather more complicated and cannot be generally used. However, approximation (2) could be used as a rough estimation.

The simple gain approximation could be very valuable for system engineers to accurately estimate antenna gain and perform EMC calculations (estimations of interferences and susceptibilities). It seems that the approximations of  $D$  with  $d_b = 32\,400$  could be safely used as higher limits for EMC calculations (interference and susceptibility estimations). On the other hand, the approximations of  $D$  with  $d_b = 26\,000$  could be used for radar equations or received power calculations of transmitting antenna.

## Acknowledgements

The paper is supported by the Czech National Institutional Research "Theory of Transport System" No. MSM 0021627505.

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