Using Volterra Series for an Estimation of Fundamental Intermodulation Products

Josef DOBEŠ

Department of Radio Engineering, Czech Technical University in Prague, Technická 2, 16627 Praha 6, Czech Republic

dobes@feld.cvut.cz

Abstract. *The most precise procedure for determining the intermodulation products is to find a steady-state period of the signal first, and then to calculate its spectrum by means of the fast Fourier transform. However, this method needs time-consuming numerical integration over many periods of the faster signal even for enhanced methods for finding the steady state. In the paper, an efficient method for fast estimation of the fundamental intermodulation products is presented. The method uses Volterra series in a simple multistep algorithm which is compatible with a typical structure of the frequency-domain part of circuit simulators. The method is demonstrated by an illustrative testing circuit first, which clearly shows possible incorrect interpretation of the Volterra series. Thereafter, practical usage of the algorithm is demonstrated by fast estimation of the main intermodulation products of a low-voltage low-power RF CMOS four-quadrant multiplier.*

Keywords

Steady-state algorithm, fast Fourier transform, numerical integration, Volterra series, CMOS, RF multiplier.

1. Introduction

A natural and accurate method for determining the intermodulation products consists in finding a steady-state response first, and then computing its spectrum by means of fast Fourier transform. This algorithm has been implemented into author's software tool C.I.A. (Circuit Interactive Analyzer [1]) with automatic identification of unknown periods of autonomous circuits. Essential theory of the steady-state analysis can be found in [2], [3], and some improvements of these classical methods—especially automating the procedure for autonomous circuits and defining reliable convergence criterion—are described in [4], [5]. As the method implemented in C.I.A. for numerical integration—which is necessary fundament for the steady-state algorithm—is very flexible (it is based on efficient recurrent form of Newton interpolation polynomial [6], [7] rather than Lagrange one [8]), calculated intermodulation products are available even for higher orders.

However, the numerical integration must be performed over many periods of the faster signal and therefore the analysis is time-consuming in many cases. For this reason, another method for fast estimation of the fundamental intermodulation products has also been implemented which is based on Volterra series. A brief introduction to using the Volterra series for such purposes is shown in [9], and a more comprehensive exposition can be found in [10].

A disadvantage of many of the implementations of the Volterra series consists in a creation of a new—and relatively large and isolated—block of program. In this paper, a form of the method is described which is compatible with the frequency-domain part of the C.I.A. program. Therefore, the algorithm is built into the AC-analysis code of the C.I.A. program and shares a relatively large portion of it.

The algorithm based on the Volterra series certainly is very dependent on accuracy of models of nonlinearities. Hence, for CMOS technology, all main types of MOSFET models—semiempirical [9], [1], BSIM [11], BSIM3 [12], [13], BSIM4 [13], and EKV [14]—should be tested especially from the point of view of their derivatives' precision.

2. Description of the Method

The system of nonlinear algebraic-differential equations of a circuit is generally defined in the implicit form

$$\boldsymbol{f}\left[\,\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t), t\,\right] = \boldsymbol{0}. \tag{1}$$

As the resulting formulae derived from the application of the Volterra series are very complicated, consider for simplicity of the explanation that the circuit system comprises only two equations, i.e., (1) can be rewritten in the simpler form

$$f_1\left(x_1, x_2, \dot{x}_1, \dot{x}_2, t\right) = 0, \quad f_2\left(x_1, x_2, \dot{x}_1, \dot{x}_2, t\right) = 0. \tag{2}$$

The Taylor expansion of the functions f_1 and f_2 with the inclusion of the second-order terms in a linearization center $^{(0)}$ is the following (certainly, higher-order terms are necessary for calculating higher-order intermodulation products):

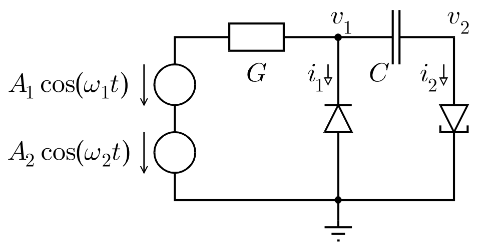

Fig. 1. Simple testing circuit with standard and tunnel diodes which is analyzed by both fast Fourier transform of its steady-state response and Volterra series.

$$
\begin{aligned}
& f_{1,2}^{(0)} + \frac{\partial f_{1,2}}{\partial x_1}^{(0)} \Delta x_1 + \frac{\partial f_{1,2}}{\partial x_2}^{(0)} \Delta x_2 + \\
& \frac{\partial f_{1,2}}{\partial \dot{x}_1}^{(0)} \Delta \dot{x}_1 + \frac{\partial f_{1,2}}{\partial \dot{x}_2}^{(0)} \Delta \dot{x}_2 + \\
& \frac{\partial^2 f_{1,2}}{\partial x_1 \partial x_2}^{(0)} \Delta x_1 \Delta x_2 + \frac{\partial^2 f_{1,2}}{\partial x_1 \partial \dot{x}_1}^{(0)} \Delta x_1 \Delta \dot{x}_1 + \\
& \frac{\partial^2 f_{1,2}}{\partial x_1 \partial \dot{x}_2}^{(0)} \Delta x_1 \Delta \dot{x}_2 + \frac{\partial^2 f_{1,2}}{\partial x_2 \partial \dot{x}_1}^{(0)} \Delta x_2 \Delta \dot{x}_1 + \qquad (3) \\
& \frac{\partial^2 f_{1,2}}{\partial x_2 \partial \dot{x}_2}^{(0)} \Delta x_2 \Delta \dot{x}_2 + \frac{\partial^2 f_{1,2}}{\partial \dot{x}_1 \partial \dot{x}_2}^{(0)} \Delta \dot{x}_1 \Delta \dot{x}_2 + \\
& \frac{1}{2}\frac{\partial^2 f_{1,2}}{\partial x_1^2}^{(0)} \Delta x_1^2 + \frac{1}{2}\frac{\partial^2 f_{1,2}}{\partial x_2^2}^{(0)} \Delta x_2^2 + \\
& \frac{1}{2}\frac{\partial^2 f_{1,2}}{\partial \dot{x}_1^2}^{(0)} \Delta \dot{x}_1^2 + \frac{1}{2}\frac{\partial^2 f_{1,2}}{\partial \dot{x}_2^2}^{(0)} \Delta \dot{x}_2^2 .
\end{aligned}
$$

A natural linearization center for this type of analysis is the operating point, i.e., the static version of (2)

$$f_1\left(x_{1_0}, x_{2_0}, 0, 0, 0\right) = 0, \quad f_2\left(x_{1_0}, x_{2_0}, 0, 0, 0\right) = 0$$

must be solved in advance—note that the C.I.A. program always computes the operating point on the basis of values of input sources at $t = 0$.

The next step is the standard (conventional) frequency analysis, i.e., solving the system of the two equations

$$
\begin{aligned}
& F_1(\omega) + \frac{\partial f_1}{\partial x_1}^{(0)} \Delta X_1 + \frac{\partial f_1}{\partial x_2}^{(0)} \Delta X_2 + \\
& j\omega \frac{\partial f_1}{\partial \dot{x}_1}^{(0)} \Delta X_1 + j\omega \frac{\partial f_1}{\partial \dot{x}_2}^{(0)} \Delta X_2 = 0, \\
& F_2(\omega) + \frac{\partial f_2}{\partial x_1}^{(0)} \Delta X_1 + \frac{\partial f_2}{\partial x_2}^{(0)} \Delta X_2 + \\
& j\omega \frac{\partial f_2}{\partial \dot{x}_1}^{(0)} \Delta X_1 + j\omega \frac{\partial f_2}{\partial \dot{x}_2}^{(0)} \Delta X_2 = 0,
\end{aligned}
$$

which must be resolved for the two frequencies ω_1 and ω_2. In this way, we obtain the first-order products $\Delta X_1(\omega_1)$, $\Delta X_1(\omega_2)$, $\Delta X_2(\omega_1)$, and $\Delta X_2(\omega_2)$. The terms $F_1(\omega)$ and $F_2(\omega)$ represent independent signal sources of the circuit.

The second-order intermodulation products can be estimated using the second-order terms in (3) as the signal sources of the circuit (instead of the independent ones), i.e., the system

$$
\begin{aligned}
& \frac{\partial f_{1,2}}{\partial x_1}^{(0)} \Delta X_1' + \frac{\partial f_{1,2}}{\partial x_2}^{(0)} \Delta X_2' + \\
& j\omega \frac{\partial f_{1,2}}{\partial \dot{x}_1}^{(0)} \Delta X_1' + j\omega \frac{\partial f_{1,2}}{\partial \dot{x}_2}^{(0)} \Delta X_2' + \\
& \frac{\partial^2 f_{1,2}}{\partial x_1 \partial x_2}^{(0)} \Delta X_1 \Delta X_2 + j\omega \frac{\partial^2 f_{1,2}}{\partial x_1 \partial \dot{x}_1}^{(0)} \Delta X_1^2 + \\
& j\omega \frac{\partial^2 f_{1,2}}{\partial x_1 \partial \dot{x}_2}^{(0)} \Delta X_1 \Delta X_2 + j\omega \frac{\partial^2 f_{1,2}}{\partial x_2 \partial \dot{x}_1}^{(0)} \Delta X_2 \Delta X_1 + \\
& j\omega \frac{\partial^2 f_{1,2}}{\partial x_2 \partial \dot{x}_2}^{(0)} \Delta X_2^2 - \omega^2 \frac{\partial^2 f_{1,2}}{\partial \dot{x}_1 \partial \dot{x}_2}^{(0)} \Delta X_1 \Delta X_2 + \\
& \frac{1}{2}\frac{\partial^2 f_{1,2}}{\partial x_1^2}^{(0)} \Delta X_1^2 + \frac{1}{2}\frac{\partial^2 f_{1,2}}{\partial x_2^2}^{(0)} \Delta X_2^2 - \\
& \omega^2 \frac{1}{2}\frac{\partial^2 f_{1,2}}{\partial \dot{x}_1^2}^{(0)} \Delta X_1^2 - \omega^2 \frac{1}{2}\frac{\partial^2 f_{1,2}}{\partial \dot{x}_2^2}^{(0)} \Delta X_2^2 = 0
\end{aligned}
$$

must be resolved for the frequencies $\omega_1 + \omega_1$, $\omega_2 + \omega_2$, $\omega_1 + \omega_2$, and $\omega_1 - \omega_2$, which gives the second-order harmonic products $\Delta X_1'(\omega_1 + \omega_1)$, $\Delta X_1'(\omega_2 + \omega_2)$, $\Delta X_2'(\omega_1 + \omega_1)$, $\Delta X_2'(\omega_2 + \omega_2)$ and intermodulation products $\Delta X_1'(\omega_1 + \omega_2)$, $\Delta X_1'(\omega_1 - \omega_2)$, $\Delta X_2'(\omega_1 + \omega_2)$, and $\Delta X_2'(\omega_1 - \omega_2)$. The third- and higher-order products can be determined in the analogical way—step by step—incorporating the third- and higher-order terms [10] to (3).

2.1 Illustrating Method Using Simple Example

The sequential steps of the analysis by the Volterra series can clearly be demonstrated by a simple testing circuit in Fig. 1.

The two signal sources have the magnitudes 0.1 V. The first source has the frequency 1 GHz and the second one 0.25 GHz, the conductance G is 0.1 S and the capacitance C is 10 pF. The standard diode is not statically opened in any part of the period due to the small magnitudes of the signal sources. Therefore, the current i_1 is only determined by the junction capacitance

$$i_1 = C_{J0}\left(1 - m v_1\right) \dot{v}_1,$$

where the term $C_{J0}\left(1 - m v_1\right)$ can be considered a simple linear approximation of the classical relation for the junction capacitance at $\phi_0 = 1$ V by Maclaurin expansion. The zero-bias junction capacitance C_{J0} is 10 pF, and the grading coefficient m changes from zero (i.e., the diode is replaced by a linear capacitor) through $0.\overline{3}$ (the diode with a linear junction) to 0.5 (the diode with an abrupt junction).

An ampère-volt characteristic of the tunnel diode can be approximated by a quadratic polynomial

$$i_2 = P_1 v_2 + P_2 v_2^2$$

in this analysis with respect to the small magnitudes of the signal sources. The coefficients of the polynomial are $P_1 = 0.2$ S and $P_2 = -1$ S/V. A current flowing through the capacitance part of the tunnel-diode model can be neglected in this task.

The **first** step of the algorithm consists in determining the operating point, which is very easy:

$$v_{1_0} = A_1 + A_2, \quad v_{2_0} = 0.$$

The **second** step of the algorithm is the standard frequency analysis, i.e., solving the system (note that $A(\omega)$ is equal to A_1 for $\omega = \omega_1$, and correspondingly equal to A_2 for $\omega = \omega_2$)

$$G[\Delta V_1 - A(\omega)] + j\omega C_{J0}[1 - m(A_1 + A_2)]\Delta V_1 + j\omega C(\Delta V_1 - \Delta V_2) = 0,$$

$$j\omega C(\Delta V_2 - \Delta V_1) + P_1 \Delta V_2 = 0.$$

This system of the two equations for the two variables $\Delta V_1(\omega)$ and $\Delta V_2(\omega)$ can be solved using Cramer rule, i.e.,

$$\Delta V_1(\omega) = \frac{(P_1 + j\omega C)\, G A(\omega)}{\{G + j\omega[C_{J0}(1 - m(A_1 + A_2)) + C]\}(P_1 + j\omega C) + \omega^2 C^2},$$

$$\Delta V_2(\omega) = \frac{j\omega C G A(\omega)}{\{G + j\omega[C_{J0}(1 - m(A_1 + A_2)) + C]\}(P_1 + j\omega C) + \omega^2 C^2}.$$

The **third** step of the algorithm is solving the system with the second derivatives for some harmonic or intermodulation product. Let us chose the product $\omega_1 + \omega_2$, e.g. From the set of the second derivatives of the function f_1, only the derivative

$$\frac{\partial^2 f_1}{\partial v_1 \partial \dot{v}_1} = -mC_{J0}$$

is nonzero. In the first equation, this derivative is multiplied by the factor

$$j(\omega_1 + \omega_2)\, \Delta V_1^2,$$

and ΔV_1 is a superposition of the frequency components ω_1 and ω_2. Therefore, it can be written in the form

$$j(\omega_1 + \omega_2)\left(\Delta V_1(\omega_1) + \Delta V_1(\omega_2)\right)^2.$$

As a source of the product $\omega_1 + \omega_2$, only the term

$$j(\omega_1 + \omega_2)\, 2\, \Delta V_1(\omega_1)\, \Delta V_1(\omega_2)$$

has meaning, but *not all* (this is just the source of frequent errors in many circuit simulators): corresponding analogy of the term $2\, \Delta V_1(\omega_1)\, \Delta V_1(\omega_2)$ in the time domain contains a factor of the type

$$\cos(\omega_1 t + \varphi_1)\cos(\omega_2 t + \varphi_2),$$

which can be expressed in the form

$$\tfrac{1}{2}[\cos((\omega_1 + \omega_2)t + \varphi_1 + \varphi_2) + \cos((\omega_1 - \omega_2)t + \varphi_1 - \varphi_2)].$$

Therefore, the term $2\, \Delta V_1(\omega_1)\, \Delta V_1(\omega_2)$ generates the intermodulation products both $\omega_1 + \omega_2$ and $\omega_1 - \omega_2$: the term

$$\Delta V_1(\omega_1)\, \Delta V_1(\omega_2)$$

is the origin of the intermodulation product $\omega_1 + \omega_2$, and the term

$$\Delta V_1(\omega_1)\, \Delta V_1^*(\omega_2)$$

is the origin of the intermodulation product $\omega_1 - \omega_2$ (the conjugate value is induced by the phase difference $\varphi_1 - \varphi_2$).

The following step is analogical—from the set of the second derivatives of the function f_2, only the derivative

$$\frac{\partial^2 f_2}{\partial v_2^2} = 2\, P_2$$

is nonzero. Therefore, the system of the linear complex equations for determining the intermodulation product $\omega_1 + \omega_2$ can be written in the form

$$G\Delta V_1' + j(\omega_1 + \omega_2)\, C_{J0}(1 - m(A_1 + A_2))\, \Delta V_1' + j(\omega_1 + \omega_2)\, C(\Delta V_1' - \Delta V_2') - j(\omega_1 + \omega_2)\, mC_{J0} \Delta V_1(\omega_1)\, \Delta V_1(\omega_2) = 0,$$

$$j(\omega_1 + \omega_2)\, C(\Delta V_2' - \Delta V_1') + P_1 \Delta V_2' + P_2 \Delta V_2(\omega_1)\, \Delta V_2(\omega_2) = 0.$$

This system can be resolved by the Cramer rule again with the following result for the second circuit variable:

$$\Delta V_2' = \frac{1}{D} \times \left\{-P_2\big[G + j(\omega_1 + \omega_2)[C_{J0}(1 - m(A_1 + A_2)) + C]\big] \times \Delta V_2(\omega_1)\, \Delta V_2(\omega_2) - (\omega_1 + \omega_2)^2 C m C_{J0} \Delta V_1(\omega_1)\, \Delta V_1(\omega_2)\right\},$$

where

$$D = \{G + j(\omega_1 + \omega_2)[C_{J0}(1 - m(A_1 + A_2)) + C]\} \times [P_1 + j(\omega_1 + \omega_2)C] + (\omega_1 + \omega_2)^2 C^2.$$

In this case, the magnitude and argument of the intermodulation product $\omega_1 + \omega_2$ have been checked by the steady-state analysis and fast Fourier transform because the resulting signal has the period only 4 ns. The comparison is shown in Tab. 1—it is clear that the estimation by the Volterra series is relatively precise for $m = 0$, and the inaccuracy is about 25 % for $m = 0.5$ (naturally, the "more nonlinear" circuit corresponds to the more inaccurate results).

m	$\lvert\Delta V_2'\rvert$ (FFT)	$\arg(\Delta V_2')$ (FFT)	$\lvert\Delta V_2'\rvert$ (Volterra)	$\arg(\Delta V_2')$ (Volterra)
0	0.63 mV	$81°$	0.59 mV	$82°$
$0.\overline{3}$	0.75 mV	$75°$	0.85 mV	$73°$
0.5	0.81 mV	$73°$	1 mV	$70°$

Tab. 1. Comparison of the results of fast Fourier transform applied to the steady-state response and estimation by the Volterra series.

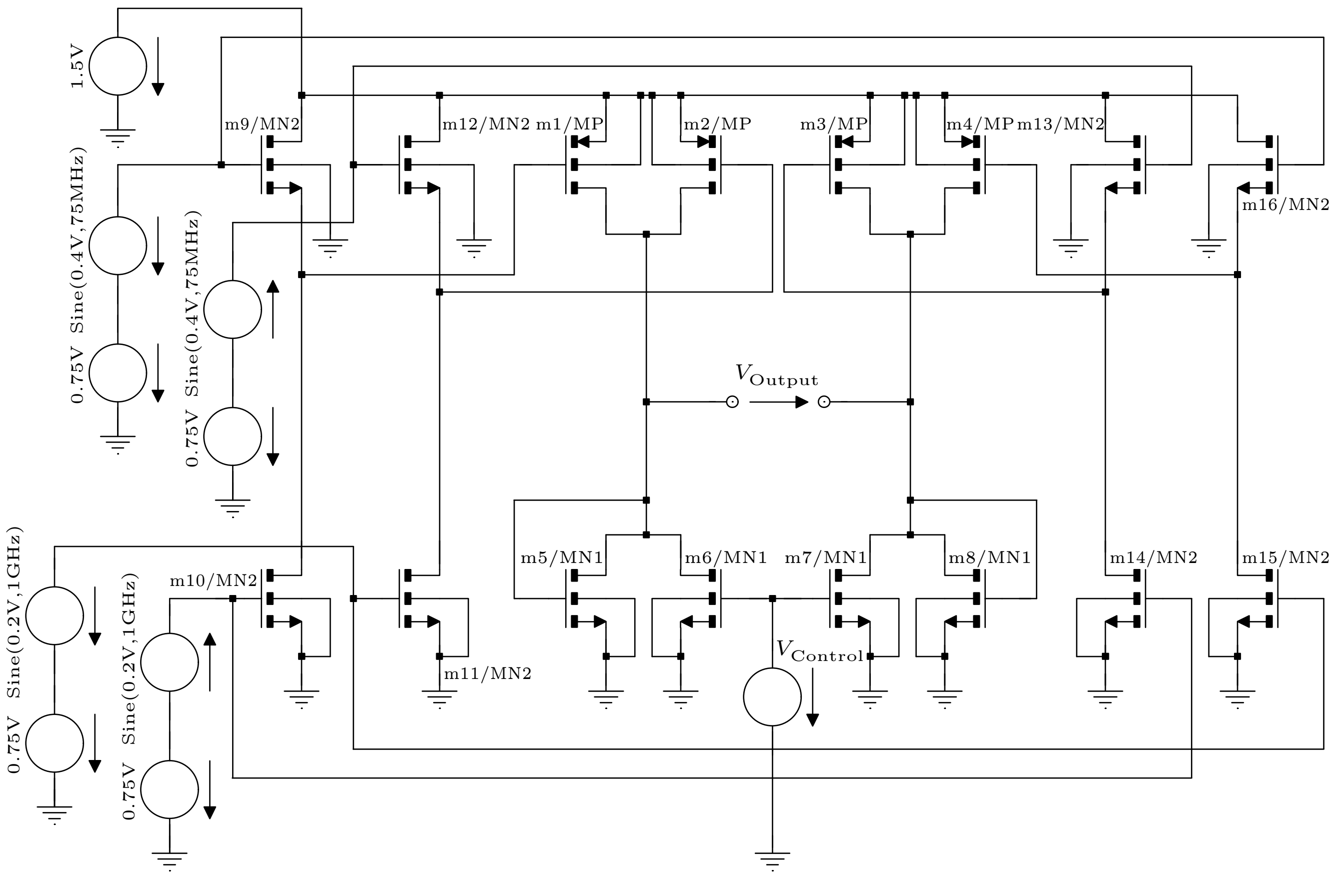

Fig. 2. Low-voltage low-power RF CMOS four-quadrant multiplier with symmetrical low-frequency (input signal) and high-frequency (local oscillator) sources.

3. Practical Example from Area of RF CMOS Integrated Circuits Design

Let us consider a four-quadrant RF CMOS multiplier in Fig. 2 [15] which has been analyzed by the C.I.A. program. All the parameters of the MOSFET BSIM/semiempirical model have kindly been granted by Prof. Salama. However, they have been slightly transformed to the new ones required by the "smoothed" gate-capacitance model with *suppressed* discontinuities [1] (certainly, possible capacitance discontinuities might have very negative influence to the precision of the simulations). The output voltage of the multiplier is strongly dependent on the controlling one which is connected to the gates of $\mathrm{m6}$ and $\mathrm{m7}$ transistors. For example, for the controlling voltages 1 and 1.5 V, the magnitudes of the output signal are about 20 and 50 mV, respectively as shown in Fig. 3 (and the "degree of nonlinearity" grows with respect to the output voltage correspondingly).

First, precise results are computed by the steady-state algorithm followed by the fast Fourier transform. The main intermodulation products are shown in Tab. 2—due to the double balancing of the multiplier, the intermodulation products $f_1 + 2f_2$, $f_1 - 2f_2$, $2f_1 + f_2$, and $2f_1 - f_2$ are negligible, which is very important. On the contrary, the intermodulation products $f_1 + 5f_2$ and $f_1 - 5f_2$ are noticeable as also shown in Tab. 2. Second, the intermodulation products $f_1 + f_2$ and $f_1 - f_2$ can be estimated much faster using the Volterra series—the results are shown in Tab. 3. The error of the estimation depends on the quality of the MOSFET models, and is acceptable for lesser magnitudes of the output signal. Unfortunately, the majority of the MOSFET models [11–14] are inaccurate regarding their derivatives, especially the higher ones. For this reason, an estimation of higher-order products is unrealistic without a model refinement. (Note that in MESFET field, the situation is better—e.g., the "realistic" model [16] has very precise derivatives.)

V_{Control}	$V_{\text{Output},1.075\,\text{GHz}}$	$V_{\text{Output},0.925\,\text{GHz}}$
1 V	8.97 mV	10.1 mV
1.1 V	10.2 mV	11.5 mV
1.2 V	11.9 mV	13.6 mV
1.3 V	14.5 mV	16.8 mV
1.4 V	18.3 mV	21.2 mV
1.5 V	22 mV	25.7 mV

V_{Control}	$V_{\text{Output},1.225\,\text{GHz}}$	$V_{\text{Output},0.775\,\text{GHz}}$
1 V	0.333 mV	0.377 mV
1.1 V	0.371 mV	0.485 mV
1.2 V	0.501 mV	0.712 mV
1.3 V	0.817 mV	1.18 mV
1.4 V	1.28 mV	1.85 mV
1.5 V	1.22 mV	1.67 mV

V_{Control}	$V_{\text{Output},3.075\,\text{GHz}}$	$V_{\text{Output},2.925\,\text{GHz}}$
1 V	0.217 mV	0.228 mV
1.1 V	0.218 mV	0.229 mV
1.2 V	0.22 mV	0.232 mV
1.3 V	0.227 mV	0.239 mV
1.4 V	0.238 mV	0.252 mV
1.5 V	0.273 mV	0.29 mV

V_{Control}	$V_{\text{Output},1.375\,\text{GHz}}$	$V_{\text{Output},0.625\,\text{GHz}}$
1 V	49.7 μV	29.9 μV
1.1 V	59.6 μV	43.8 μV
1.2 V	79 μV	72.3 μV
1.3 V	125 μV	145 μV
1.4 V	75.9 μV	120 μV
1.5 V	387 μV	773 μV

Tab. 2. Intermodulation products determined accurately by the steady-state algorithm and fast Fourier transform: $f_1 + f_2$, $f_1 - f_2$, $f_1 + 3f_2$, $f_1 - 3f_2$, $3f_1 + f_2$, $3f_1 - f_2$, $f_1 + 5f_2$ and $f_1 - 5f_2$. The third-order products $f_1 + 2f_2$, $f_1 - 2f_2$, $2f_1 + f_2$, and $2f_1 - f_2$ are negligible, which is important and expected due to the double balancing of the multiplier—see also Tab. 4.

$\lvert V_{\text{Output}}\rvert_{\max}$	V_{Control}	$V_{\text{Output},1.075\,\text{GHz}}$	$V_{\text{Output},0.925\,\text{GHz}}$
19.3 mV	1 V	7.02 mV	7.57 mV
22.6 mV	1.1 V	7.59 mV	8.29 mV
26.4 mV	1.2 V	8.38 mV	9.27 mV
33.1 mV	1.3 V	9.47 mV	10.6 mV
41.9 mV	1.4 V	11 mV	12.5 mV
49 mV	1.5 V	13 mV	15 mV

Tab. 3. Main intermodulation products $f_1 + f_2$ and $f_1 - f_2$ estimated by the Volterra series.

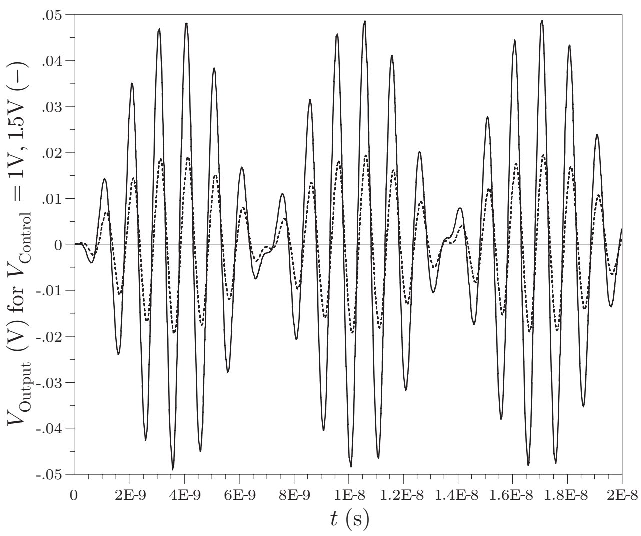

Fig. 3. Dependence of the output voltage of the multiplier on the controlling voltage.

4. Conclusion

An algorithm for the fast estimation of the fundamental intermodulation (and harmonic) products has been presented. The algorithm can easily be implemented as an add-on to the standard frequency-analysis routine, and can efficiently reuse a part of its code. In this way, the algorithm has been implemented to the Circuit Interactive Analyzer (C.I.A.) program. This program is also able to calculate the intermodulation products in the most precise but time-consuming way: to find a steady-state response of the circuit first, and then to calculate its spectrum by means of fast Fourier transform.

In the paper, a simple testing circuit has been analyzed first. The analytic derivation shows a possibility of incorrect implementation of the formulae of the Volterra series. Comparing the results of the Volterra series with those obtained by the steady-state analysis and fast Fourier transform clearly shows that with growing degree of nonlinearity grows the error of the estimation, too.

A technical utilization of the algorithm has been illustrated by an analysis of an RF CMOS four-quadrant multiplier. Again, for a lesser degree of nonlinearities, the estimation can be used as a fast—let say approximative—analysis.

The analysis of the RF CMOS multiplier also illustrates necessity of the precision of model derivatives. As the majority of the main MOSFET models do not have sufficiently precise derivatives, the third- and higher-order products cannot be computed accurately. Therefore, the MOSFET models need another refinement from this point of view. However, in the MESFET field, the state-of-the-art is better, and the third-order products can be calculated precisely enough.

Acknowledgements

This paper has been supported by the Czech Technical University in Prague Research Project MSM 6840770014 (distortion analysis), and by the Grant Agency of the Czech Republic, grant No. 102/08/0784 (steady-state analysis).

V_{Control}	$V_{\mathrm{Output},1.150\,\mathrm{GHz}}$	$V_{\mathrm{Output},0.850\,\mathrm{GHz}}$
1 V	0.17 μV	0.149 μV
1.1 V	0.403 μV	0.525 μV
1.2 V	0.895 μV	0.81 μV
1.3 V	0.761 μV	1.05 μV
1.4 V	0.658 μV	1.37 μV
1.5 V	0.257 μV	0.904 μV

V_{Control}	$V_{\mathrm{Output},2.075\,\mathrm{GHz}}$	$V_{\mathrm{Output},1.925\,\mathrm{GHz}}$
1 V	0.513 μV	0.877 μV
1.1 V	0.243 μV	0.683 μV
1.2 V	0.597 μV	0.845 μV
1.3 V	0.195 μV	0.561 μV
1.4 V	0.229 μV	0.772 μV
1.5 V	1.27 μV	0.461 μV

V_{Control}	$V_{\mathrm{Output},2.150\,\mathrm{GHz}}$	$V_{\mathrm{Output},1.850\,\mathrm{GHz}}$
1 V	0.147 μV	0.0758 μV
1.1 V	0.127 μV	0.248 μV
1.2 V	0.471 μV	0.301 μV
1.3 V	0.629 μV	0.227 μV
1.4 V	0.547 μV	0.893 μV
1.5 V	0.114 μV	0.106 μV

V_{Control}	$V_{\mathrm{Output},1.525\,\mathrm{GHz}}$	$V_{\mathrm{Output},0.475\,\mathrm{GHz}}$
1 V	9.38 μV	13.5 μV
1.1 V	9.93 μV	15.7 μV
1.2 V	10.6 μV	22.3 μV
1.3 V	12.9 μV	36.9 μV
1.4 V	78 μV	110 μV
1.5 V	116 μV	161 μV

Tab. 4. Some other intermodulation products determined by the steady-state algorithm and fast Fourier transform: $f_1 + 2f_2$, $f_1 - 2f_2$, $2f_1 + f_2$, $2f_1 - f_2$, $2f_1 + 2f_2$, $2f_1 - 2f_2$, $f_1 + 7f_2$ and $f_1 - 7f_2$. The third-order products and the products $2f_1 + 2f_2$ and $2f_1 - 2f_2$ are mostly below 1 μV (thus they can be considered neglectable)—on the contrary, the products $f_1 + 7f_2$ and $f_1 - 7f_2$ are still noticeable.

Also many thanks to Prof. Salama for emailing the MOSFET model parameters.

References

[1] DOBEŠ, J. Reliable CAD analyses of CMOS radio frequency and microwave circuits using smoothed gate capacitance models. *AEÜ—International Journal of Electronics and Communications,* 2003, vol. 57, no. 6, p. 372–380.

[2] SKELBOE, S. Computation of the periodic steady-state response of nonlinear networks by extrapolation methods. *IEEE Transactions on Microwave Theory and Techniques,* 1980, vol. 27, p. 161–175.

[3] VLACH, J., SINGHAL, K. *Computer Methods for Circuit Analysis and Design.* New York: Van Nostrand Reinhold Company, 1982.

[4] DOBEŠ, J., BIOLEK, D., POSOLDA, P. An efficient steady-state analysis of microwave circuits. *International Journal of Microwave and Optical Technology,* 2006, vol. 1, no. 2, p. 284–289.

[5] DOBEŠ, J. A steady-state add-on to the algorithm for implicit numerical integration. In *Proceedings of the 51st Midwest Symposium on Circuits and Systems.* Knoxville (Tennessee), 2008, p. 511–514.

[6] PETRENKO, A. I., VLASOV, A. I., TIMTSCHENKO, A. P. *Tabular Methods of Computer-Aided Modeling.* (In Russian.) Kiyv: Higher School, 1977.

[7] BRENAN, K. E., CAMPBELL, S. L., PETZOLD, L. R. *Numerical Solution of Initial-Value Problems in Differential-Algebraic Equations.* Philadelphia: SIAM, 1996.

[8] CHUA, L. O., LIN, P.-M. *Computer-Aided Analysis of Electronic Circuits.* Englewood Cliffs (New Jersey): Prentice-Hall, 1975.

[9] MASSOBRIO, G., ANTOGNETTI, P. *Semiconductor Device Modeling With SPICE.* 2nd ed. New York: McGraw-Hill, 1993.

[10] WAMBACQ, P., SANSEN, W. *Distortion Analysis of Analog Integrated Circuits.* Boston: Kluwer Academic Publishers, 1998.

[11] SHEU, B. J., SCHARFETTER, D. L., KO, P. K., JENG, M.-C. BSIM: Berkeley short-channel IGFET model for MOS transistors. *IEEE Journal of Solid-State Circuits,* 1987, vol. 22, no. 8, p. 558–566.

[12] CHENG, Y., HU, C. *MOSFET Modeling & BSIM3 User's Guide.* Boston: Kluwer Academic Publishers, 1999.

[13] LIU, W. *MOSFET Models for SPICE Simulation Including BSIM3v3 and BSIM4.* New York: John Wiley & Sons, 2001.

[14] BUCHER, M., THÉODOLOZ, F., KRUMMENACHER, F. *The EKV MOSFET Model for Circuit Simulation.* Lausanne: EPFL, 1998.

[15] SALAMA, M. K., SOLIMAN, A. M. Low-voltage low-power CMOS RF four-quadrant multiplier. *AEÜ—International Journal of Electronics and Communications,* 2003, vol. 57, no. 1, p. 74–78.

[16] PARKER, A. E., SKELLERN, D. J. A realistic large-signal MESFET model for SPICE. *IEEE Transactions on Microwave Theory and Techniques,* 1997, vol. 45, no. 9, p. 1563–1571.

About Author...

Josef DOBEŠ received the Ph.D. degree in microelectronics at the Czech Technical University in Prague in 1986. From 1986 to 1992, he was a researcher of the TESLA Research Institute, where he performed analyses on algorithms for CMOS Technology Simulators. Currently, he works at the Department of Radio Electronics of the Czech Technical University in Prague. His research interests include the physical modeling of radio electronic circuit elements, especially RF and microwave transistors and transmission lines, creating or improving special algorithms for the circuit analysis and optimization, such as time- and frequency-domain sensitivity, poles-zeros or steady-state analyses, and creating a comprehensive CAD tool for the analysis and optimization of RF and microwave circuits.