Modeling of Atmospheric Turbulence Effect on Terrestrial FSO Link

Aleš PROKEŠ

Dept. of Radio Electronics, Brno University of Technology, Purkyňova 118, 612 00 Brno, Czech Republic

prokes@feec.vutbr.cz

Abstract. *Atmospheric turbulence results in many effects causing fluctuation in the received optical power. Terrestrial laser beam communication is affected above all by scintillations. The paper deals with modeling the influence of scintillation on link performance, using the modified Rytov theory. The probability of correct signal detection in direct detection system in dependence on many parameters such as link distance, power link margin, refractive-index structure parameter, etc. is discussed and different approaches to the evaluation of scintillation effect are compared. The simulations are performed for a horizontal-path propagation of the Gaussian-beam wave.*

Keywords

FSO, atmospheric turbulence, scintillation, Gaussian beam, Rytov variance.

1. Introduction

A number of phenomena in the atmosphere such as scattering, absorption and turbulence affect laser beam propagation; for free space optics (FSO) only turbulence and scattering are appropriate to be taken into consideration due to the negligible absorption effect obtained at certain, currently used optical wavelengths.

The most important effects of atmospheric turbulence on the laser beam are: phase-front distortion, beam broadening, beam wander and redistribution of the intensity within the beam. The temporary redistribution of the intensity, known as scintillation, results from the chaotic flow changes of air and from thermal gradients within the optical path caused by the variation in air temperature and density. Zones (often referred to as eddies) of various sizes and differing densities act as lenses scattering light off its intended path. Particular parts of the laser beam then travel on slightly different paths and combine. The recombination is destructive or constructive at any particular moment and results in spatial redistribution of signal and consequently in lowering the received optical power.

This paper deals with the application of a heuristic theory of scintillation [1], [2], that has been developed at the University of Central Florida by L. Andrews, L. Philips and Y. Hopen, on the model of real FSO systems, with the aim of evaluating the effect of scintillation on correct operation of the link. The laser beam with an ideal Gaussian intensity profile corresponding to the theoretical TEM00 mode [3] propagating along a horizontal path through the clear atmosphere (without scattering by the particles present in the atmosphere) has been assumed for the calculation.

2. Gaussian Beam Theory

The Gaussian beam that propagates along the z axis can be characterized at the transmitter ($z = 0$) by the *beam spot radius* W_0, at which the optical intensity falls off to the $1/e^2$ of the maximum on the beam axis, by the *wave number* $k = 2\pi/\lambda$ (where λ is the wavelength), and by the *radius of curvature* F_0, which specifies the beam forming. The cases $F_0 = \infty$, $F_0 > 0$ and $F_0 < 0$ correspond to collimated, convergent and divergent beam forms, respectively. These parameters are usually used to describe the beam at a given position $z = L$ by the so-called *input-plane beam parameters* [1], [2] and [3]

$$\Theta_0 = 1 - \frac{L}{F_0}, \quad \Lambda_0 = \frac{2L}{kW_0^2}. \tag{1}$$

The symbol Θ_0 denotes the *curvature parameter* and Λ_0 is the *Fresnel ratio* at the input plane. Similarly, *output-plane beam parameters* corresponding to the input-plane beam parameters were introduced in the form

$$\Theta = \frac{\Theta_0}{\Theta_0^2 + \Lambda_0^2} = 1 - \frac{L}{F}, \qquad \Lambda = \frac{\Lambda_0}{\Theta_0^2 + \Lambda_0^2} = \frac{2L}{kW^2} \tag{2}$$

where F is the *radius of curvature* at the receiver. The *beam spot radius* at the receiver can be expressed by using (1) and (2) as

$$W = W_0\left(\Theta_0^2 + \Lambda_0^2\right)^{1/2}. \tag{3}$$

An additional parameter useful for beam description is the *divergence half-angle*, which defines the spreading of the beam when it propagates towards infinity. It is given by

$$\theta = \frac{\lambda}{\pi W_B} \tag{4}$$

where

$$W_B = \frac{W_0}{\left[\left(\frac{kW_0^2}{2F_0}\right)^2 + 1\right]^{1/2}} \tag{5}$$

is the spot size radius at the beam waist, i.e. at the minimum beam radius along the path.

2.1 Intensity and Power in Beam

The *irradiance,* or *intensity* of the optical wave, is the square amplitude of the optical field [1]. At radial distance r from the optical axis the irradiance can be expressed by

$$I^0(r,L) = I_0\left[\frac{W_0}{W(L)}\right]^2 \exp\left[-\frac{2r^2}{W^2(L)}\right] \tag{6}$$

where $I_0 = I^0(0,0)$ is the transmitter output irradiance at the centerline of the beam and the superscript 0 denotes the irradiance in a free space (without turbulence).

The relation between the intensity of the optical wave and the total power in the beam for the case $r = 0$ can be written in the form [4]

$$I^0(0,L) = I_0\frac{W_0^2}{W^2(L)} = \frac{2P_0}{\pi W^2(L)} \tag{7}$$

where P_0 is the total power transmitted by the beam. Power P incident on the circular receiver lens of aperture diameter D situated at distance L is

$$P(D,L) = P_0\left[1-\exp\left(-\frac{D^2}{2W^2(L)}\right)\right]. \tag{8}$$

Definition of some parameters mentioned above is shown in Fig. 1.

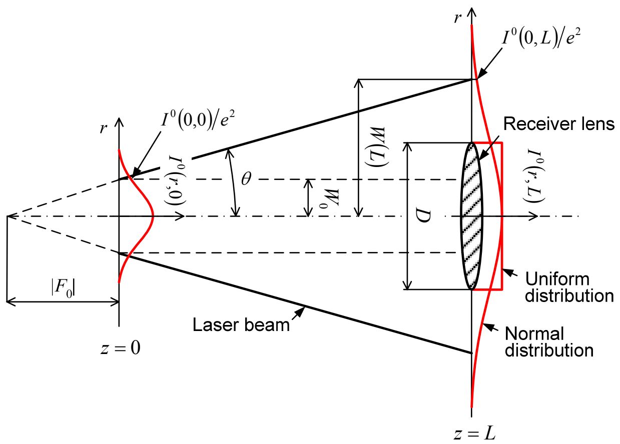

Fig. 1. Definition of the parameters describing the Gaussian beam.

2.2 Possible Simplification for Real FSO

In a real situation, a relatively large divergence angle (1 mrad ≤ θ ≤ 10 mrad) causes that $kW_0^2/2F_0 >> 1$, and the divergence calculation using (4) and (5) reduces to $\theta = W_0/|F_0|$. Similarly, equation (3) could be expressed in a simplified form, $W = W_0 + L\theta$.

On the assumption that the beam radius at the receiver position is much greater than the diameter of the receiver lens, i.e. $W(L) >> D$, the optical intensity at the lens can be regarded as uniformly distributed (see Fig. 1). The received optical power is then

$$P(D,L) = I^0(0,L)\frac{\pi D^2}{4} = \frac{D^2 P_0}{2(W_0 + L\theta)^2}. \tag{9}$$

3. Optical Scintillation Modeling

The intensity of the optical wave I propagating through turbulent atmosphere is a random variable. The normalized variance of optical wave intensity, referred to as the *scintillation index,* is defined by

$$\sigma_I^2 = \frac{\langle I^2\rangle - \langle I\rangle^2}{\langle I\rangle^2} = \frac{\langle I^2\rangle}{\langle I\rangle^2} - 1 \tag{10}$$

where the angle brackets denote an ensemble average. The scintillation index indicates the strength of irradiance fluctuations. For weak fluctuations, it is proportional, and for strong fluctuations, it is inversely proportional to the *Rytov variance* for a plane wave [1]

$$\sigma_1^2 = 1.23\, C_n^2 k^{7/6} L^{11/6} \tag{11}$$

where C_n^2 [m$^{-2/3}$] is the *refractive-index structure parameter*. The refractive-index structure parameter is very difficult to measure as it also depends on the temperature, wind strength, altitude, humidity, atmospheric pressure, etc. Some methods leading to its calculation are presented in [5], [6] or [7]. For a homogenous turbulent field, which can be assumed for near-ground horizontal-path propagation, the refractive-index structure parameter is constant.

The scintillation levels are usually divided into three regimes in dependence on the Rytov variance [8]: a weak-fluctuations regime ($\sigma_1^2 < 0.3$), a moderate-fluctuations regime (focusing regime) ($0.3 \le \sigma_1^2 < 5$), and a strong-fluctuations regime (saturation regime) ($\sigma_1^2 \ge 5$).

3.1 Modified Rytov Theory for Gaussian Beam

An appropriate model of the scintillation index usable for all three regimes is given by the modified Rytov theory, in which intensity I is defined as a modulation process [2] in the form

$$I = xy \tag{12}$$

where x and y are statistically independent unit mean random variables representing fluctuations caused by small-scale (diffractive) and large-scale (refractive) turbulence eddies.

The scintillation index in the Rytov theory was derived from the log-intensity ln(I) of the optical wave. On the assumption that $\langle x\rangle = \langle y\rangle = \langle I\rangle = 1$ and $\langle I^2\rangle = \langle x^2\rangle\langle y^2\rangle$ and using (10) and (12) the scintillation index can be expressed in the form

$$\sigma_I^2 = \left(1+\sigma_x^2\right)\left(1+\sigma_y^2\right)-1 = \exp\left(\sigma_{\ln x}^2 + \sigma_{\ln y}^2\right)-1 \quad (13)$$

where σ_x^2 and σ_y^2 are normalized variances and $\sigma^2_{\ln x}$ and $\sigma^2_{\ln y}$ are log-irradiance variances of x and y [1].

Optical scintillations can be reduced by increasing the collecting area of the receiver lens by the integration of various intensities incident on particular parts of the lens. This phenomenon is known as *aperture averaging*.

The scales of turbulence eddies range from a lower limit l_0 (in the order of millimeters), called *inner-scale*, to an upper limit L_0 (in the order of meters), called *outer-scale*. In the simplified case $l_0 = 0$, $L_0 = \infty$ and when aperture averaging is assumed, the log-irradiance variances are given by [2]

$$\sigma_{\ln x}^2(D,L) = 0.49\,\sigma_1^2\left(\frac{\Omega_G-\Lambda_1}{\Omega_G+\Lambda_1}\right)^2 \times\left(\frac{1}{3}-\frac{1}{2}\overline{\Theta}+\frac{1}{5}\overline{\Theta}^2\right)\frac{\eta_x^{7/6}}{\left(1+0.40\,\eta_x\dfrac{2-\overline{\Theta}}{\Omega_G+\Lambda_1}\right)^{7/6}} \quad (14)$$

and

$$\sigma_{\ln y}^2(D,L) = \frac{1.27\,\sigma_1^2\eta_y^{-5/6}}{1+0.40\,\eta_y/\left(\Omega_G+\Lambda_1\right)},\quad \eta_y >> 1 \quad (15)$$

where $\overline{\Theta} = 1-\Theta$, $\Omega_G = 16L/kD^2$ is a *Fresnel ratio* characterizing the radius of Gaussian lens,

$$\eta_x = \frac{\left(\frac{1}{3}-\frac{1}{2}\overline{\Theta}+\frac{1}{5}\overline{\Theta}^2\right)^{-6/7}\left(\frac{\sigma_B^2}{\sigma_1^2}\right)^{6/7}}{1+0.56\,\sigma_B^{12/5}} \quad (16)$$

and

$$\eta_y = \left(\frac{\sigma_B^2}{\sigma_1^2}\right)^{6/5}\left(1+0.69\,\sigma_B^{12/5}\right) \quad (17)$$

are the artificial variables, and

$$\sigma_B^2 = 3.86\,\sigma_1^2\left\{\frac{2}{5}\left[\left(1+2\Theta\right)^2+4\Lambda^2\right]^{5/12} \times\cos\left[\frac{5}{6}\tan^{-1}\left(\frac{1+2\Theta}{2\Lambda}\right)\right]-\frac{11}{16}\Lambda^{5/6}\right\} \quad (18)$$

is the Rytov variance for a Gaussian-beam wave [1].

For the case where both the inner-scale and the outer-scale effects are assumed, i.e. $l_0 > 0$ and $L_0 < \infty$, relatively complex equations were derived for the calculation of log-irradiance variances. They can be found in chapters 6.5.2 and 6.5.3. of [2].

4. Intensity Distribution

Under weak fluctuations, the intensity is lognormally distributed. The probability density function (PDF) valid for $I > 0$ is of the form [2]

$$p(I) = \frac{1}{I\sqrt{2\pi\sigma_I^2}}\exp\left\{-\frac{\left[\ln\left(I/\langle I\rangle\right)+\sigma_I^2/2\right]^2}{2\sigma_I^2}\right\}. \quad (19)$$

For the moderate-to-strong fluctuation regime, the gamma-gamma distribution is preferable, as it provides a good fit to the irradiance fluctuations collected by very small or point apertures. The PDF of the gamma-gamma distribution is

$$p(I) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)I}\left(\frac{I}{\langle I\rangle}\right)^{\frac{\alpha+\beta}{2}}K_{\alpha-\beta}\left(2\sqrt{\frac{\alpha\beta I}{\langle I\rangle}}\right) \quad (20)$$

where α and β are positive parameters given by

$$\alpha^{-1} = \sigma_x^2 = \exp\left(\sigma_{\ln x}^2\right)-1, \quad \beta^{-1} = \sigma_y^2 = \exp\left(\sigma_{\ln y}^2\right)-1, \quad (21)$$

and $K_n(\cdot)$ is the Bessel function of the second kind and n-th order.

Both intensity distributions were studied and compared by using an experimental data in [8], [9], [10], and [11]. It was shown that the lognormal distribution is a good approximation for many weak-to-strong scenarios if the aperture averaging is used.

On the assumption that scintillation is a redistribution of intensity without loss of power and that beam broadening due to turbulence is not considered (in the strongly divergent beam it can be ignored), the mean value of intensity needed in (19) and (20) is equal to the intensity without turbulence i.e. $\langle I\rangle = I^0(0,L)$.

5. Probability of Fade

The probability of fade is calculated using so called *threshold approach*. It is based on the assumption that due to the intensity fading the data reception is interrupted within a certain time intervals in which the received intensity I drops below the intensity threshold I_T (or the received optical power drops below the receiver sensitivity). The threshold approach offers simple calculation of the probability of fade as it does not require a detailed investigation of the specific receiver performance. The probability of fade is then given by the cumulative distribution function (CDF)

$$\Pr[I \le I_T(0,L)] = \int_0^{I_T(0,L)} p(I)dI \cdot \qquad (22)$$

The intensity threshold $I_T(0, L)$ can be obtained by solving (7) for P_0, substituting result into (8) and then solving (8) for intensity. Under assumption that the received power $P(D,L)$ match with the receiver sensitivity P_r, the intensity threshold is given by

$$I_T(0,L) = \frac{2P_r}{\pi W^2(L)}\left[1-\exp\left(-\frac{D}{2W^2(L)}\right)\right]^{-1} . \qquad (23)$$

6. Simulation Results

All simulations were realized in the MATLAB environment for three typical FSO representatives**,** whose parameters are summarized in Tab. 1.

The beam parameters, intensities and powers were calculated according to exact equations (1) to (8). All powers in this paragraph are expressed in dBm units.

The laser power, beam divergence, receiver sensitivity and area of the receiver lens define how the FSO is able to eliminate atmospheric path losses or turbulence effects. They are summarized in the *link margin* $M(L)$, which results from (9), taking into consideration the condition $W_0 << L\theta$. It is given by

$$M(L) = P_0 - 20\log\left(\frac{\sqrt{2}L\theta}{D}\right) - P_r \quad [\mathrm{dB}] \qquad (24)$$

or $M(L) = M0 - 20\log L$, where $M0$ includes all constant values in (24) given by the FSO design. The link margins for all three representatives of FSO links intended for a data rate of 1 Gbps, calculated according to approximate relation (24) and according to an exact relation based on (8), are shown in Fig. 2.

	FSO A	FSO B	FSO C
P_0 [dBm]	10	10	13
P_r [dBm]	-27	-30	-30
W_0 [mm]	10	10	20
F_0 [m]	-5	-10	-20
D [mm]	75	100	150
2θ [mrad]	4	2	2
M_0 [dB]	65.5	77.0	83.5

Tab. 1. Parameters of the FSO systems used for calculations.

The scintillation index shown in Fig. 3 was evaluated for a scenario with $\lambda = 850$nm, $C_n^2 = 0.25\cdot10^{-13}$ m$^{-2/3}$ and with the two different scales of turbulence eddies. In the case of $l_0 = 0$ and $L_0 = \infty$, relations (13) to (18) were used, while in the case of $l_0 = 2$ mm and $L_0 = 10$ m the scintillation index was calculated using the relations given in [2]. The results obtained demonstrate the influence of the aperture averaging effect. It is evident that the largest receiver lens area (FSO C) exhibits the lowest scintillations. The lower scintillation index (solid waveforms) is generally caused by the smaller scale of turbulence eddies [2].

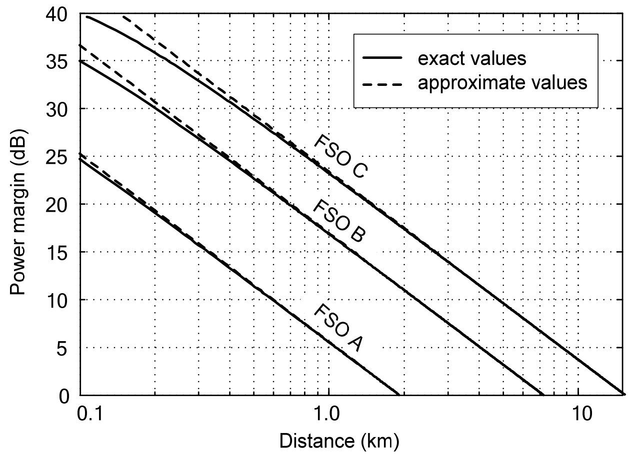

Fig. 2. Power link margin vs. link distance.

The fade probability corresponding to the two different scales of turbulence eddies, calculated using (19) and (22) on the assumption that the optical irradiance is lognormally distributed, is depicted in Fig 4. The integral in (22) was evaluated numerically. It is evident from Fig. 2 and Fig. 4 that the probability of fade reaches its maximum at the point where the power link margin approaches zero, and that it falls off relatively quickly with decreasing link distance.

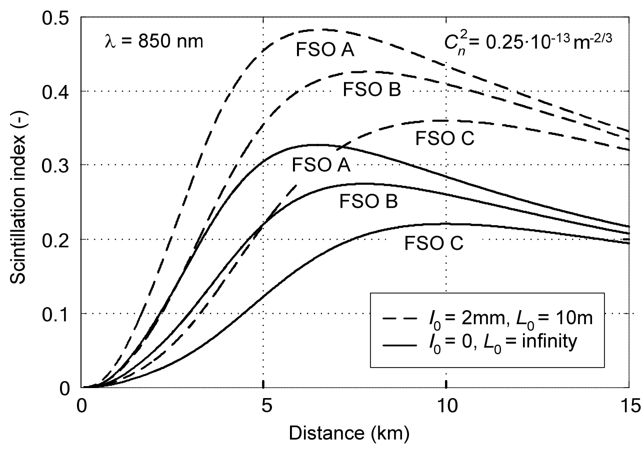

Fig. 3. Scintillation index in dependence on the link distance.

A comparison of both the lognormal and the gamma-gamma distributions is shown in Fig 5. The large diameter of the receiving aperture and the large value of the refractive-index structure parameter cause that both coefficients α and β reach values exceeding one hundred. This causes that the calculation of the gamma-gamma distribution produces undefined results due to numerical format overflowing. That is why the probabilities of fade were not compared for the FSO C system.

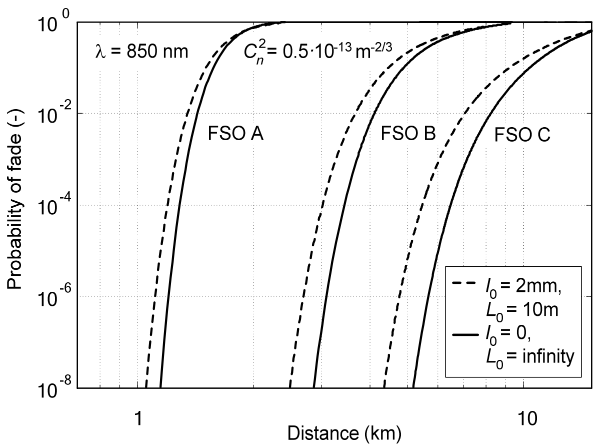

Fig. 4. Probability of fade vs. link distance calculated for different inner and outer scales of turbulence eddies.

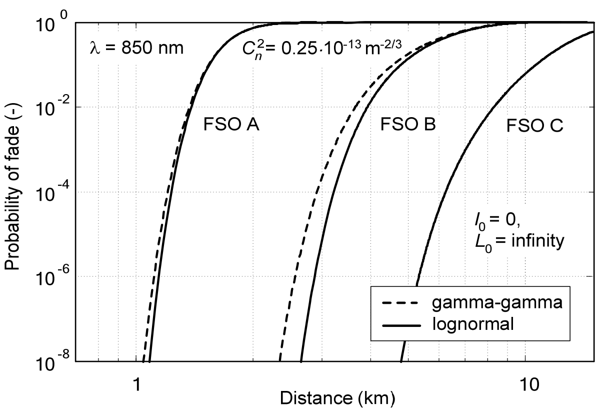

Fig. 5. Probability of fade vs. link distance calculated for both the gamma-gamma and the lognormal intensity distributions.

It is evident from Fig. 5 that with decreasing scintillation index (link distance decreasing below 5 km) the difference between both approximations decreases.

Although there are a few optical wavelengths suitable for transmission through the atmosphere, FSO designers usually use near-IR spectral windows around 850 nm, 1300 nm or 1550 nm. The reason for this choice is the good availability of lasers and photodiodes for these wavelengths. Which spectral window is more appropriate for reliable operation of FSO in turbulence can be found in Fig. 6 where the calculation was performed for FSO B.

It is evident that the dependence of the probability of fade on wavelength is relatively small. The best result was obtained for the spectral window around 850 nm, which is a typical result for strong scintillation, where the longer wavelengths correspond to the stronger scintillation [12].

The refractive-index structure parameter, which is used to determine the level of scintillation, varies over a wide range of values ($10^{-16} \le C_n^2 \le 10^{-13}$ [$m^{-2/3}$]). It is minimal at sunrise and sunset and reaches its maximum at midday and midnight, when the ground is warmer than the overlying air and the temperature gradient is generally the greatest.

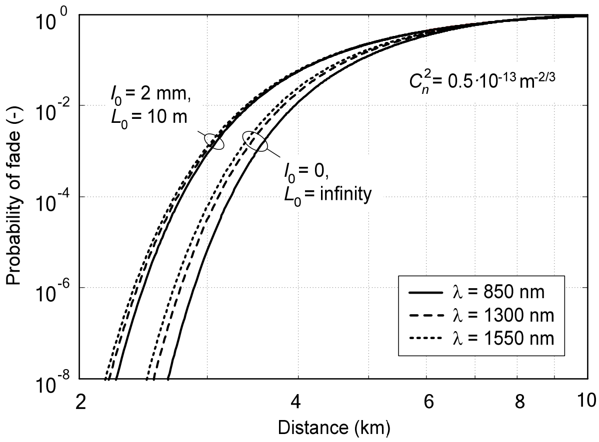

Fig. 6. Probability of fade vs. link distance calculated for different wavelengths.

The influence of C_n^2 on the probability of fade as calculated for FSO B is shown in Fig. 7. It can be seen that the probabilities of fade vary within a range of more than three decades.

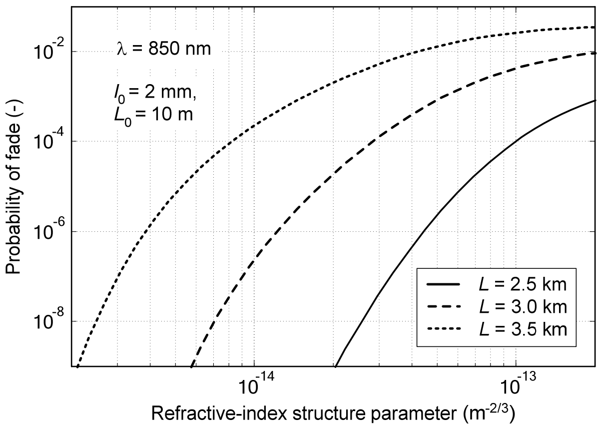

Fig. 7. Probability of fade vs. the refractive-index structure parameter.

The probability of fade depends on many parameters characterizing the atmospheric turbulence and FSO systems. Differences in results obtained for particular representatives of FSO links (Figs. 4 and 5) are caused mainly by considerably differing power link margin. But as it can be seen from Figs. 3, 6, and 7 very important role is also played by the scale of turbulence eddies, the effect of aperture averaging, and magnitude of the refractive-index structure parameter.

7. Conclusion

The aim of the paper has been to describe the influence of turbulence on terrestrial laser-beam communication, to evaluate the probability of fade of optical signal propagated through turbulent atmosphere, and to determine the influence of system parameters such as beam divergence, diameter of the receiving aperture, power margin, link distance and wavelength on eliminating the turbulence effect.

In this paper, unlike other works, the probability of fade was calculated for typical representatives of FSO links in dependence on the link distance. This approach takes into account two simultaneously changing parameters affecting the probability of fade. The first is the received optical power, which decreases with increasing link distance, and the second is the scintillation index, which with increasing link distance initially grows rapidly in the weak-fluctuation regime and then decreases slowly in the saturation regime. The results presented here give an idea of the reliability of free space communication ensured by a particular FSO deployed at various distances.

Many FSO systems are designed for correct operation in foggy conditions. These systems need a large power link margin to overcome the fluctuations of beam attenuation, which can reach tens of decibels per kilometer. Since the required power link margin can in many cases be ensured only by short link distances and the fade caused by scintillations appears only if the link distance is relatively large, these systems cannot be affected by turbulence. However, in many localities in the world such as desert or semi-desert areas, fog (or rain) occurs rarely. In these localities FSO systems can be deployed over long distances and the turbulence effect has to be taken into consideration.

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About Author…

Aleš PROKEŠ was born in Znojmo in 1963. He received the M.S. and the Ph.D. degree from the Brno University of Technology in 1988 and 2000, respectively. Since 2006 he has been working as an associate professor at the Department of Radio Electronics, Brno University of Technology. His research interests include signal processing, contactless velocity measurement and free space optical communication.