Invited paper

3D Metamaterial Based on a Regular Array of Resonant Dielectric Inclusions

Irina VENDIK, Mikhail ODIT, Dmitryi KOZLOV

St. Petersburg Electrotechnical University, 5 Prof. Popov Str., St. Petersburg, 197376, Russia

IBVendik@eltech.ru, MAOdit@mail.eltech.ru

Abstract. *The 3D regular lattice of bi-spherical dielectric resonant inclusions arranged in a cubic lattice as two sets of spheres made from the same dielectric material having different radii and embedded in a host dielectric material with lower dielectric permittivity was carefully investigated. The magnetic resonance corresponding to the first Mie resonance in the spherical particles is followed by forming a regular array of effective magnetic dipoles, and the structure of the identical spherical dielectric resonators can be designed as an isotropic μ-negative 3D-metamaterial. For the electric resonance it was found experimentally and by the simulation that the resonant response of the electric dipole was weakly pronounced and the μ-negative behavior was remarkably suppressed. To enhance the electric dipole contribution we considered another kind of the symmetry of the bi-spherical arrangement of the particles corresponding to the body-centered cubic symmetry instead of the symmetry of NaCl analog considered previously. Electromagnetic properties of a volumetric structure based on a regular lattice of identical cubic dielectric particles is also considered and analyzed as μ-negative metamaterial. The cubic particle based 3D-metamaterial is preferable for practical realization as compared with the spherical inclusions.*

Keywords

Metamaterial, resonant inclusion, Mie resonance, magnetic dipole, electric dipole, isotropic, backward wave, cubic resonators.

1. Introduction

Medium with simultaneously negative permittivity and permeability or so-called double negative medium (DNG) can be formed by a regular lattice of dielectric resonant inclusions, providing excitation of electric and magnetic dipoles. Dielectric disk, cylindrical, or spherical resonators are suitable for establishing the dipole moments. Metamaterials with desired values of permeability ±μ and permittivity ±ε are developed by exciting electric and magnetic resonant modes. In many practical cases, isotropic DNG structure is very attractive. Different ways to create the 3D isotropic DNG medium based on a regular lattice of resonant inclusions have been considered and published [1-6]. The 3D regular lattice of bi-spherical dielectric resonant inclusions was suggested in [2]. In this structure, the metamaterial medium is composed of two sets of spherical particles made of the same dielectric material embedded in a host dielectric material. The spheres differ by radius. The dielectric constant of the spherical particles is much larger than that of the host material. By combining two sets of the spheres with suitable radii, different modes can be simultaneously exited in the spheres: the magnetic resonance mode giving rise to the magnetic dipole momentum and the electric resonance mode being responsible for the electric dipole momentum. These moments create the negative permeability and permittivity in a limited frequency range near the resonant frequency. In [3-5], it was suggested that the dielectric resonators do not interact and for the 3D structure the responses of the both spherical particles are superimposed. By full-wave simulation and experimental investigation, it was found that the resonance response of the magnetic dipole is very effective and the μ -negative isotropic metamaterial can be designed as a regular array of dielectric spherical particles with the first Mie-resonance. At the same time, the electric dipole corresponding to electric resonance mode is weakly pronounced. As a consequence, the ε -negative behavior is blurred [5].

Detailed theoretical description based on full-wave analysis was performed in [6] for different all-dielectric structures of metamaterial: single-negative (SN) medium based on the spherical particles; bi-spherical DNG medium based on an array of the dielectric spherical particles of the same radius made of two materials differing in dielectric permittivity; a set of identical discs and discs made of two different dielectric materials etc. The structures based on the discs form 2D SN and DNG metamaterials. Interesting 2D structures based on cylindrical resonator arrays situated in a parallel-plate waveguide have been discussed and experimentally verified in [7-8]. In these structures, the DNG properties are provided by magnetic resonance in the cylinders (magnetic dipole, μ -negative behavior) and by ε -negative response of electromagnetic wave in a parallel-plate metallic waveguide with TE_n modes below the cut-off frequency.

Recently the experimental investigation of isotropic metamaterials based on resonant dielectric inclusions has been reported [9-11]. The resonant μ-negative response was registered in the 3D structure based on a regular array of dielectric cubes in [9] and the 2D array of cubes in [10]. The 3D DNG material was realized as a set of dielectric spherical particles regularly distributed in a metallic wire frame exhibiting cubic symmetry. The wire frame provides an environment with evanescent waves followed by the effective negative permittivity, which in combination with the resonant dielectric particles giving rise to magnetic dipoles and negative permeability leads to a propagating wave with a negative phase velocity i.e. forms the 3D isotropic DNG medium.

All these achievements support the fruitful idea of a realization of the isotropic metamaterial using dielectric resonant inclusions. In order to improve the performance of the all dielectric DNG medium with cubic symmetry based on spherical dielectric particles, we suggest the new structures obtained by changing the symmetry of the single cell of the bi-spherical structure to enhance the contribution of the electric resonance in the effective dielectric permittivity. In this paper, we consider the 3D DNG bi-spherical structure based on the single cell belonging to the body-centered cubic symmetry exhibiting higher packing density for the same distance between adjacent resonant particles as compared with the face-centered cubic lattice previously analyzed in [2-5]. The regular 3D-structure based on cubic resonant inclusions is also considered as the candidate for a practical realization of the μ-negative isotropic metamaterial.

2. 3D Isotropic Bi-spherical Metamaterial of Cubic Symmetry with High Packing Density

Single cubic cells of isotropic metamaterial based on two types of spherical resonators are presented in Fig 1. Different ways of particle packing are possible. The face-centered NaCl like structure (Fig. 1a), body-centered structure (Fig. 1b) and other type of face-centered cubic lattice (Fig. 1c) are all members of the cubic system of symmetry pertaining to the class m3m [3, 12].

In the case of cubic symmetry, the second rank tensors of all physical parameters of the media are diagonal and have the components of the same values [12]. Thus the permittivity and permeability tensors are written in the following form:

$$\varepsilon = \begin{vmatrix} \varepsilon_{eff} & 0 & 0 \\ 0 & \varepsilon_{eff} & 0 \\ 0 & 0 & \varepsilon_{eff} \end{vmatrix}, \quad \mu = \begin{vmatrix} \mu_{eff} & 0 & 0 \\ 0 & \mu_{eff} & 0 \\ 0 & 0 & \mu_{eff} \end{vmatrix} \qquad (1)$$

where the sub-indices *eff* are introduced to stress that the permittivity and permeability are obtained as a result of averaging electric and magnetic polarization of spherical particles embedded in the matrix. The result of averaging the polarization of the spherical particles embedded in the matrix depends on the volume of the matrix falling on each particle considered.

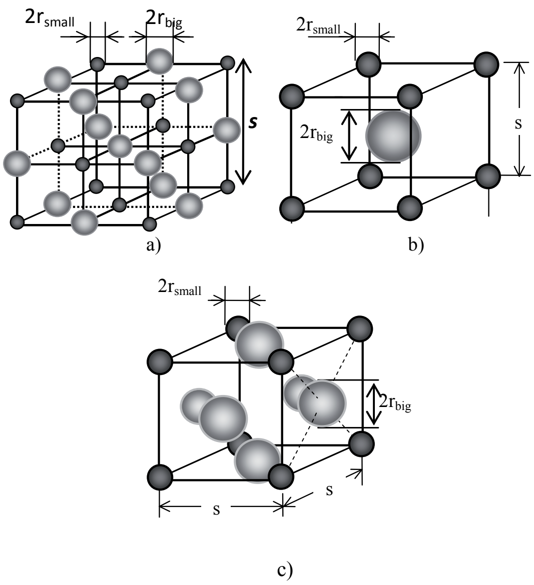


Fig. 1. a) Face centered NaCl cell; b) body-centered single cell; c)face-centered single cell.

The structures considered are characterized by different minimal distances between the particles of the same/different dimensions. The comparative Tab. 1 shows difference in these parameters for different packing factors for the same distance *a* between adjacent particles. Here *s* is the crystallographic lattice constant. The volume fraction is defined as the ratio of the volume of the sphere and the total volume of cube limiting the space around one sphere and $v_f = 4/3\ \pi r^3/s^3$ is the volume fraction for one sphere in the space limited by the cube with the side *s*, *r* is the radius of the spherical particle.

The distance between the resonant particles is of crucial importance. Mutual interaction of closely positioned particles leads to shift of the resonance frequency and distortion of the resonant characteristic. The analytical model

Structure	Lattice constant, s	Minimal distance between the big particles	Minimal distance between the small particles	Minimal distance between different particles	Volume fraction for the big particles	Volume fraction for the small particles
Face-centered (NaCl-like)	$2a$	$s\sqrt{2}/2$	$s\sqrt{2}/2$	$s/2$	$4v_f$	$4v_f$
Body-centered	$2a/\sqrt{3}$	s	s	$s\sqrt{3}/2$	v_f	v_f
Face-centered	$2a/\sqrt{2}$	$s\sqrt{2}/2$	s	$s\sqrt{2}/2$	$3v_f$	v_f

Tab. 1. Distances between particles for different structures.

of the diffraction of a plane electromagnetic wave on a dielectric spherical particle [5] describes the effective parameters of the bi-spherical isotropic metamaterial under the assumption that there is no mutual interaction between the particles. Neglecting mutual coupling is reasonable in case of appropriate distance between the particles. Simulation of the electromagnetic waves outside the spherical particles demonstrates the strong *e*-times attenuation of the electromagnetic wave outside the sphere over the distance equal to its diameter. At the same time, to strengthen the DNG effect the particles should be placed as close as possible to each other. If particles are positioned too far, the DNG effect becomes negligibly small.

Simulation showed that the optimal distance between the resonant particles is achieved for the body-centered structure (Fig. 1b). For this type of isotropic structure there is maximum packing density and the distance between the adjacent particles (of different kind) is still big enough to neglect the effect of their interaction.

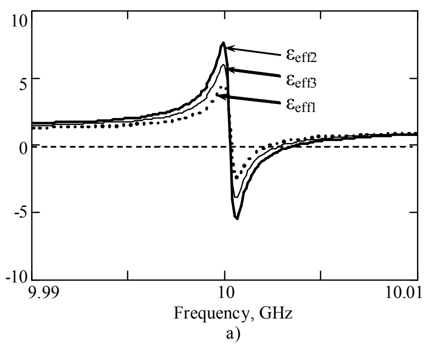
Frequency dependent effective permittivity and permeability were calculated for three different structures:

$$\varepsilon_r^{(eff)}(f) = \frac{N}{a^3}\varepsilon_p \frac{3}{2} F_{eps}(f, r_{big}) \cdot b^{(t)}(f, r_{big}) + \varepsilon_h , \quad (2)$$

$$\mu_r^{(eff)}(f) = \frac{n}{a^3}\sqrt{\frac{\varepsilon_p}{\varepsilon_h}}\frac{3}{2} F_{mu}(f, r_{small}) \cdot a^{(t)}(f, r_{small}) + \mu_h . \quad (3)$$

Here a is the distance between closest particles of different radii, ε_p (μ_p), ε_h (μ_h) are the permittivity (permeability) of the particle and the host material respectively, r_{small} and r_{big} are the radii of the resonators, f is the incident electromagnetic wave frequency, $a^{(t)}$ and $b^{(t)}$ are the amplitudes of spherical wave functions, F_{eps} and F_{mu} are the results of integration of the electric and magnetic field components over the particle volume, N = 0.5, 0.65, 0.35 and n = 0.5, 0.65, 0.65 for the structures in Fig. 1a, 1b, 1c respectively.

In Fig. 2, the results of calculation of the effective parameters of the bi-spherical metamaterial are presented for three different structures with parameters: ε_p = 400, ε_h = 1, $\mu_p = \mu_h$ = 1, r_{small} = 0.748 mm, r_{big} = 1.069 mm, and s = 4 mm, no losses were taken into account. The results are obtained by a consideration of the diffraction of the plane electromagnetic wave on a single dielectric spherical particle.



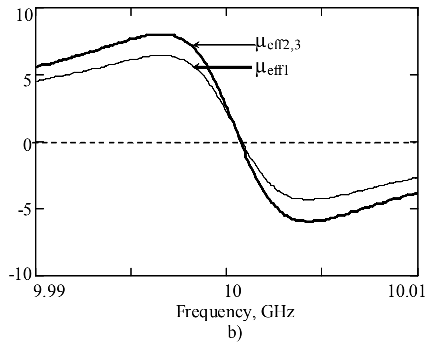


Fig. 2. Effective a) permittivity and b) permeability for face-centered Na-Cl structure (sub-index 1) the face-centered structure (sub-index 2) and body-centered structure (sub-index 3).

In order to demonstrate the effectiveness of the body-centered lattice, let us consider the simplified case of one-dimensional structure (Fig. 3) using full-wave analysis. The single cell is centered by the spherical particle of bigger value of the radius r_{big} = 1.05 mm and is surrounded by the particles of smaller value of the radius r_{small}=0.748 mm. The distance between the particles of the same size is s=4 mm. To fit the conditions of the translation symmetry, the one-dimensional structure is bounded by a perfect electric conductor (PEC) and a perfect magnetic conductor (PMC). The particle dimensions are chosen to provide the magnetic resonance in the smaller spheres and the electric resonance in the bigger particles at the same frequency. It results in creation of magnetic and electric dipoles followed by the negative effective permeability and effective permittivity in a limited frequency range. That leads to appearance of transmission of electromagnetic wave in the frequency range near the resonant frequency (Fig. 4).

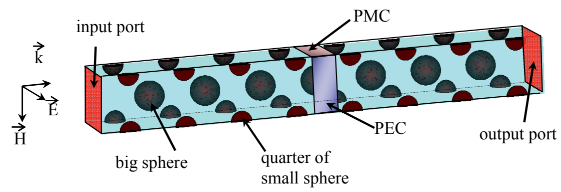


Fig. 3. 1-D structure of bi-spherical isotropic metamaterial. Radii of small and big spheres are 0.748 mm and 1.05 mm respectively. Boundary conditions are provided by perfect magnetic (PMC) and perfect electric (PEC) walls.

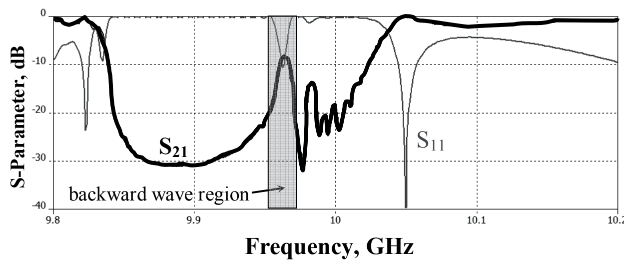


Fig. 4. Transmission S_{21} and reflection S_{11} coefficients for the structure depicted in Fig. 3. Gray area is the frequency range, where the resonant frequencies of two types of resonances coincide and backward wave is observed.

The full-wave analysis of the electromagnetic wave propagation confirms the existence of the backward wave. In Fig. 5, the magnetic field pattern is shown for four different moments of the period of time. It is clearly seen that the wave propagates from the right side to the left, whereas the incident wave enters the structure from the left side.

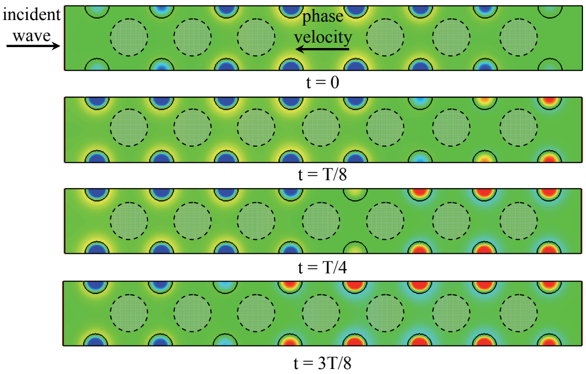


Fig. 5. Magnetic field patterns for four different moments of the period of time. The incident wave enters the structure from the left side, whereas the wave inside structure propagates from the right side to the left.

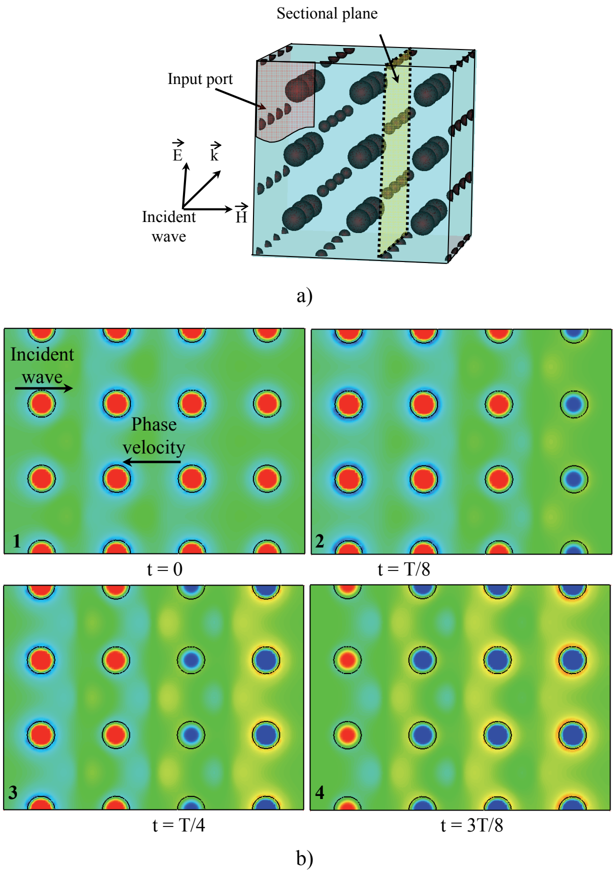


Fig. 6. a) Bi-spherical structure of metamaterial with body-centered cubic symmetry. Radii of small and big spheres are 0.748 mm and 1.05 mm respectively. Distance between the same particles is 4 mm. b) Magnetic field patterns for the section plane shown by yellow in (a) for 4 different moments of the period of time.

The 3D volumetric structure corresponding to the body-centered symmetry is shown in Fig. 6. The small and big spherical particles are situated in different planes. The same is related to the magnetic and electric dipoles. The PECs and PMCs are used for providing boundary conditions for this limited in volume structure. The dimensions of the spherical particles and spacing between the identical particles are the same as in the previous one-dimensional case. Magnetic field pattern for the section plane shown by yellow in Fig. 6 a) is depicted in Fig. 6 b) for four different moments of the time period. Again the backward wave is observed. The response of the structure to the incident plane electromagnetic wave is presented in Fig. 7. In the limited frequency range one can observe the wave transmitting through the DNG structure. Hence, the increasing packing factor leads to enhanced electric dipole contribution as compared with our previously obtained results for NaCl structure [4], [5].

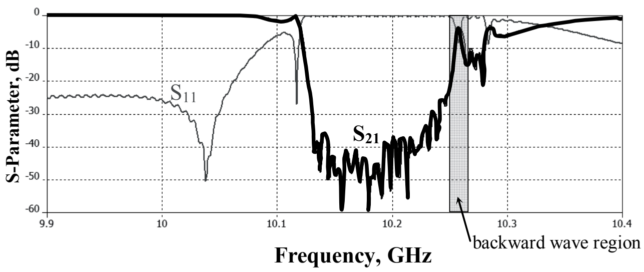


Fig. 7. Transmission S_{21} and reflection S_{11} coefficients for the modeled structure depicted in Fig. 5. Gray area is the frequency range where two types of resonances coincide and backward wave is observed.

3. 3D Metamaterial Based on Cubic Dielectric Resonant Inclusions

As it was shown, the 3D regular lattice based on resonant spherical inclusions exhibits properties of the DNG material. Unfortunately the practical realization of such a structure is very problematic. As we know, there is the only publication demonstrating the experimental results of measuring transmission characteristic of the DNG medium based on the spherical inclusions inserted in the wire frame [11]. The experimental investigation of the 3D structure [9] and 2D structure [10] of metamaterial based on cubic resonant dielectric inclusions confirmed the μ-negative properties of these artificial media. In Fig. 8, the regular lattice of the dielectric cubes is shown. The cubic particles are distantly positioned and the mutual coupling is negligibly small. In this case the frequency response of the structure is the same as for the material based on the identical spherical inclusions: the stop-band occurs near the resonant frequency, which corresponds to the μ-negative behavior of the material. The magnetic field distribution in the resonant particles for the first magnetic resonance in the spherical and cubic particles is shown in Fig. 9. Evidently the magnetic dipole in the both cases is nearly the same. The results of full-wave simulation of the transmission through this structure are presented in Fig. 10 exhibiting the stop-band near the resonant frequency.

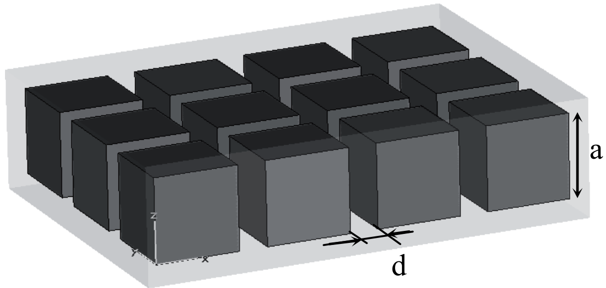


Fig. 8. The lattice of dielectric high permittivity cubes in the dielectric matrix of lower permittivity. The following parameters of cubes are used: size a = 3 mm, ε_r = 1000, dielectric cubes tan δ = 0.001, the distance between the cubes d = 1 mm.

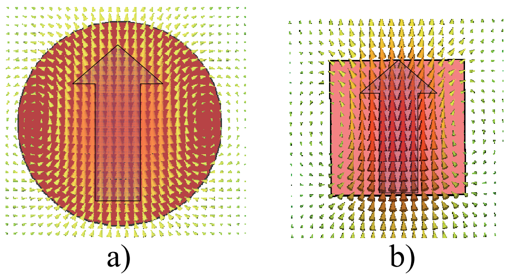
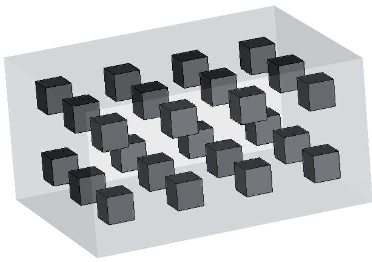
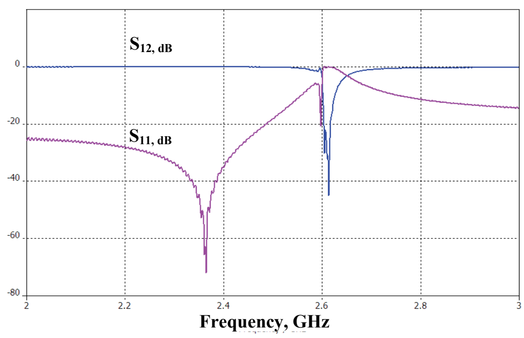


Fig. 9. Magnetic resonance in a) the dielectric spherical particle b) the dielectric cubic particle.



a)

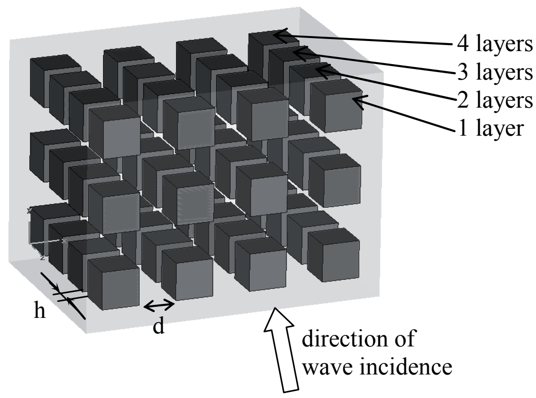


b)

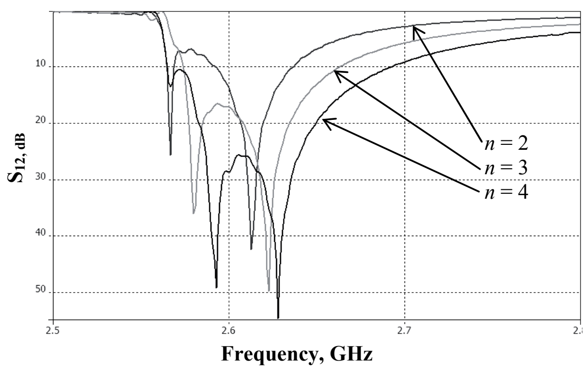
Fig. 10. a) 2 layer dielectric cube structure. b) Transmission and reflection coefficient for this structure d = 5 mm, h = 5 mm.

It was shown by the simulation that for the structure with a finite number of the particles the resonant response depends on the number of layers: the bigger number of the layers used, the higher is the resonant frequency and the wider is the resonant response. That means that the coupling between the cubic particles influences the characteristics of the material.

The regular lattice of the cubic particles can be realized as a multilayer structure (for example multilayer structure based on low-temperature cofired ceramic, LTCC). In general case, the 3D lattice can be designed being regular in plane with desired spacing between the layers. This spacing is determined by the ceramic layer thickness, whereas the lattice period in plane is determined by the mask dimensions. The results of modeling of the resonant characteristics of the structure based on the cubic dielectric resonators with the cube edge 1 mm and the distance between the cubes 3 mm in the plane are shown for different number of the layers. The spacing between the layers is 1.2 mm. The resonant characteristics are split due to strong coupling between the layers containing regular 2D lattice of the cubes (Fig. 11). The more number of the layers containing the cubic resonant inclusions is used, the more effective is the reflection from this artificial material. That can be used as an effective impedance surface without using any conducting or magnetic materials. We can expect that the cubic inclusion based metamaterial will be realized in multilayer technology in the nearest future.



a)



b)

Fig. 11. a) Dielectric cubes structure with several number of cube layers. b) Transmission coefficient for different number n of layers, d = 3 mm, h = 1.2 mm.

4. Conclusions

A double negative artificial material based on bi-spherical dielectric spherical resonators has been investigated. A new structure with body-centered unit cell of higher packaging factor has been introduced in order to strengthen the DNG effect. The isotropy of the metamaterial – main challenging advantage for the material manufacturing – has been kept. Backward wave existence in the new structure has been confirmed by the numerical analysis. A concept of metamaterial based on cubic dielectric inclusions is more preferable for a practical realization. Full wave simulation results revealed the μ-negative behavior of the structures based on the cubic dielectric resonators. The multilayer ceramic technology can be used for manufacturing of this metamaterial.

References

[1] HOLLOWAY, C., KUESTER, E. A double negative composite medium composed of magnetodielectric spherical particles embedded in a matrix. *IEEE Trans. on Antennas and Propagation*, 2003, vol. 51, no. 10, p. 2596 - 2603.

[2] VENDIK, O. G., GASHINOVA, M. S. Artificial double negative (DNG) media composed by two different dielectric sphere lattices embedded in a dielectric matrix. In *Proceedings of European Microwave Conference EuMC34*. Paris (France), 2004, p. 1209.

[3] VENDIK, I., VENDIK, O., GASHINOVA, M. Artificial dielectric medium possessing simultaneously negative permittivity and magnetic permeability. *Tech. Phys. Lett.,* 2006, vol. 32, p. 429-433.

[4] VENDIK, I., VENDIK, O., KOLMAKOV, I., ODIT, M. Modeling isotropic DNG media for microwave applications. *Opto-Electronics Review,* 2006, vol. 14, p. 179.

[5] ODIT, M., VENDIK, I., VENDIK, O. 3D isotropic metamaterial based on dielectric resonant spheres. In *Proceedings of Metamaterials 2007.* Rome (Italy), 2007, p. 946.

[6] AHMADI, A., MOSALLAEI, H. Physical configuration and performance modeling of all-dielectric metamaterials. *Phys. Rev. B*, 2008, vol. 77, 045104 (1-11).

[7] SEMOUCHKINA, E. A., SEMOUCHKIN, G. B., LANAGAN, M., RANDALL, C. A. FDTD study of resonance processes in metamaterials. *IEEE Trans. Microwave Theory Tech*., 2005, vol. 53, p. 1477.

[8] UEDA, T., LAI1, A., ITOH, T. Negative refraction in a cut-off parallel-plate waveguide loaded with two-dimensional lattice of dielectric resonators. In *Proceedings of European Microwave Conferenc EuMC36.* 2005, p. 435.

[9] QIAN ZHAO, LEI KANG, DU, B., ZHAO, H., XIE, Q., HUANG, X., LI, B., ZHOU, J. , LI, L. Experimental demonstration of isotropic negative permeability in a three-dimensional dielectric composite. *Phys. Rev. Lett*., 2008, vol. 101, p. 027402 (1-4).

[10] SHIBUYA, K., TAKANO, K., MATSUMOTO, N., IZUMI, K., MIYAZAKI, H., JIMBA, Y., HANGYO, M. Terahertz metamaterials composed of TiO_2 cube arrays. In *Proc. Metamaterial 2008.* Pamplona (Spain), 2008, p. 777.

[11] XIAOBING CAI, RUI ZHU, GENGKAI HU Experimental study for metamaterials based on dielectric resonators and wire frame. *Metamaterials*, 2008, vol. 2, no. 4, December 2008, p. 220-226.

[12] NYE, J. F. *Physical Properties of Crystals*. Oxford at the Clarendon Press, 1964.

About Authors ...

Irina B. VENDIK received the electronics engineer diploma and the candidate of Sc. (Ph.D.) degree from the Leningrad Electrical Engineering Institute (now St. Petersburg Electrotechnical University), St.-Petersburg, Russia, in 1959 and 1964 respectively, and the D. Sc. (Phys.) degree from A.F. Ioffe Physicotechnical Institute, St.-Petersburg, Russia, in 1990. She is currently a Professor in the Dept. of Microelectronics and Radio Engineering, and the Head of the Microwave Microelectronics Laboratory, St.-Petersburg Electrotechnical University, St.-Petersburg, Russia. Her general research interests have been in solid state physics and microwave electronics. The last years she has been a leader of the group of the ETU involved in FP6 and FP7 projects of the EC. I. Vendik is a member of IEEE (USA) from 1996 and EuMA (2006). I. Vendik is honored educator of the Russian Federation (1999).

Mikhail ODIT was born in 1982 in St. Petersburg. He was graduated in Radio Engineering from St. Petersburg Electrotechnical University "LETI" in 2005. His research interests are concentrated in the field of artificial materials and microwave applications of metamaterials. The last years he was involved to the project of Network of Excellence “Metamorphose” FP6 EC.

Dmitryi KOZLOV was born in Borisov in 1988. His research interests include metamaterials for microwave applications.