SPICE Model of Memristor with Nonlinear Dopant Drift

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Abstract. *A mathematical model of the prototype of memristor, manufactured in 2008 in Hewlett-Packard Labs, is described in the paper. It is shown that the hitherto published approaches to the modeling of boundary conditions need not conform with the requirements for the behavior of a practical circuit element. The described SPICE model of the memristor is thus constructed as an open model, enabling additional modifications of non-linear boundary conditions. Its functionality is illustrated on computer simulations.*

Keywords

Memristor, drift, window function, SPICE.

1. Introduction

On May 1st, 2008, a research group from Hewlett-Packard laboratories, led by Dr. Williams, published a report [1] about the manufacture of a memristor, the so-called fourth elementary passive element. Its existence was predicted in 1971 by professor Chua in his famous paper [2]. The future of this circuit element seems to be promising. Intensive research is under way in HP labs, focusing on revolutionary applications of the memristor in ultra-dense memory cells (RRAM, Resistive Random-Access Memory), when this element acts as a crossbar switch [3]. Also, the utilization of memristors in the analog mode as synapses in a neural computing architecture is anticipated because such a technology seems to be dense enough to simulate processes in the human brain [4].

Over the 10 months since the announcement of the above break-through in [1], numerous papers have appeared with the aim to analyze the elementary attributes of the memristor as a passive circuit element, manufactured in HP labs [5], [6]. The input data for such analyses, i.e. information about the physical model of the memristor, were taken particularly from the original work [1]. Since real samples of the memristor are inaccessible to most researchers, it is useful to have a computer model of the memristor as a tool for speeding-up the analysis of the behavior and developing applications of this interesting circuit element via simulation experiments.

With regard to the fact that memristor is specified as the fourth, newly discovered passive circuit element, supplementing the well-known *R*, *C*, *L* triplet (resistor, capacitor, inductor), it is logical to suggest extending the model libraries of SPICE-family simulating programs just with the model of the memristor.

The purpose of this paper is a description of the working SPICE model of the memristor, manufactured in HP labs. The paper structure is as follows: Section 2, which follows this Introduction, summarizes the information about the physical and mathematical models of the memristor, already published in [1] and [5]. Section 3 introduces the SPICE model of the memristor, which starts from the mathematical model in Section 2. Section 4 is devoted to the demonstrations of SPICE simulations that are based on the proposed model. Some open problems and pending issues of the boundary effects modeling are discussed in Section 5.

2. Model of the Memristor from HP Labs

The physical model of the memristor from [1], shown in Fig. 1, consists of a two-layer thin film (size $D \approx 10$ nm) of TiO_2, sandwiched between platinum contacts. One of the layers is doped with oxygen vacancies and thus it behaves as a semiconductor. The second, undoped region, has an insulating property. As a consequence of complex material processes, the width w of the doped region is modulated depending on the amount of electric charge passing through the memristor. With electric current passing in a given direction, the boundary between the two regions is moving in the same direction. The total resistance of the memristor, R_{MEM}, is a sum of the resistances of the doped and undoped regions,

$$R_{MEM}(x) = R_{ON} x + R_{OFF}(1-x), \qquad (1)$$

$$\text{where } x = \frac{w}{D} \in (0,1) \qquad (2)$$

is the width of the doped region, referenced to the total length D of the TiO_2 layer, and R_{OFF} and R_{ON} are the limit values of the memristor resistance for $w=0$ and $w=D$. The ratio of the two resistances is usually given as 10^2 - 10^3.
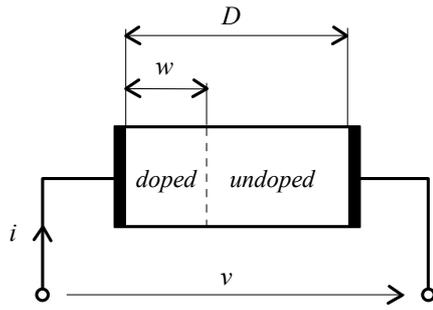
Fig. 1. Memristor model according to [1].

Fig. 2. The window function (5) for different values of integer p.

The Ohm's law relation is applicable between the memristor voltage and current:

$$v(t) = R_{MEM}(w)\, i(t). \qquad (3)$$

The speed of the movement of the boundary between the doped and undoped regions depends on the resistance of doped area, on the passing current, and on other factors according to the state equation [1]

$$\frac{dx}{dt} = k\, i(t)\, f(x), \quad k = \frac{\mu_v R_{ON}}{D^2} \qquad (4)$$

where $\mu_v \approx 10^{-14}$ m^{2}s^{-1}V^{-1} is the so-called dopant mobility. As mentioned in [1], in nanoscale devices, small voltages can yield enormous electric fields, which can secondarily produce significant nonlinearities in ionic transport. These nonlinearities manifest themselves particularly at the thin film edges, where the speed of the boundary between the doped and undoped regions gradually decreases to zero. This phenomenon, called nonlinear dopant drift, can be modeled by the so-called window function $f(x)$ on the right side of (4). A concrete window function that would correspond to the memristor from HP labs is not available at this moment.

The paper [5] proposes the window function in the following form:

$$f(x) = 1 - (2x - 1)^{2p} \qquad (5)$$

where p is a positive integer.

The form of function (5) guarantees zero speed of the x-coordinate when approaching either boundary. Moreover, the differences between the models with linear and nonlinear drift disappear when p increases.

3. SPICE Model

State equation (4) and port equation (3) of the memristor can be modeled by the block-oriented diagram in Fig. 3. The memory effect of the memristor is modeled via a feedback-controlled integrator. With regard to the limiting boundary conditions, it stores the effects of the passing current, and controls the memristor resistance via modifying the boundary position. The nonlinear drift and the influence of the boundary conditions are modeled by the feedback via the nonlinear window function $f(\ )$.
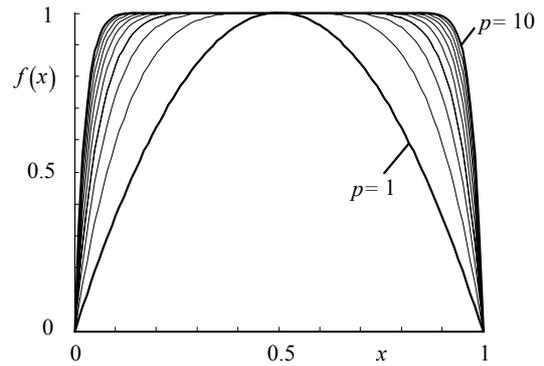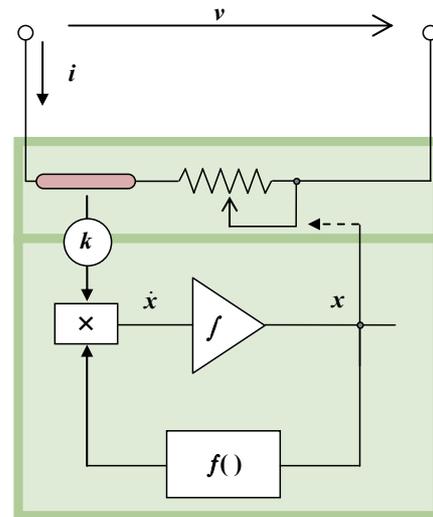

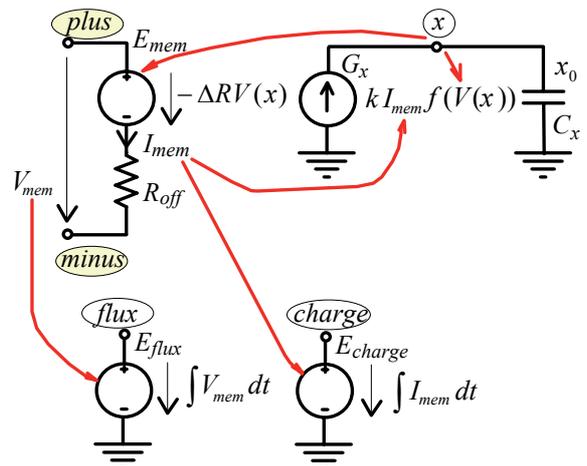**Fig. 3.** Block diagram of the memristor model.

Fig. 4. Structure of the SPICE model.

The structure of the SPICE model is shown in Fig. 4. The relation between the memristor voltage and current is modeled on the basis of the modified equation (1):

$$R_{MEM}(x) = R_{OFF} - x\Delta R, \ \ \Delta R = R_{OFF} - R_{ON}. \qquad (6)$$

In Fig. 4, equation (6) corresponds to the R_{OFF} resistor in series with the E-type voltage source whose terminal volt-

age is controlled according to the formula "$-x\Delta R$". The normalized width x of the doped layer is modeled by the voltage $V(x)$ of the capacitor C_x, which serves as an integrator of the quantities on the right side of state equation (4). The initial state of the normalized width of the doped layer x_0, which is modeled as initial voltage of the capacitor, is determined by the initial resistance R_{INIT} of the memristor according to the formula, derived from (6):

$$x_0 = \frac{R_{OFF} - R_{INIT}}{\Delta R}. \tag{7}$$

The model is implemented as a SPICE subcircuit with parameters which can pass the following values into the subcircuit as arguments: the initial R_{INIT} resistance, the R_{OFF} and R_{ON} resistances, the width of the thin film D, the dopant mobility μ_v, and the exponent p of the window function. The list of the SPICE subcircuit mentioned below includes conventional model of the window function according to Joglekar [5], which is open to any modifications of the functions describing the nonlinear drift, including the import of experimentally acquired data.

The SPICE model is also complemented with direct computation of the integral quantities which define the memristor, i.e. the time integrals of electrical voltage (flux) and of electric current (charge). These quantities belong to the results of the SPICE analysis, being available as voltages of the internal controlled sources E_{flux} and E_{charge}.

4. Demonstrations of SPICE Analyses

The SPICE model from Section 3 was used for the simulation of experiments described in [1]. The appropriate results are shown in Figs 5 (a), (b), and (c). In each case, the memristor is driven by a voltage source.

As follows from the waveforms of voltages V_x in Figs 5 (a, b), the memristor works in a regime such that the boundary between the doped and undoped layers does not approach the edges with dominant nonlinear effects. More detailed simulations prove the fact from [1] that the typical loops in I-V curves are gradually suppressed when increasing the frequency of the applied voltage.

Fig. 5 (c) demonstrates the simulation results for lower R_{OFF}/R_{ON} ratios which cause, in conjunction with the sufficiently high amplitude of the applied voltage, the hard-switching cases [1]. As shown in Fig. 5 (c), the normalized width x of the doped region is switched between the low and high levels near the limiting values of 0 and 1. The corresponding typical pattern of the I-V characteristic in Fig. 5 (c) is also in agreement with [1].

The charge&flux curves in Fig. 5, i.e. relationship between the time-domain integrals of electric current&voltage of the memristor confirm the well known fact that there is a one-to-one correspondence between them, in spite of the I-V hysteresis effects.

The simulation results in Fig. 5 agree well with the graphs published in [1]. This, however, cannot be claimed for the remaining results, given in [1] in Figs 3 (a, b), which show the simulations of the memristor behavior with and without dynamic negative differential resistance in conditions of linear drift. An analysis, given in the following Section, indicates that the cause can consist in the current methodology of modeling the boundary conditions via the window functions.

* HP Memristor SPICE Model * For Transient Analysis only * created by Zdenek and Dalibor Biolek ************************ * Ron, Roff - Resistance in ON / OFF States * Rinit - Resistance at T=0 * D - Width of the thin film * uv - Migration coefficient * p - Parameter of the WINDOW-function * for modeling nonlinear boundary conditions * x - W/D Ratio, W is the actual width * of the doped area (from 0 to D) * .SUBCKT memristor Plus Minus PARAMS: + Ron=1K Roff=100K Rinit=80K D=10N uv=10F p=1 ** * DIFFERENTIAL EQUATION MODELING * ** Gx 0 x value={ I(Emem)*uv*Ron/D^2*f(V(x),p)} Cx x 0 1 IC={(Roff-Rinit)/(Roff-Ron)} Raux x 0 1T	* RESISTIVE PORT OF THE MEMRISTOR * **************************** Emem plus aux value={-I(Emem)*V(x)*(Roff-Ron)} Roff aux minus {Roff} *** *Flux computation* *** Eflux flux 0 value={SDT(V(plus,minus))} *** *Charge computation* *** Echarge charge 0 value={SDT(I(Emem))} *** * WINDOW FUNCTIONS * FOR NONLINEAR DRIFT MODELING * *** *window function, according to Joglekar .func f(x,p)={1-(2*x-1)^(2*p)} *proposed window function ;.func f(x,i,p)={1-(x-stp(-i))^(2*p)} .ENDS memristor

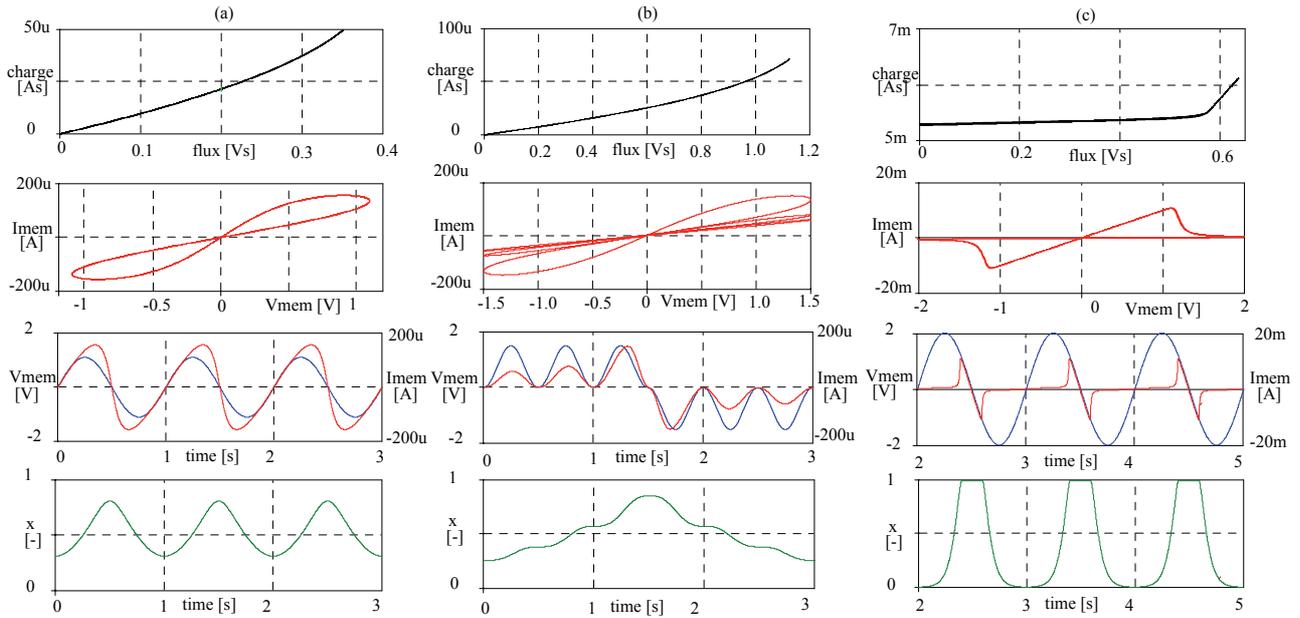**Tab. 1.** SPICE model of memristor. SDT and STP are standard PSPICE functions (time-domain integration, unity step).

Fig. 5. Memristor with the parameters R_{ON}=100 Ω, p=10, driven by a voltage: (a) harmonic with an amplitude of 1.2 V and a frequency of 1 Hz, (b) $\pm V_0\,(\sin\omega_0 t)^2$ with V_0= 1.5 V and f_0=1 Hz, (c) harmonic with an amplitude of 2 V and a frequency of 1 Hz. Other parameters are: (a) R_{OFF}=16 kΩ, R_{INIT}=11 kΩ, (b) R_{OFF}=38 kΩ, R_{INIT}=28 kΩ, (c) R_{OFF}=5 kΩ, R_{INIT}=1 kΩ. Simulation in Fig. (c) confirms the hard switching effects [1].

5. Modeling the Boundary Effects – the Pending Issue

The testing of the developed SPICE model of the memristor manufactured in HP laboratories under hard switching conditions pointed out two problems, associated with the so-called boundary effects, both related to the way of defining window function (5).

The first problem consists in the fact that when setting the memristor to the terminal state R_{ON} or R_{OFF}, no external stimulus can change this state back to another value. In other words, such a memristor would be bound to hold its state forever. This conclusion is a direct consequence of state equation (4) and the zero-value of the window function in either boundary state.

According to the currently available information, the memristor from HP labs "remembers" the x-coordinate of the boundary between the two layers, not the amount of electric charge that passed through it. This coordinate is proportional to the charge only within the active area of the memristor. However, the second problem of window function (5) consists in modeling the memristor as a component which exactly remembers the entire charge which is passing through. This conclusion also follows from state equation (4): if window function (5) is only a function of the x variable, then the electric charge

$$q_1 = \int_{x_0}^{x_1} \frac{d\xi}{k\,f(\xi)} \tag{8}$$

is necessary for transposing the memristor state from x_0 to x_1, and the same charge but with different sign,

$$q_2 = \int_{x_1}^{x_0} \frac{d\xi}{k\,f(\xi)} = -\int_{x_0}^{x_1} \frac{d\xi}{k\,f(\xi)} = -q_1, \tag{9}$$

is required for its return from x_1 to x_0. When driving the memristor by a constant current within a time interval, e.g. one minute, the same time interval, one minute, would be necessary for restoring the state before this driving, regardless of the fact that the memristor could be in its terminal state all the time when the current flows. Such delays are unambiguously confirmed by SPICE simulations of the current-driven memristors. Unfortunately, such an operating regime of the memristor is not referenced in the current literature.

As results from the above, the window function can be considered a measure of how precisely the memristor stores the amount of electric charge: the memory effect is lost at the boundaries. When the current direction is reversed at this moment, the boundary starts to move in the opposite direction regardless of the past, which is lost, i.e. along another curve.

The above discrepancy between the behavior of the model and the requirements for the operation of a real circuit element can be resolved by designing a modified window function which models the fact that the boundary speeds of approaching and receding from the thin film limits are different.

The following function (10) with the graph in Fig. 6 meets the above requirements:

$$f(x) = 1 - \left(x - \text{stp}(-i)\right)^{2p} \tag{10}$$

where p is a positive integer, i is the memristor current, and

$$\text{stp}(i) = \begin{cases} 1 & \text{pro } i \geq 0 \\ 0 & \text{pro } i < 0 \end{cases}. \qquad (11)$$

The current is considered to be positive if it increases the width of the doped layer, or $x \rightarrow 1$. Function (10) is zero at either edge. Increasing the p yields a flat window function with steep throughs to zeros at $x = 0$ and $x = 1$.
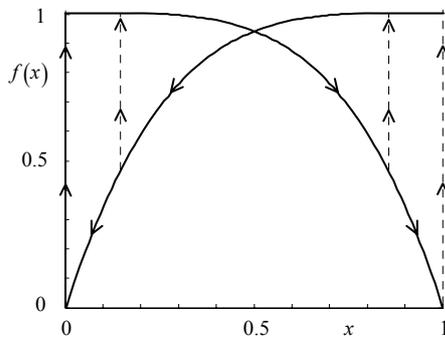

Fig. 6.   Proposal of a new window function, see (10), $p = 2$.

Our hitherto results indicate that all the conclusions from [1] can be proved by means of the above approach, but with the exception of the hard switching effects governed by nonlinear ionic drift which is characterized by the symmetrical hysteresis loop in Fig. 5 (c). On the other hand, this case is truly modeled via the window function according to Joglekar whose general utilization is problematic in the light of the above analysis. It has turned out that subsequent development of the memristor model should consist in a proper modification of the window function which should not depend only on the state variable x. However, an acceleration of such a development may be expected as soon as detailed information about the nonlinear ionic drift is released.

6.   Conclusions

The SPICE model of the memristor, made up on the basis of state equations and equations modeling the boundary effects, offers results which are in good agreement with a part of the hitherto published experiments. It should be noted that the differences between the behavior of the model in some concrete regimes of the operation and the anticipated behavior of the real circuit element may be due to the current methods of modeling nonlinear dopant drift. That is why the SPICE model of the memristor is designed such that it enables easy modification of the nonlinear relations describing the boundary effects. Such a modification can be done when the relevant data are specified by the manufacturer. The usefulness of the model is amplified by the fact that samples of the memristor will be probably not available for a long time. Today, a lot of institutions are involved in research into the basic features of the memristor. The results of SPICE simulations from Section 4 point to the possibility of conducting interesting simulation experiments which can be of great importance for such a research.

Acknowledgements

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About Authors...

Zdeněk BIOLEK received the M.Sc. degree in 1983, and the Ph.D. degree in 2001 from the Brno University of Technology (BUT), Czech Republic, both in Electrical Engineering with a view to thermodynamics. Until 1992 he was the principal research worker at Czech Semiconductor Company TESLA Rožnov. He is currently with the SŠIEŘ Rožnov p. R. He has authored several special electronic instruments for manufacturing and testing integrated circuits. He is also the author and co-author of papers from the area of the utilization of variational principles in circuit theory and the stability testing of resistive circuits.

Dalibor BIOLEK received the M.Sc. degree in Electrical Engineering from the BUT, in 1983, and the Ph.D. degree in Electronics from the Military Academy Brno, in 1989. He is with the Dept. of EE, University of Defense Brno (UDB), and with the Dept. of Microelectronics, BUT, as professor in Theoretical Electrical Engineering. His scientific activity is directed to the areas of general circuit theory, frequency filters, and computer simulation of electronic systems. For years, he has been engaged in algorithms of the symbolic and numerical computer analysis of electronic circuits with a view to the linear continuous-time and switched filters. He has published over 250 papers and a book on circuit analysis and simulation. At present, he is member of the CAS/COM Czech National Group of IEEE, and the president of Commission C of the URSI National Committee for the Czech Republic.

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