Comparison of Pilot Symbol Embedded Channel Estimation Algorithms

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Abstract. *In the paper, algorithms of the pilot symbol embedded channel estimation are compared. Attention is turned to the Least Square (LS) channel estimation and the Sliding Correlator (SC) algorithm. Both algorithms are implemented in Matlab to estimate the Channel Impulse Response (CIR) of a channel exhibiting multi-path propagation. Algorithms are compared from the viewpoint of computational demands, influence of the Additive White Gaussian Noise (AWGN), an embedded pilot symbol and a computed CIR over the estimation error.*

Keywords

Channel estimation, Least Squares, Sliding Correlator, AWGN channel, Hadamard function.

1. Introduction

Thanks to the dynamic development of mobile systems, requirements on effective utilization of frequency spectrum rise rapidly. That is the main reason why the Code Division Multiple Access (CDMA), when all users share the same frequency bandwidth at the same time, became the most frequently used type of the multiple access [1].

The mobile radio channel is of the multi-path fading nature: the received signal consists of the direct path signal (Line of Sight, LOS) and several reflected signals. Each signal propagates with a different delay and attenuation. Therefore, the analysis of wireless channel parameters is very important to decode the received signal correctly [4].

Today's detectors of 3G mobile systems require knowledge of CIR which can be computed from the sequence of known bits embedded in each transmitted communication burst. So called sliding correlator and least square method belong to the most widely used types of pilot embedded channel estimators [2], [3].

There are also blind estimators which do not need pilot symbols for their operation. Blind estimators acquire information about channel parameters [6]. Blind estimators are out of the scope of this paper.

In last years several papers about the wireless channel estimation algorithms were published:

- A description of the sliding correlator and its improvement was published in [4]. It describes a utilization of the sliding correlator in the UMTS.
- The least squares channel estimation is briefly described in [2]. Attention is turned to suppress the estimation error.
- Iterative version of the least squares channel estimation is discussed in [8]. It is compared with the basic least squares algorithm.
- New estimation technique based on correlation blending was published in [9]. This paper also mentions the least squares algorithm.

This paper brings newly a comparative study of the pilot embedded channel estimation algorithms for CDMA systems - sliding correlator and least squares. In Section 2 the system model of the proposed signal transmission is developed. Section 3 describes main steps of both channel estimation algorithms. Section 4 compares a performance of these channel estimators. Finally, Section 5 summarizes results of the whole paper.

2. System Model

The communication system is assumed to consist of a transmitter, a channel and a receiver. In the transmitter, information bits are multiplied by a unique spreading code (Hadamard functions in our case). Then, the resultant signal is scrambled by a random function. In order to de-spread the signal at the receiver side, the same Hadamard function and the same scrambling code have to be applied [4].

In order to achieve a maximum simplicity of the system model, the transmission of the base-band signal without any modulation is assumed throughout the paper (the stream of +1 and –1). In order to minimize computational demands of simulations, only pilot and no data symbols are transmitted. As a receiver, a chip matched filter is used [4].

The radio wireless channel can be described by several ways [4]. In this paper, the multi-path fading channel is assumed.

The received signal $y(t)$ can be computed as the convolution of the transmitted signal $x(t)$ and the channel impulse response $h(t, \tau)$ [2]:

$$y(t) = x(t) * h(t, \tau) + v(t) \tag{1}$$

where $v(t)$ is the Additive White Gaussian Noise (AWGN) specified by a Signal to Noise Ratio (SNR).

In order to compare the investigated channel estimation algorithms, a typical channel impulse response was constructed - each coefficient of the CIR represents a reflected path or a scattered path with an appropriate delay and transmission. The first sample is the LOS and takes usually the value 1 (no attenuation). Following CIR coefficients vary from 0 (totally attenuated) to 1 [2].

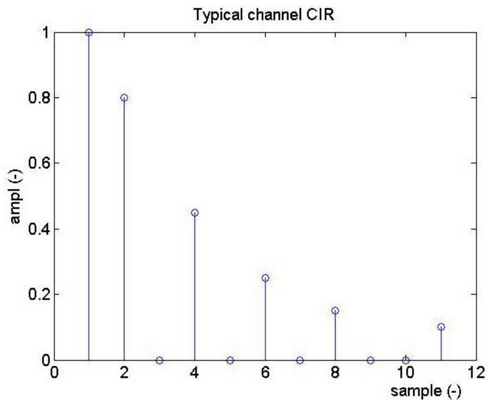

Fig. 1. Typical CIR for UMTS system (mostly used for simulations).

The UMTS system operates in the frequency band of 2 GHz with the channel bandwidth 5 MHz. For these values, the channel delay spread typically achieves 2.5 µs. Then, the CIR can be divided into 11 samples, e.g. (Fig. 1):

$$\mathbf{h}_{typ} = [1 \quad 0.8 \quad 0 \quad 0.45 \quad 0 \quad 0.25 \quad 0 \quad 0.15 \quad 0 \quad 0 \quad 0.1]$$

This CIR corresponds to the measured CIR for large outdoor cells [5].

3. Description of Algorithms

In this section, the Sliding Correlator and the Least Square algorithm are briefly reviewed. Both algorithms are based on transmitting a pilot symbol, which is known for the estimator at the receiving side. This technique decreases efficiency of a channel utilization on one hand, and is more accurate and simpler than the blind estimation on the other hand.

3.1 Sliding Correlator

The sliding correlator is based on the comparison of the auto-correlation function of the transmitted signal and the cross-correlation function of the transmitted signal and the received signal. Fig. 2 shows the detail of these functions around their maximum.

Comparing positions of peaks of the both functions, the delay of each path can be determined. Dividing matching coefficient values of the cross-correlation function by its maximum, the transmission of the path of interest is obtained. If all paths are analyzed, the approximate estimate of the channel impulse response is obtained.

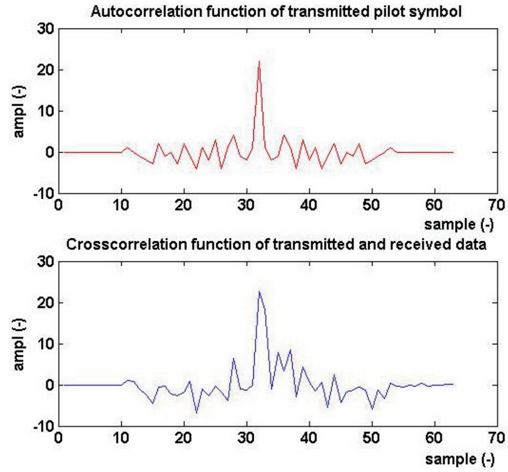

Fig. 2. Auto-correlation function of the transmitted pilot symbol *opt22* (top, defined in Section 4.3). Cross-correlation function of the transmitted and received data (bottom). Simulated for AWGN SNR = 40 dB.

The shape of the autocorrelation function of the transmitted pilot symbol significantly influences the accuracy of the algorithm. Ideally, the autocorrelation function should obtain a single maximum only, and the rest of the function should be zero. In reality, autocorrelation functions exhibit "side lobes" as depicted in Fig. 2.

Fig. 3 shows the comparison of the CIR set in simulation and the computed CIR. The response of the estimated CIR is the same as the searched one. Nevertheless, several samples of the computed CIR are incorrect due to the simulation with insufficiently high SNR = 40 dB for the AWGN. These errors are caused by false side lobes in the autocorrelation function.

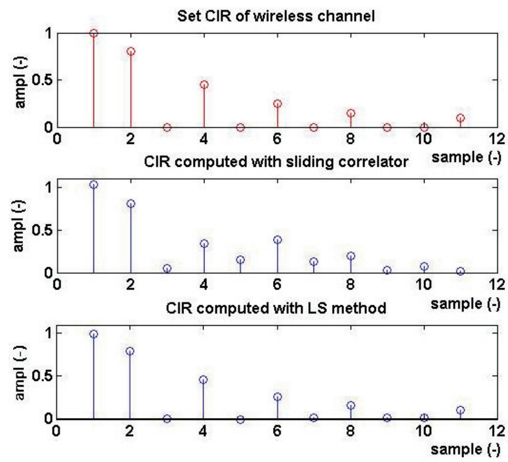

Fig. 3. Comparison of the CIR set in a simulation (top) with CIR estimated by sliding correlator (middle) and LS method (bottom). Simulated for AWGN SNR = 40 dB.

3.2 Least Squares

Over short time intervals, the time varying wireless channel can be described by a discrete time impulse response. Since the received signal is given by the convolution of the transmitted signal, $x(n)$, and the impulse response, h_k, corrupted by the AWGN, the CIR should be infinite. For a short batch of data, the CIR is assumed to be time invariant (FIR) of length M. The pilot data signal, $x(n)$, has to be of length $N \geq 2M$. Now, the model can be described by [2]:

$$y(n) = \sum_{k=0}^{M-1} h_k \, x(n-k) + v(n) \ . \tag{2}$$

The time impulse response, h_k, is going to be computed from (2) as accurately as possible using the Least Squares algorithm. First, the Toeplitz matrix containing transmitted data of size (N-M+1, M) is composed [2]:

$$\mathbf{X} = [\mathbf{x}(N-M) \quad \mathbf{x}(N-M+1) \quad \ldots \quad \mathbf{x}(N)] \tag{3}$$

where individual vectors from **X** are given by [2]:

$$\mathbf{x}(n) = [x(n) \quad x(n-1) \quad \ldots \quad x(n-M+1)]^T \tag{4}$$

where n goes from N - M + 1 to N.

Equations (3) and (4) show how individual columns of the Toeplitz matrix X are given by shifting one position transmitted data.

In the next step, the Moore-Penrose inverse of the matrix **X** is computed [2]:

$$\mathbf{X}^{\Psi} = \left(\mathbf{X}^{\mathrm{T}} \mathbf{X}\right)^{-1} \mathbf{X}^{\mathrm{T}} \ . \tag{5}$$

Here, $()^{\mathrm{T}}$ denotes transposition of the matrix and $()^{-1}$ denotes the matrix inverse.

For the known vector of received data

$$\mathbf{y}(n) = [y(N-M+1) \quad \ldots \quad y(N)] \tag{6}$$

and the known vector of noise samples

$$\mathbf{v}(n) = [v(N-M+1) \quad \ldots \quad v(N)] \ , \tag{7}$$

the channel impulse response of length M can be computed [2]:

$$\mathbf{h}_e = \mathbf{X}^{\Psi}\mathbf{y} = \mathbf{h} + \mathbf{X}^{\Psi}\mathbf{v} \ . \tag{8}$$

Here, $\mathbf{h}_e$ is the CIR estimate, and $\mathbf{X}^{\Psi}$ **v** represents the error of the estimate. At the receiving side, nothing about the size of the noise vector **v** is usually known.

Fig. 3 shows that the error of the LS estimation for the AWGN SNR = 40 dB is close to zero because of very low values of the noise vector **v**. With the decreasing signal to noise ratio of the AWGN, the error of the method increases significantly. For the method, the formation of the Toeplitz matrix **X** according to (3) and (4) is crucial. The Toeplitz matrix may not be singular. In that case, its determinant becomes zero and the inverse matrix in (5) cannot be computed.

4. Comparison of Algorithms

In this section, the properties of the sliding correlator and least squares are compared from the viewpoint of their computational demands and the influence of system model components on the estimate error. In order to achieve statistically credible results, estimations have to be performed several times (usually 2 000 times). Then, the average channel impulse response and the estimation error can be computed. All the discussed results are summarized in Tab. 1 at the end of Section 4.

4.1 Computational Demands

Mathematical analysis of all procedures, which are used to compute a tap of CIR of the length M using the batch of data of the length N, shows that computational demands of the Least Squares algorithm are about three times higher compared to demands of the sliding correlator.

Using Matlab profiler, a similar computational time for both algorithms was measured. This is caused by the fact that the sliding correlator has to find positions of maxima in the correlation functions, which slows down the algorithm significantly.

4.2 Influence of AWGN

To express the average error of the estimation the known CIR, **h**, set for the simulation is compared with that one obtained from the simulation, $\mathbf{h}_e$. The result is then related to a unit coefficient of the CIR:

$$e = \frac{100}{M} \sum_{m=1}^{M} \left| h_e(m) - h(m) \right| \ . \tag{9}$$

For R repetitions of the same CIR estimation, the average percentage error of the estimation per a unit coefficient of the CIR can be evaluated:

$$e_{avg} = \frac{1}{R} \sum_{r=1}^{R} e(r) \ . \tag{10}$$

Now, the relationship between the computational error of both algorithms and the AWGN signal to noise ratio can be displayed (Fig. 4).

For least squares, e_{avg} exponentially decreases in the whole range of SNR, and approaches zero for the higher values of the SNR. On the contrary, the sliding correlator estimate error decreases to a given SNR only, and then stays constant. Remarkably, the sliding correlator is more precise than the least squares technique for SNR < 12 dB. The SNR value determining which algorithm is more precise depends on the used pilot symbol and the searched CIR especially.

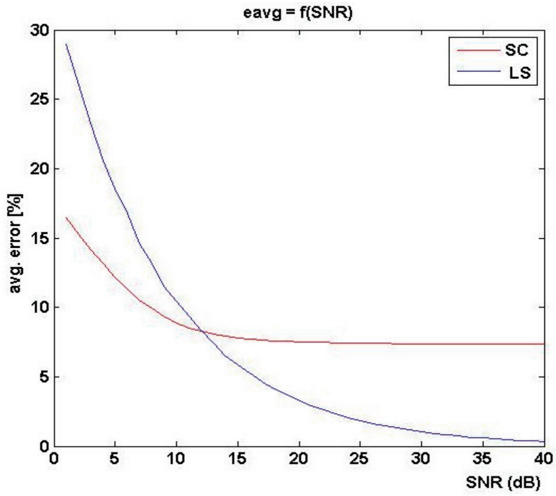

Fig. 4. Relationship between computational error, e_{avg}, of the SC and LS algorithms and SNR. Averaged over 2 000 estimates, used pilot symbol *opt22*.

4.3 Influence of Pilot Symbol

The choice of the transmitted pilot symbol exhibits the strongest influence on the behavior of both algorithms and especially on the sliding correlator technique. As already discussed, the size of parasitic side lobes in the auto-correlation function of the pilot symbol negatively affects the estimation error.

Pseudorandom functions with a required autocorrelation function were deeply researched. In [6], a function of length $N = 22$ with the best autocorrelation properties was published:

$$opt_{22} = [+1 \;\; +1 \;\; +1 \;\; +1 \;\; -1 \;\; +1 \;\; -1 \;\; -1 \;\; -1 \;\; \ldots \;\; +1 \;\; -1 \;\; -1 \;\; +1 \;\; -1 \;\; -1 \;\; -1 \;\; +1 \;\; +1 \;\; -1 \;\; \ldots \;\; -1 \;\; -1 \;\; +1]$$

Also the Barker code, which is widely used in radiolocation systems, exhibits good autocorrelation properties. Unfortunately, the longest known Barker code is of length 13 only:

$$bak_{13} = [-1 \;\; -1 \;\; -1 \;\; -1 \;\; -1 \;\; +1 \;\; +1 \;\; -1 \;\; -1 \;\; \ldots \;\; +1 \;\; -1 \;\; +1 \;\; -1]$$

Therefore, the Barker code can be used for the estimation of CIR consisting of 6 paths or less. Finally, three randomly generated codes are introduced:

$$rnd_{122} = [-1 \;\; -1 \;\; -1 \;\; +1 \;\; +1 \;\; -1 \;\; +1 \;\; +1 \;\; -1 \;\; \ldots \;\; +1 \;\; +1 \;\; +1 \;\; +1 \;\; +1 \;\; +1 \;\; -1 \;\; +1 \;\; +1 \;\; -1 \;\; \ldots \;\; +1 \;\; -1 \;\; -1],$$

$$rnd_{222} = [+1 \;\; +1 \;\; -1 \;\; +1 \;\; -1 \;\; +1 \;\; +1 \;\; +1 \;\; -1 \;\; \ldots \;\; +1 \;\; -1 \;\; -1 \;\; +1 \;\; +1 \;\; -1 \;\; -1 \;\; -1 \;\; +1 \;\; +1 \;\; \ldots \;\; +1 \;\; +1 \;\; -1]$$

and

$$rnd_{322} = [+1 \;\; -1 \;\; -1 \;\; -1 \;\; -1 \;\; +1 \;\; +1 \;\; -1 \;\; -1 \;\; \ldots \;\; +1 \;\; -1 \;\; +1 \;\; -1 \;\; -1 \;\; -1 \;\; +1 \;\; -1 \;\; -1 \;\; -1 \;\; \ldots \;\; -1 \;\; +1 \;\; -1]$$

Both algorithms are compared from the viewpoint of the average error to a unit sample of CIR considering pilots opt_{22}, bak_{13} and rnd_{122}, rnd_{222} and rnd_{322}.

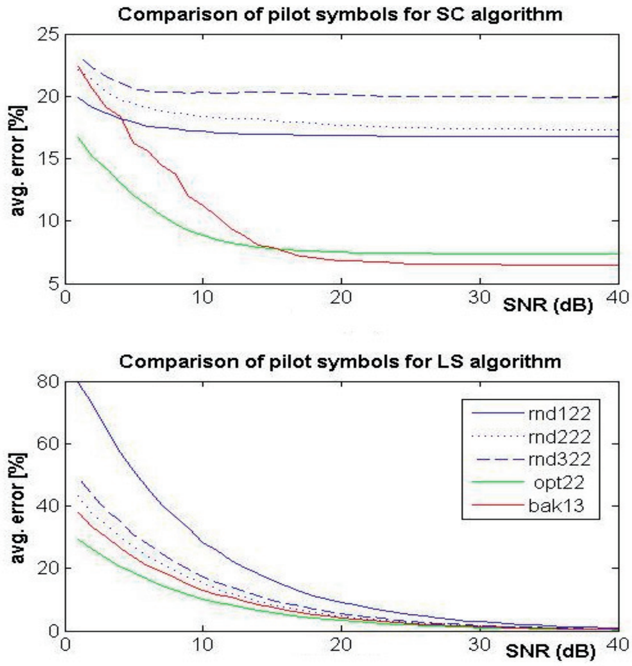

Fig. 5. Comparison of pilot symbols from the viewpoint of error of algorithms SC (top) and LS (bottom). Percentage error is averaged over 2 000 estimations.

Simulation results for both algorithms (Fig. 5) show that the sliding correlator reaches the minimum error for the Barker code; the error is about three times smaller than the error in case of the random pilot symbol. Least squares reach the best results for the opt_{22} pilot, but the choice of the pilot symbol is not so crucial (at higher values of SNR, the estimation error is almost the same for all used pilots).

4.4 Influence of Searched CIR

Results from the previous subsection show that both investigated algorithms achieve good results using the pilot symbol opt_{22}. Therefore this function is used to find out the influence of the searched CIR on the accuracy of algorithms. Results of the simulation using h_{opt} can be found in previous Sections 4.2 and 4.3. Sum of simulations made showed that the accuracy of the least squares technique does not practically depend on the searched CIR. Therefore, attention is turned to the sliding correlator only.

Results of a simulation scenario are presented here to show the influence of the searched CIR on the sliding correlator accuracy. Let us consider a wireless channel with a LOS path and a single reflected and very low attenuated path only:

$$\mathbf{h} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9 & 0 & 0 \end{bmatrix}.$$

Since the modeled scenario is simpler than in case of $\mathbf{h}_{typ}$, the estimate should be more precise. Surprisingly, Fig. 6 shows a lot of incorrect coefficients in the estimated CIR. The parasitic side lobes in the correlation function are again the reason for that.

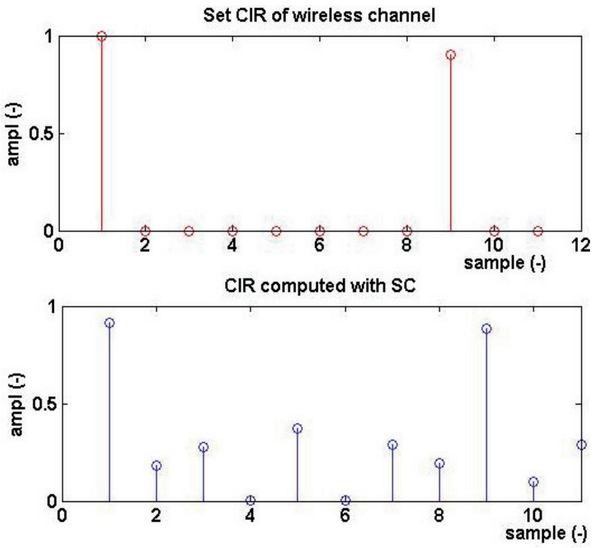

Fig. 6. Comparison of CIR set in simulation (top) with CIR estimated by sliding correlator (bottom). Simulated for AWGN SNR = 40 dB.

There are of course other options in comparing both algorithms than those discussed in previous subsections 4.1–4.4. Tab. 1 summarizes the comparison of the most important properties of both computational methods for the wireless channel parameters estimation.

	least squares	**sliding correlator**
robustness	strong	weak
minimum achieved e_{avg}	0.35 %	6.00 %
AWGN influence	strong in whole range of SNR	strong to certain SNR only
pilot symbol influence	strong	crucial
optimum pilot	opt_{22}	bar_{13}
CIR influence	very weak	strong

Tab. 1. Comparison of most important properties of the Sliding Correlator and Least Squares.

5. Conclusions

A comparative study of two pilot embedded channel estimators widely used in CDMA systems was presented in this paper. Attention was turned to the computational demands, accuracy and the influence of model parameters (used pilot symbol, searched channel impulse response) to the sliding correlator and least squares technique.

Simulation results showed that the average error per the unit CIR coefficient of both algorithms depends mainly on the SNR value. The error of least squares decreases in the whole range of SNR and approaches zero. On the other hand, the error of the sliding correlator decreases to the certain value of SNR only and stays constant from this value.

The accuracy of the least squares algorithm does not depend on the searched CIR and is weakly depending on the used pilot symbol. The best results were achieved with the opt_{22} symbol. For the sliding correlator, the choice of the pilot symbol is crucial. Attention has to be paid to correlation properties of the pilot symbol. Parasitic side lobes should be suppressed as much as possible. The best results were achieved with the Barker code in the role of the pilot symbol.

Considering simulation results, the least square estimator is much more robust and its results are more accurate than results produced by the sliding correlator. Furthermore, computational demands are almost the same for both algorithms. The sliding correlator is more accurate than least squares for small values of the AWGN SNR only. Therefore, the least square algorithm can be recommended for precise CIR estimations.

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