

Modeling of Transmission Functions and Crosstalk in Metallic Cables for Implementation of MIMO Concept

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Abstract. *The new promising wireless networks based on multi-carrier modulations (MCM) and multiple-input multiple-output concept (MIMO) will soon offer high-speed digital connections. Their access points are mostly connected by fixed metallic lines to core data and telecommunication networks. That is why it will also be necessary to increase the transmission speed and overall performance of these fixed access networks adequately in order to meet the expected requirements of wireless connections. It would be possible to use VDSL2 digital subscriber lines and implement MIMO concept into the existing metallic networks for this purpose, but before that it will be necessary to solve several problems first. The transmission capacity of present VDSL2 digital lines is limited mainly by crosstalk occurring in metallic cables. This paper describes a new method for modeling of transmission functions and crosstalk in multi-pair and multi-quad metallic cables including its mathematical implementation, and it also gives an example of results obtained so far. The presented model is based on statistical evaluations of measured values, generation of pseudo-random components of frequency response and subsequent filtration process.*

Keywords

Multi-carrier modulation, multiple-input multiple-output, vectored DMT modulation, crosstalk, modeling.

1. Introduction

The initial idea leading to establishing wireless networks was to provide connection for a specific area of a territory where other telecommunication solutions (such as optical or metallic lines) would be very expensive and also very unprofitable for the operators. However, wireless networks were gradually established also for the purpose of cheap and easy connection of common households in cities and their suburbs to the Internet. Along with this fast development, there emerged the necessity to establish a network of access points (AP) for covering the selected areas and to connect them to core data and telecommunication networks. The constant growth of

demands for transmission speed in wireless networks and for their overall performance also brings additional requirements – to increase the transmission speed and the overall performance of fixed access networks adequately to meet the expected load of wireless connections. There are several possibilities for their implementation, depending on specific demands and total costs. It can be very promising to use VDSL2 digital subscriber lines and the existing metallic cables, mainly because of low costs of such solution. In the article [1], a proposal based on VDSL2 digital subscriber line was presented, using the concept of active access network and Vectored DMT modulation [2] for crosstalk and other disturbances cancelation. This solution could reach transmission speeds of hundreds of Mbps at distances up to one kilometer using the existing metallic lines.

The principle of modulation with multiple carriers (MCM) has been successfully used not only for wireless networks, but also for xDSL digital subscriber lines as a DMT modulation. DMT modulation allows more effective use of the allocated frequency band in a metallic cable and partially eliminates the influence of interferences and disturbances. Vectored DMT modulation will bring full crosstalk suppression in future applications. Crosstalk is the most serious source of disturbance in present xDSL lines and it mostly limits the maximum achievable transmission speeds. It originates from unbalanced capacitive and inductive couplings between single copper pairs, their quads and multi-quads as well as from ground unbalances. Near-end crosstalk (NEXT) can be well limited by separating transmission directions using different frequency bands, but the elimination of far-end crosstalk (FEXT) is not so easy, and therefore FEXT is the dominant source of disturbance in present xDSL lines.

The overall capacity of metallic lines could also be increased by using MIMO principles together with VDMT modulation for crosstalk elimination. However, the vectored DMT modulation is not available for coordination of transmissions in large metallic cable systems due to insufficient computational capacities of the existing terminals. One of the possibilities how to simplify this process would be performing the vectored DMT modulation only for a limited number of pairs or even only for xDSL sub-channels. These pairs could be selected according to their contribution of crosstalk [6], [7]. This

method would decrease the number of disturbing systems to be coordinated [3] and simplify the whole process. The new concept of modeling the transmission functions for a specific MIMO environment in multi-pair metallic cables is proposed in this paper. It is based on theoretical assumptions concerning the allocation of disturbing sources and verified by measurements performed with a specific multi-pair metallic cable. This model could serve for determining the FEXT contributions of each transmission system in a cable and to prepare the necessary parameters for implementing VDMT modulation together with MIMO principles.

2. Standard Model and New Concept

2.1 Standard Model

The standard model for transmission channel and crosstalk comes from the derivation of interactions between channels and it results in an elementary formula:

$$|H_{FEXT}(f)|^2 = K_{FEXT} \cdot f^2 \cdot |H(f)|^2 \quad (1)$$

where $|H_{FEXT}(f)|^2$ represents the transmission function of crosstalk between channels, $|H(f)|^2$ is the transmission function of a channel and K_{FEXT} is a crosstalk parameter (a constant for the given combination of transmission channels). This parameter presents the rate of interactions between selected pairs and it is different (unique) for all combinations. In [4] and [5] it is shown that this parameter depends on the mutual position of the disturbing and disturbed channels in a metallic cable. However, for most applications the value of this parameter is unknown, and only one mean value is usually given for the whole cable. It is obvious that this model with only one parameter cannot be very accurate and that it provides only approximate results, as presented in [4]. The accuracy of these results can be sufficient for some specific applications (for example, summation of many contributions), but this simple model is not very useful for a precise modeling of perspective VDSL2 lines with MIMO concept. Individual modeling of channels is usually required in MIMO systems for all combinations of transmission channels and their interactions. That is why a new modeling method is proposed. To express the attenuation of FEXT crosstalk in dB, which is more typical, the logarithm of the formula (1) is used.

2.2 New Idea of Advanced Crosstalk Modeling

Several changes and improvements to the standard model for transmission channels and crosstalk are proposed. These upgrades are based on previous results obtained by measurements of specific metallic cables, presented in [4] and [5]. The resulting advanced model takes into consideration the relative position of disturbing

and disturbed pairs in a cable (channels in MIMO system for possible future applications). The idea of this new method for modeling was further extended and a new process for its generation was developed, moreover the idea was also compared with conclusions given in [9]. This model allows the generation of crosstalk transmission functions (interaction between different transmitters and receivers in MIMO systems) and simulations of systems using metallic cables. This procedure allows generating pseudorandom characteristics under different and various conditions. The model helped to prepare the necessary results for examining the impact of VDMT modulation on the suppression level of crosstalk without performing time-consuming measurements of real metallic cables.

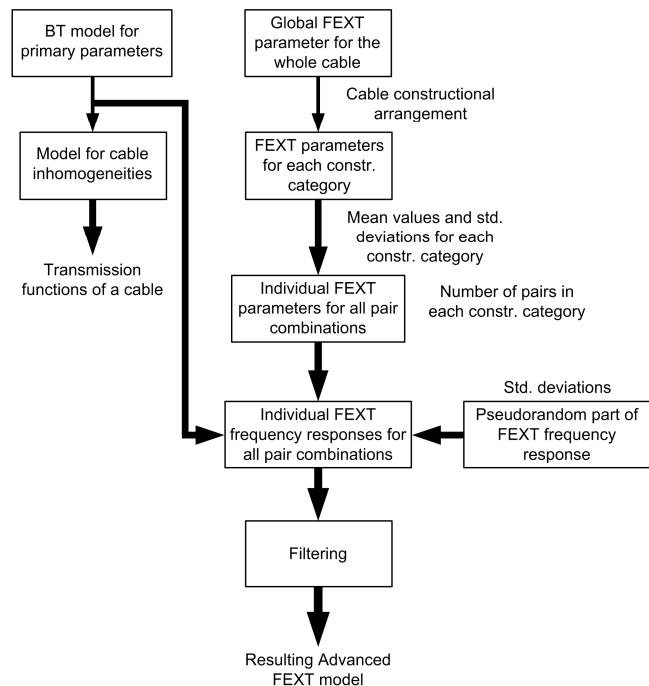


Fig. 1. The idea of FEXT transmission functions generator.

Channel transmission functions (interaction between the same transmitters and receivers in MIMO system) are generated using a model for homogenous metallic pairs based on the British Telecom model. They are supplemented by pseudorandom components, which simulate channel inhomogeneities. K_{FEXT} parameters of crosstalk between the channels are needed for all constructional categories to calculate transmission functions. These parameters are expressed as mean values and standard deviations of normal distribution for all constructional categories. These statistical parameters were determined by measurements followed by statistical evaluations for a specific metallic cable. The model is further supplemented by pseudorandom parts of frequency responses, which are also generated using statistical parameters. The necessary parameters were obtained by comparing the model with measured results. In the last step, the model with this pseudorandom element is filtered in order to eliminate spectral components exceeding real characteristics. Previous conclusions concerning the cable

constructional arrangement were used during the statistical evaluations. This enabled a reduction of the number of parameters by grouping them according to their relative positions in the cable. The model and its parameters were calculated and derived from measurements performed for a specific multi-pair metallic cable. However, the mathematical model for the new modeling method presented in the next section is valid for any metallic cable with multi-pair or multi-quad construction.

3. Advanced Model

A list of mathematical operations and several rules for mathematical notation were created for the purpose of the following description. Scalar variables are written in uppercase italics (only Greek letters are of standard lower case font type in formulas of statistical evaluations), matrices and vectors are described by bold uppercase symbols. The number of transmission channels is marked as n , while the number of frequency steps (often used as a dimension of a frequency vector) is m .

It is possible to determine the output matrix of frequency vectors of DMT symbols for n transmission channels if cyclic prefix (CP) has sufficient length according to [7]. Perfect synchronization of input symbols is necessary and the channel must have pulse response shorter than CP:

$$\mathbf{Y} = \mathbf{H}^C \cdot \mathbf{X} + \mathbf{w} \quad (2)$$

$$\left([Y_n(i)]_{i=1}^m \right)_{n,1} = \mathbf{H}^C \cdot \left([X_n(i)]_{i=1}^m \right)_{n,1} + \left([w_n(i)]_{i=1}^m \right)_{n,1}$$

where

$$\mathbf{Y} = \left([Y_n(i)]_{i=1}^m \right)_{n,1}$$

represents the output one-column matrix of frequency vectors of received DMT symbols for all n channels,

$$\mathbf{X} = \left([X_n(i)]_{i=1}^m \right)_{n,1}$$

stands for one-column matrix of frequency vectors of input DMT symbols,

$$\mathbf{w} = \left([w_n(i)]_{i=1}^m \right)_{n,1}$$

is one-column matrix of noise vectors, containing frequency vectors of the disturbance from all surrounding channels, which are not coordinated by vectored DMT modulation. Finally, matrix \mathbf{H}^C (generally complex) is a square matrix of dimension n, n , containing frequency vectors of transmission functions (channels and FEXT crosstalk), in which the vectors:

$$\mathbf{H}_n^{Ch} = \left[H_n^{Ch}(i) \right]_{i=1}^m$$

placed diagonally represent vectors of frequency response of n channels with the length m .

$$\mathbf{H}_{n-1n}^F = \left[H_{n-1n}^F(i) \right]_{i=1}^m$$

remaining frequency vectors represent FEXT crosstalk between all relative combinations of transmission channels (here e.g. combination of channels $n-1$ and n). Matrix \mathbf{H}^C can be divided into a diagonal matrix of transmission channels \mathbf{H}_{Ch}^C and a remaining matrix of FEXT crosstalk \mathbf{H}_F^C :

$$\mathbf{H}^C = \mathbf{H}_{Ch}^C + \mathbf{H}_F^C \quad (3)$$

A mathematical operation called Hadamard product was implemented for the purpose of the following description and calculations. This operation is defined for objects containing multiple elements (typically vectors, matrices) and implements the multiplication of elements on the same relative positions (i.e. with the same index). A necessary condition is the same dimension of multiplied objects, the mathematical symbol is usually an asterisk (*). If this operation is used for multiplication of two matrices where elements are vectors, each vector in a matrix is multiplied by each other.

Symmetric pairs in typical metallic cables usually provide similar transmission characteristics and their primary parameters are supposed to be identical for all channels. Minor differences will be implemented in further process of pseudorandom generation and simulations of channel inhomogeneities. Thanks to this assumption, frequency vectors of all n channels are identical and the matrix of transmission channels can be expressed using the Hadamard operation described above:

$$\mathbf{H}_{Ch}^C = \left[H^{Ch}(i) \right]_{i=1}^m * \mathbf{E}_{n,n} \quad (4)$$

where $\mathbf{E}_{n,n}$ is an identity matrix of dimension n, n defined as follows:

$$\mathbf{E}_{j,k}^n = 1 \text{ for } j=k \text{ and } \mathbf{E}_{j,k}^n = 0 \text{ for } j \neq k$$

$$p=1 \quad n$$

where j stands for column and k for row index of the matrix. The matrix of transmission channels \mathbf{H}_{Ch}^C can also be derived from matrix \mathbf{H}^C :

$$\mathbf{H}_{Ch}^C = \sum_{p=1}^n \mathbf{A}_p \cdot \mathbf{H}^C \cdot \mathbf{A}_p \quad (5)$$

where the square-type matrix \mathbf{A}_p of dimension n , n is defined as:

$$\mathbf{A}_{p,j,k}^n = 1 \text{ for } j \wedge k = p \text{ and } \mathbf{A}_{p,j,k}^n = 0 \text{ for } j \vee k \neq p$$

$$p=1 \quad n$$

where p stands for an index of a pair.

The matrix of transmission functions of FEXT crosstalk can be expressed using the same matrices:

$$\mathbf{H}_F^C = \sum_{p=1}^n (\mathbf{E} - \mathbf{A}_p) \cdot \mathbf{H}^C \cdot \mathbf{A}_p. \quad (6)$$

The matrix of transmission functions of FEXT crosstalk \mathbf{H}_F^C can also be calculated using the formula for the standard model (1) for all combinations of disturbing and disturbed channels:

$$\left[H_{12}^F(i) \right]_{i=1}^m = K_{12}^{FEXT} \cdot l \cdot \left[f^2(i) \right]_{i=1}^m * \left[H^{Ch}(i) \right]_{i=1}^m. \quad (7)$$

Several operations for generating pseudorandom parameters and results were used during the process of the presented generation of transmission functions in Sec. 2.2. These operations are based on statistical parameters of normal distribution (mean value and deviation) and they require adequate mathematical description. The frequency vector of randomizing parameters Θ contains vectors of randomizing parameters for all n channels (pairs):

$$\begin{aligned} \Theta &= \left[\left[\Theta_n^{Ch}(i) \right]_{i=1}^m \right]_n \\ \Theta &= \left[\left[\Theta_1^{Ch}(i) \right]_{i=1}^m, \left[\Theta_2^{Ch}(i) \right]_{i=1}^m, \dots, \left[\Theta_n^{Ch}(i) \right]_{i=1}^m \right]_n \end{aligned} \quad (8)$$

The matrix of transmission functions of the channels can further be modified using (4), (5) and (8):

$$\begin{aligned} \widehat{\mathbf{H}}_{Ch}^C &= \sum_{p=1}^n \left[\Theta_p^{Ch}(i) \right]_{i=1}^m * \mathbf{A}_p \cdot \mathbf{H}^C \cdot \mathbf{A}_p \\ \widehat{\mathbf{H}}_{Ch}^C &= \left[H^{Ch}(i) \right]_{i=1}^m * \left[\left[\Theta_n^{Ch}(i) \right]_{i=1}^m \right] * \mathbf{E}_{n,n} \end{aligned} \quad (9)$$

where symbol $\widehat{\mathbf{H}}_{Ch}^C$ represents matrix \mathbf{H}_{Ch}^C with randomized part of vector Θ .

Pseudo-randomly generated matrices and vectors can be calculated as follows:

$$\widehat{\mathbf{K}}_{F_{n,n}} = \left(Rnd \left(K_{cat}^{FEXT}, \sigma_{cat}^{FEXT} \right)_{cat} \right)_{n,n}, \quad (10)$$

$$\left[\left[\Theta_n^{Ch}(i) \right]_{i=1}^m \right]_n = \left[\left[Rnd_{ff} \left(\sigma^{Ch} \right) \right]_{i=1}^m \right]_n, \quad (11)$$

$$\Delta_{F_{n,n}} = \left(\left[Rnd_{ff} \left(\sigma_{cat}^F \right)_{cat} \right]_{i=1}^m \right)_{n,n} \quad (12)$$

where symbols $\widehat{\mathbf{H}}_F^C$, $\widehat{\mathbf{K}}_{F_{n,n}}$ represent previous matrices supplemented by random components. Matrix $\Delta_{F_{n,n}}$ contains pseudorandom components of frequency responses of crosstalk between channels. Function Rnd was used for generating all random or pseudorandom components of all characteristics. Necessary parameters (mean value and deviation) of normal distribution were determined using measured results and statistical evaluations. Index cat stands for constructional category of the selected cable and

index ff indicates the process of frequency generating (for each frequency step) followed by subsequent process of filtration.

The matrix of transmission functions of FEXT crosstalk can be specified using the definitions (6), (7), (10) and (12):

$$\widehat{\mathbf{H}}_F^C = \left[\sum_{p=1}^n (\mathbf{E} - \mathbf{A}_p) \cdot \mathbf{H}^C \cdot \mathbf{A}_p \right] + \Delta_{F_{n,n}}. \quad (13)$$

$$\widehat{\mathbf{H}}_F^C = l \cdot \left[f^2(i) \right]_{i=1}^m * \left[H^{Ch}(i) \right]_{i=1}^m * \widehat{\mathbf{K}}_{F_{n,n}} + \Delta_{F_{n,n}}$$

Finally, the output matrix defined as (2) can be expressed from (3), (9) and (13):

$$\mathbf{Y} = \widehat{\mathbf{H}}^C \cdot \mathbf{X} + \mathbf{w} = (\widehat{\mathbf{H}}_F^C + \widehat{\mathbf{H}}_{Ch}^C) \cdot \mathbf{X} + \mathbf{w} \quad (14)$$

$$\begin{aligned} \widehat{\mathbf{H}}^C &= \left\{ l \cdot \left[f^2(i) \right]_{i=1}^m * \left[H^{Ch}(i) \right]_{i=1}^m * \widehat{\mathbf{K}}_{F_{n,n}} + \Delta_{F_{n,n}} \right\} + \\ &+ \left\{ \left[H^{Ch}(i) \right]_{i=1}^m * \left[\left[\Theta_n^{Ch}(i) \right]_{i=1}^m \right]_n * \mathbf{E}_{n,n} \right\} \end{aligned}$$

$$\widehat{\mathbf{H}}^C = \left[H^{Ch}(i) \right]_{i=1}^m * \left\{ l \cdot \left[f^2(i) \right]_{i=1}^m * \widehat{\mathbf{K}}_{F_{n,n}} + \left[\left[\Theta_n^{Ch}(i) \right]_{i=1}^m \right]_n * \mathbf{E}_{n,n} \right\} + \Delta_{F_{n,n}}. \quad (15)$$

4. Results of Advanced Modeling

Equations (14), (15) represent the final advanced model for transmission channels respecting the influence of FEXT crosstalk in VDSL2 digital subscriber lines. This model was developed according to the procedure presented in Sec. 2.2 and described by several steps in Fig. 1, its mathematical implementation is presented in Sec. 3.

The graphs in Fig. 2 and Fig. 3 give an example of results obtained using the presented method for generating transmission functions of channels and FEXT crosstalk. Unlike the standard simple model (1) of FEXT crosstalk (presented in the graphs as a black solid line), the proposed advanced modeling method includes also the dependence on the mutual relative position of disturbing and disturbed channels within the metallic cable. This solution provides more precise results, which can be used for accurate simulations of transmission channels and subsequent implementation of MIMO principles into VDSL2 digital systems. The standard model (1) comes only from average values for the whole cable and provides very approximate estimations only. The proposed advanced model brings more accurate results and reaches more realistic shapes of transmission and crosstalk characteristics in MIMO systems. This model, together with generated pseudorandom component (grey line in Fig. 2) after filtering (grey line in Fig. 3), provides final results very close to the characteristics in real applications.

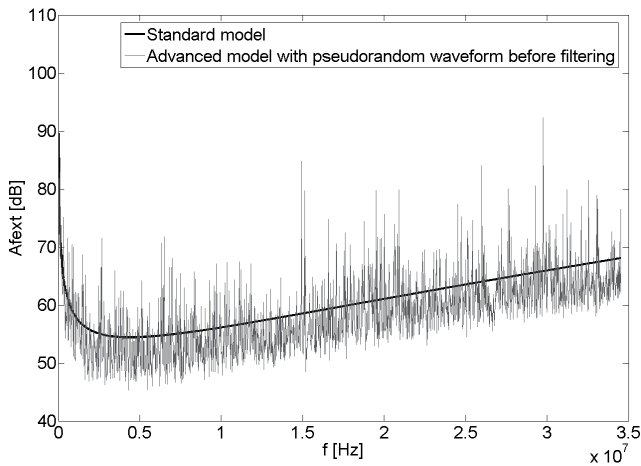


Fig. 2. Comparison of Standard and Advanced model before the filtering.

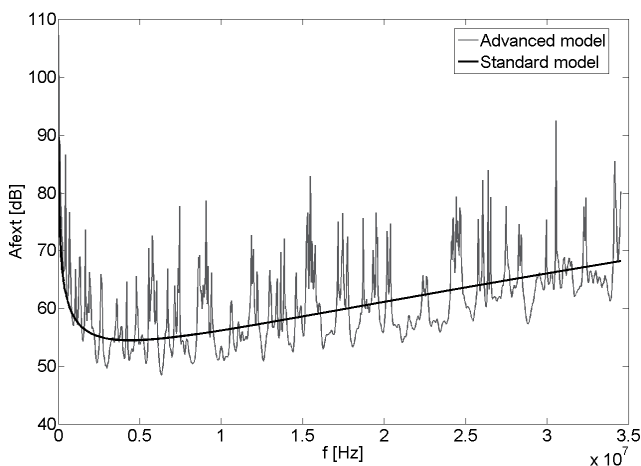


Fig. 3. Comparison of Standard and Advanced model after the filtering.

One of the most important parameters in both models is the crosstalk parameter K_{FEXT} , which determines the resulting FEXT characteristics. During the process of designing the advanced model, a normal distribution of this parameter was assumed for all constructional categories. Mean values and standard deviations were calculated by performing statistical evaluations of measured results according to formulas of normal distribution. These parameters were subsequently used for the process of generating pseudorandom outputs and characteristics. That is why it is necessary to verify the initial assumptions made about the type of distribution of K_{FEXT} parameters and to compare them with the measured results and generated characteristics. Fig. 4 presents the result of a statistical evaluation of K_{FEXT} parameters from measured characteristics.

It is obvious that the previous assumptions about the type of distribution of the K_{FEXT} parameter were correct and the values of this parameter really correspond to a normal distribution behavior. The result also demonstrates that there are significant crosstalk differences between the proposed constructional categories of the selected cable, which was described in previous publications [4], [5].

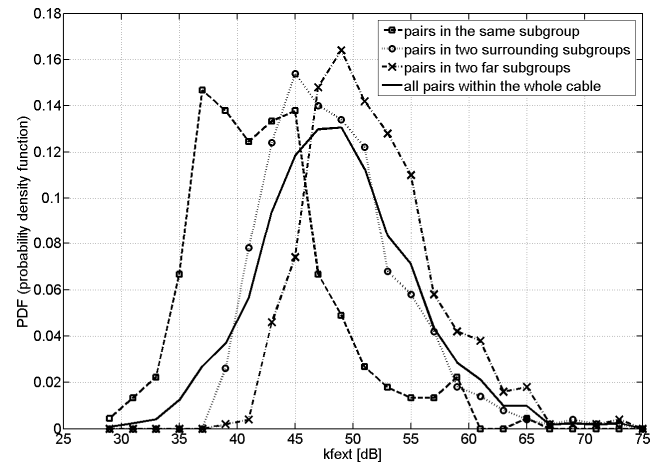


Fig. 4. The result of statistical processing of K_{FEXT} parameter for measured characteristics.

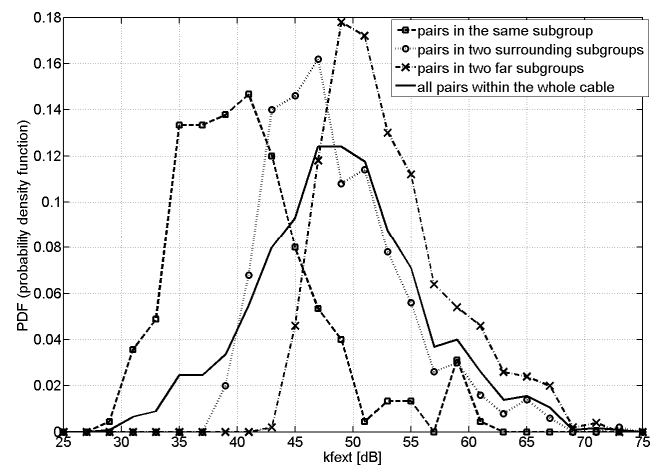


Fig. 5. The result of statistical processing of K_{FEXT} parameter for generated characteristics.

Fig. 5 shows the same statistical evaluation performed for pseudo-randomly generated characteristics. The statistical values and resulting graphs are practically the same as for the measured results, and this confirms the accuracy of the mathematical formulas and assumptions of the advanced model presented in previous sections.

This advanced modeling method was developed during a systematic and long-term process; many measurements were performed and the model was compared with a standard one in many specific situations to verify the results. The authors have been publishing the partial results and the progress in the development of this advanced model continuously, for example in [8], [4] or [5].

5. Conclusions

The most limiting factor in the current VDSL2 systems is the crosstalk coming from the systems operating within one metallic cable. The influence of near-end crosstalk (NEXT) could be eliminated by separating the frequency bands for both transmission directions, but the

suppression of far-end (FEXT) crosstalk is not so easy, even when its value is decreased by line attenuation. The reduction of crosstalk by VDMT modulation is not possible in present systems due to its overall complexity and demands on computational units in DSLAMs. Therefore, the possibility of using space selection of disturbing sources is proposed, which would allow the use of vectored DMT modulation only for a limited number of the most disturbing channels in a particular system.

A new method was proposed and designed for simulating and modeling FEXT transmission functions in VDSL2 digital lines. This solution uses VDMT modulation for crosstalk elimination as well as the MIMO concept for reaching the maximum transmission speed and serves for the last mile of access networks. The model is based on detailed analyses of the transmission environment and its parameters, statistical evaluation of measured results, generation of pseudorandom characteristics and the processing of their filtering. The necessary mathematical description was prepared, implemented and presented in this paper. The exact description allows the use of the advanced model for various types of environments and situations under different conditions. The calculated results and generated characteristics obtained using this method are very realistic and very close to real crosstalk characteristics in real transmission channels. These results also allow the analysis of the VDMT modulation ability to reduce FEXT crosstalk under different conditions without performing necessary measurements of real metallic cables. The model could serve for simulations and calculations of FEXT crosstalk and to prepare realistic results for implementing VDMT modulation together with MIMO principles into VDSL2 digital lines.

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References

- [1] VODRÁŽKA, J., HUBENÝ, T. Wired core network for local and premises wireless networks. *Personal Wireless Communications*. New York: Springer, 2007, p. 332-340.
- [2] BRADY, M. H., CIOFFI, J. M. The worst-case interference in

DSL systems employing dynamic spectrum management. *Journal on Applied Signal Processing*, 2006, vol. 2006, p. 1-11.

- [3] VODRÁŽKA, J. Multi-carrier modulation and MIMO principle application on subscriber lines. *Radioengineering*, 2007, Vol. 16, No. 4, p. 33-37.
- [4] LAFATA, P., VODRÁŽKA, J. Simulations and statistical evaluations of FEXT crosstalk in xDSL systems using metallic cable constructional arrangement. In *TSP - 31st International Conference Telecommunications and Signal Processing*. Budapest (Hungary), 2008.
- [5] LAFATA, P., VODRÁŽKA, J. Practical application of FEXT models for VDMT modulation. In *Proceedings of VIIIth Conference KTTO 2008*. Ostrava (Czechia), 2008, p. 85-88.
- [6] CENDRILLON, R., GINIS, G., VAN DEN BOGAERT, E., MOONEN, M. A near-optimal linear crosstalk canceller for upstream VDSL. *IEEE Transactions on Signal Processing*, 2006, vol. 54, no. 8, p. 3136-3146.
- [7] CENDRILLON, R., GINIS, G., MOONEN, M., ACKER, K. Partial crosstalk precompensation in downstream VDSL. *Signal Processing*, 2004, vol. 84, no. 11, p. 2005-2019.
- [8] VODRÁŽKA, J., LAFATA, P. Transmission environment modeling for VDMT system simulation. In *Proceedings CD-ROM of Digital Technologies 2007*. Žilina (Slovakia), 2007.
- [9] STARR, T., CIOFFI, J. M., SILVERMAN, P. J. *Understanding Digital Subscriber Line Technology*. Upper Saddle River (USA): Prentice Hall PTR, 1999.

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Jiří VODRÁŽKA was born in Prague in 1966. He joined the Department of Telecommunication Engineering, FEE, CTU in Prague in 1996 as a research assistant and received his Ph.D. degree in electrical engineering in 2001. He has been the head of the Transmission Media and Systems scientific group since 2005 and became an associate professor in 2008. He participates in numerous projects in cooperation with external bodies. Currently he also acts as vice-head of the Department.