Estimating a Spectral Correlation Function under the Conditions of Imperfect Relation between Signal Frequencies and a Sampling Frequency

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Abstract. *This paper is devoted to the estimation of the spectral correlation function. The examined signal is a simple DSB-SC signal. The aim of the paper is discovering events connected with nonstandard relations between signal frequency parameters and sampling frequency. They manifest by descending the spectral correlation function modulus. The events has been described analytically and verified using computer experiments. It is shown, that influence of the inaccuracy of the frequency is stronger for modulating frequency than for carrier frequency. Small inaccuracies of the frequencies have acceptable consequences.*

Keywords

Spectral correlation function, feature detector, cyclic frequency, spectral frequency, frequency offset.

1. Introduction

The main task in the cognitive radio systems is to reliably detect an existence of a primary user in the given frequency band [1]. A method of the detection and corresponding quality of detection are dependent on the stage of knowledge of the detected signal properties. The waveform of the detected signal is known in the case of the matched filter detection. No special knowledge is necessary for the energy detection. The feature detection lies between both mentioned. It consists in exploitation of the build-in periodicity of the received signal [2], [3]. Its main tool is a spectral correlation function [4], [5].

The cyclostationary feature detection is suitable for detecting digital signals [6]. A right relation among carrier frequency, a symbol period and sampling frequency is necessary for the right function of the feature detector [7]. The aim of this paper is to describe implications of the infraction of the mentioned relations.

The spectral correlation function S_x can be calculated using formulas published in [1]:

$$S_x(k,k_\alpha)=\frac{1}{N}\sum_{n=1}^{N}X_L\left(n,k+\frac{k_\alpha}{2}\right)X_L^*\left(n,k-\frac{k_\alpha}{2}\right),\ k,k_\alpha,n\in\mathbf{Z} \quad (1)$$

where

$$X_L(n,k)=\frac{1}{\sqrt{L}}\sum_{l=n-L/2}^{n+L/2-1}x(l)\exp(-j2\pi kl/L),\quad l\in\mathbf{Z}, \quad (2)$$

N is a number of the sliding DFT windows,
L is a width of the DFT window,
n is a shift of the sliding DFT window,
$X_L(n,k)$ is the DFT transform of the sequence $x(l)$,
α is cyclic frequency and quantity

$$k_\alpha=\frac{\alpha L}{F_s}, \quad (3)$$

f is spectral frequency and quantity

$$k=\frac{fL}{F_s} \quad (4)$$

where F_s is sampling frequency.

A very simple signal is chosen for analysis of the detection. Simplicity allows discovering events allied to changing the frequency signal parameters. The analyzed signal is a double side band – suppressed carrier (DSB-SC) signal given by

$$x(l)=\cos[(2\pi f_m/F_s)l+\varphi_m]\cos[(2\pi f_c/F_s)l+\varphi_c],\ l\in\mathbf{Z} \quad (5)$$

where $x(l)$ is DSB-SC signal,
f_c is carrier frequency,
f_m is modulating frequency,
φ_c is initial phase of the carrier,
φ_m is initial phase of the modulating signal.

Quantity watched in this paper is the value of the spectral correlation function for the spectral frequency $f = f_c$ and the cyclic frequency $\alpha = 2f_m$.

Let the base case of the relation between the carrier frequency and/or the modulating frequency and the sampling frequency is given this way

$$f_{\mathrm{c}} = \frac{k_{\mathrm{c}} F_{\mathrm{s}}}{L}, \quad k_{\mathrm{c}} \in \mathbf{N} \tag{6}$$

and

$$\alpha = \frac{2 k_{\mathrm{m}} F_{\mathrm{s}}}{L}, \quad k_{\mathrm{m}} \in \mathbf{N}\,. \tag{7}$$

Quality of the spectral correlation function estimation is investigated first for imperfect f_{c}, then for imperfect f_{m} and subsequently for both $f_{\mathrm{c}}, f_{\mathrm{m}}$ imperfect. Section 2 presents analytic description of effects evoked by deviations of the frequencies. Theoretical findings are verified and discussed for three cases of the frequency deviations in Section 3. Finally, a conclusion is given in Section 4.

2. Theory

2.1 DFT of the Harmonic Signal Using a Sliding Window

Let a harmonic signal is given by the equation

$$g(l) = \cos(\omega_{\mathrm{o}} l + \Phi), \quad l \in \mathbf{Z}, \tag{8}$$

where ω_0 is the normalized angular frequency and Φ is the initial phase.

DFT in (2) employs a window

$$w(l) = \begin{cases} 1 & \text{for } n - L/2 \le l \le n + L/2 - 1, \\ 0 & \text{otherwise}, \end{cases} \qquad n \in \mathbf{N}\,. \tag{9}$$

The examined signal is then defined as

$$\gamma(l) = \begin{cases} \cos(\omega_{\mathrm{o}} l + \Phi), & n - L/2 \le l \le n + L/2 - 1, \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

Affinity of DFT and DTFT allows using DTFT as a tool for determination of some relations [8].

DTFT of the window $w(l)$ is given by the equation

$$W(\mathrm{e}^{j\omega}) = \exp\left[-j\omega\left(n - \frac{1}{2} \right) \right] \frac{\sin\left(\omega \frac{L}{2} \right)}{\sin\left(\frac{\omega}{2} \right)} \tag{11}$$

and DTFT of the signal $g(l)$ is

$$G(\mathrm{e}^{j\omega}) = \pi \sum_{m=-\infty}^{\infty} \left[\mathrm{e}^{j\Phi} \delta(\omega - \omega_{\mathrm{o}} + 2\pi m) + \mathrm{e}^{-j\Phi} \delta(\omega + \omega_{\mathrm{o}} + 2\pi m) \right], \tag{12}$$

$m \in \mathbf{Z}$.

DTFT of the signal $\gamma(l)$ can be obtained using convolution:

$$\begin{aligned} \Gamma(\mathrm{e}^{j\omega}) = {} & \frac{1}{2} \exp\left[j\Phi - j(\omega - \omega_{\mathrm{o}})(n - \frac{1}{2}) \right] \frac{\sin[(\omega - \omega_{\mathrm{o}})L/2]}{\sin[(\omega - \omega_{\mathrm{o}})/2]} + \\ & + \frac{1}{2} \exp\left[-j\Phi - j(\omega + \omega_{\mathrm{o}})(n - \frac{1}{2}) \right] \frac{\sin[(\omega + \omega_{\mathrm{o}})L/2]}{\sin[(\omega + \omega_{\mathrm{o}})/2]}. \end{aligned} \tag{13}$$

An element $X_L(n,k)$ is for the signal $x(l)=g(l)$ and for the normalized angular frequency

$$\omega_k = \frac{2\pi}{L} k \tag{14}$$

given as

$$X_L(n,k) = \frac{1}{\sqrt{L}} \Gamma(\mathrm{e}^{j\omega}) \Big|_{\omega = \omega_k}. \tag{15}$$

The value of $X_L(n,k)$ is determined mainly by the first term of the right side of (13) for ω_k close to ω_0. The second term is then vanished.

The quantity

$$\varepsilon = \frac{L}{2\pi} (\omega_{\mathrm{o}} - \omega_k) \tag{16}$$

is introduced for characterization of the frequency deviation. The modulus of $X_L(n,k)$ can be found to be

$$|X_L(n,k)| \approx \frac{1}{2\sqrt{L}} \left| \frac{\sin(\pi\varepsilon)}{\sin(\pi\varepsilon / L)} \right|. \tag{17}$$

It can be shown that the relative modulus of $X_L(n,k)$ is practically independent on L.

Argument of $X_L(n,k)$ is approximately given by

$$\arg[X_L(n,k)] = \Phi - (\omega_k - \omega_{\mathrm{o}})(n - \frac{1}{2}) = \Phi - 2\pi\varepsilon(n - \frac{1}{2}) \frac{1}{L}. \tag{18}$$

The argument is dependent on the sliding parameter n for $\omega_k \neq \omega_0$.

2.2 Impacts on the Spectral Correlation Function

A summand in the sum in the right side of (1) is given by

$$S_x(k, k_\alpha, n) = X_L\left(n, k + \frac{k_\alpha}{2} \right) X_L^*\left(n, k - \frac{k_\alpha}{2} \right). \tag{19}$$

The signal DSB-SC has an upper harmonic component and a lower harmonic component. Let the amplitudes of the components are equal to 1, like in equation (8). Then for $k, k_\alpha \in \mathbf{Z}$ and simultaneously the nearest to $f_{\mathrm{c}} L/F_{\mathrm{s}}$ and $2 f_{\mathrm{m}} L/F_{\mathrm{s}}$, respectively,

$$|S_x(k, k_\alpha, n)| \approx \frac{1}{4L} \left| \frac{\sin(\pi\varepsilon)}{\sin(\pi\varepsilon / L)} \right|^2. \tag{20}$$

Initial phases of the components are $\varphi_{\mathrm{c}} + \varphi_{\mathrm{m}}$ and $\varphi_{\mathrm{c}} - \varphi_{\mathrm{m}}$. If the deviations of the components are the same, $\varepsilon_{\mathrm{u}} = \varepsilon_{\mathrm{l}} = \varepsilon$, then

$$\begin{aligned} \arg[S_x(k, k_\alpha, n)] \approx {} & \varphi_{\mathrm{c}} + \varphi_{\mathrm{m}} - 2\pi\varepsilon(n - \frac{1}{2}) \frac{1}{L} \\ & - \left[\varphi_{\mathrm{c}} - \varphi_{\mathrm{m}} - 2\pi\varepsilon(n - \frac{1}{2}) \frac{1}{L} \right] = 2\varphi_{\mathrm{m}}. \end{aligned} \tag{21}$$

The argument is given as

$$\arg\left[S_x(k,k_\alpha,n)\right] \approx \varphi_\mathrm{c} + \varphi_\mathrm{m} - 2\pi\varepsilon(n-\frac{1}{2})\frac{1}{L} - \left[\varphi_\mathrm{c} - \varphi_\mathrm{m} + 2\pi\varepsilon(n-\frac{1}{2})\frac{1}{L}\right] = 2\varphi_\mathrm{m} - 4\pi\varepsilon(n-\frac{1}{2})\frac{1}{L} \tag{22}$$

when the deviations of the components have the opposite sign, $\varepsilon_\mathrm{u} = -\varepsilon_\mathrm{l} = \varepsilon$. In this case $\arg[S_x(k,k_\alpha,n)]$ depends on the sliding parameter n. It could lead to additional descent of $|S_x(k,k_\alpha)|$ in (1).

3. Computer Experiments

3.1 The Base Case

The equations (6) and (7) are satisfied for the base case. The chosen set of the parameters of the signal (5) is $f_\mathrm{c} = 2.75$, $f_\mathrm{m} = 0.25$ and $\varphi_\mathrm{c} = \varphi_\mathrm{m} = 0$. The sampling frequency $F_\mathrm{s} = 8$, the DFT parameter $L = 64$. The value 0.5 has been chosen as cyclic frequency $\alpha = 2f_\mathrm{m}$. Coordinates of the spectral correlation function peak are spectral frequency f_c and cyclic frequency α. The corresponding integer number frequency coefficients, see (3) and (4), are $k_\alpha = 4$ for the cyclic frequency and $k = 22$ for the spectral frequency. Let the number of the DFT windows $N = 82$. The spectral correlation function is calculated using (2) and (1) and its modulus is depicted in Fig. 1. The peak $|S_x(22, 4)| = 4$ is labeled with an arrow.

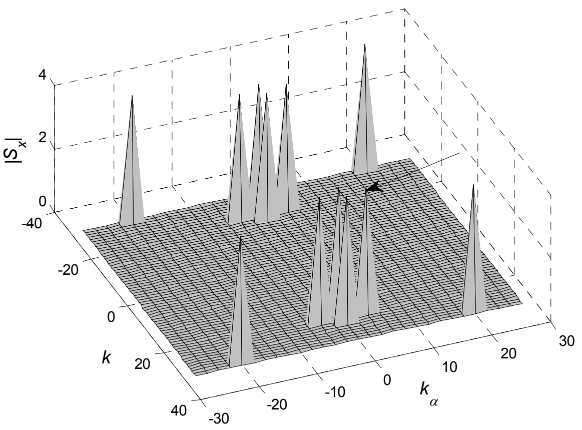

Fig. 1. A spectral correlation function. The base case.

3.2 A Carrier Frequency Offset

All parameters have the same values as in the previous case with the exception of the carrier frequency. Now the carrier frequency is given by the relation

$$f_\mathrm{c} = (22+\varepsilon)\frac{F_\mathrm{s}}{L}$$

where $\varepsilon F_\mathrm{s}/L$ is the frequency deviation of the carrier frequency.

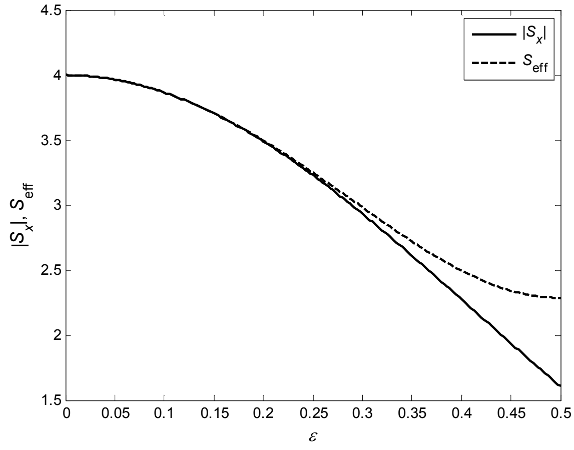

Fig. 2. An impact of the carrier frequency deviation.

Behavior of $|S_x(22, 4)|$ versus ε is shown in Fig. 2 by a solid line. It is in harmony with (20).

The dashed line presents values of the function $S_\mathrm{eff}(\varepsilon) = \sqrt{|S_x(22,4)|^2 + |S_x(23,4)|^2}$ defined to exploit the neighboring value of the spectral correlation function in order to slightly improve the detector performance for larger frequency deviation.

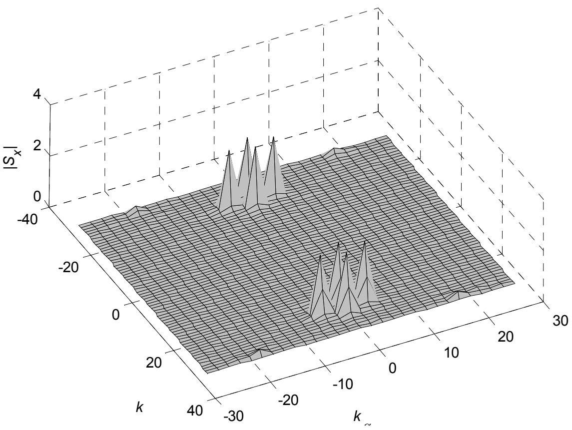

Fig. 3. A spectral correlation function. An impact of the carrier frequency deviation, $\varepsilon = 0.4$.

3.3 A Cyclic Frequency Offset

All parameters have the same values as in the base case (Section 3.1) with the exception of the modulating frequency f_m. Now the cyclic frequency α is given by the relation

$$\alpha = (4+2\varepsilon)\frac{F_\mathrm{s}}{L}$$

where $2\varepsilon F_\mathrm{s}/L$ is frequency deviation of the cyclic frequency.

The cyclic frequency offset is equal to 2ε. It corresponds to the modulating frequency deviation ε and simultaneously to the deviation ε of one spectral line. Downtrend of the spectral correlation modulus in Fig. 4 is more remarkable than equation (20) shows. Equation (22) gives explanation. The upper harmonic component of the DSB-SC signal is shifted towards infinity and the lower harmonic component is shifted towards minus infinity for the positive value of ε. The impact is dependent on the values of ε and N.

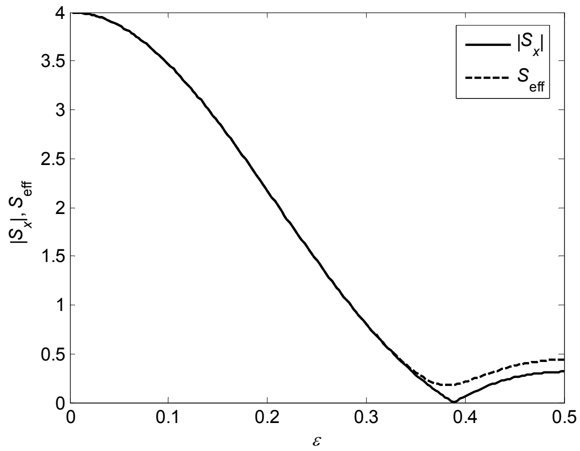

Fig. 4. An impact of the cyclic frequency deviation 2ε.

3.4 Conjoined Deviations of the Carrier Frequency and the Cyclic Frequency

The sampling frequency $F_s = 8$ and the DFT parameter $L = 64$ are set similarly to all previous cases. The nominal cyclic frequency α has the same value 0.5 as in the base case. The number of the DFT windows $N = 82$. Frequency coefficients are $k_\alpha = 4 + 2\varepsilon$ for the cyclic frequency and $k = 22 + \varepsilon$ for the spectral frequency. The spectral correlation modulus $|S_x(22, 4)|$ versus ε is displayed in Fig. 5. Both equations, (20) and (22), invoke their influence.

In the studied example the value of $|S_x(23, 4)|$ was too small and the quantity S_{eff} was not used.

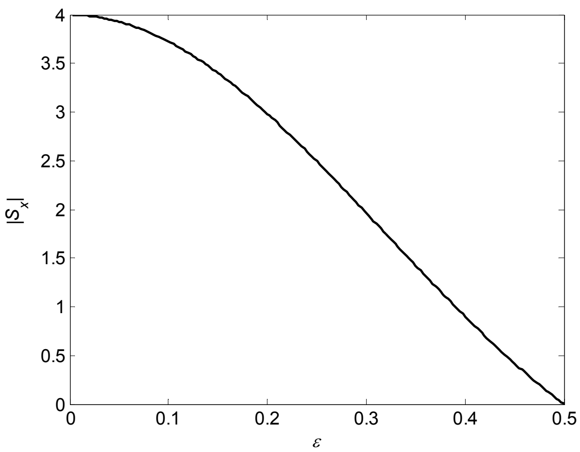

Fig. 5. A simultaneous offset of the cyclic frequency and the modulating frequency.

4. Conclusions

In this paper, estimating the spectral correlation function under conditions of irregular relations between a carrier frequency and a sampling frequency as well as between a modulating frequency and the sampling frequency has been investigated.

Theoretical analysis shows that the base sake of the spectral correlation function module decline is described by equation (20). It is valid in the case of inaccurate relation between the carrier frequency and the sampling frequency. Certain lessening the described effect can be achieved using exploitation of the neighboring value of the spectral correlation function.

An added possible cause of the added spectral correlation function module decline implies equation (22). It fully invokes in the case of irregular relation between the modulating frequency and the sampling frequency. The effect can be suppressed via reducing or changing the sliding window number N. But such step needs additional investigations.

Influence of the inaccuracy of the frequency is stronger for the modulating frequency than for the carrier frequency. Small inaccuracies of the frequencies are acceptable. To keep decreasing the $|S_x(k, k_\alpha)|$ less than 10%, the normalized frequency deviation ε (equation (16)) have to be smaller than 0.176 in the case of inexact carrier frequency and smaller than 0.086 in the case of inaccurate modulating frequency and $N = 82$.

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