Modeling of Human Head Surface by Using Triangular B-Splines

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Abstract. The paper deals with modeling of human head surface on the basis of quadratic triangular B-splines for the standard videocodec MPEG-4 SNHC. First, we define a triangular B-spline surface of any degree and then we present its calculation for modeling of the quadratic form. Next, we apply the quadratic triangular B-splines on modeling of the human head surface using 3D polygonal model Candide 3-1-6. For allocation of knots belonging to vertices of triangulation of the model, the limiting conditions are specified in the method of Monte-Carlo used for their generation. To achieve smoothness of the modeled human head surface, we propose an algorithm for calculating control points. Then for the proposed net of control points as well as the allocation of knots, the results of the modeled human head surface and its textured version using a texture of selected avatar are presented.

Keywords

Triangular B-splines, modeling, human head surface, 3D polygonal model, control net.

1. Introduction

Recently, information and communication technologies have extended from the real environment to virtual one [1]. In these environments, an important object is human who can be represented in their interiors as an avatar (virtual) or a clone [2]. The avatar is its coarse or symbolic representation without the need to express its real shape and motion. On the other side, the clone represents its authentic real shape including motion. In general, inside the interiors, avatars and clones may occur together. Dialog between them is carried out by using their cloned or virtual heads. Then virtual videocommunications enable fardistant participants to see the dialog inside the same virtual environment in which they are indirectly presented.

SNHC (Synthetic Natural Hybrid Coding) [3] is a subgroup of MPEG-4, which is specialized in coding of graphical models of real or virtual three dimensional (3D) objects. Standardization of the coding extends the range of initial applications of MPEG-4, because it enables a combination of real and synthetic objects in the virtual environment. A very important 3D object in real as well as in virtual environment is the human head. SNHC introduces new algorithms of human head coding based on modeling of its surface [4], animation [5] and texture [6].

Analysis and synthesis of the human head in the videocodec MPEG-4 SNHC use its polygonal (wireframe) 3D models from the computer graphics. The models are, however, a coarse approximation of human head surface, but by using suitable techniques of modeling [7] the one can be done more precisely with very good smoothness. Recently, the area of graphical modeling has been a subject of research interest, especially from the new construction methods of 3D objects point of view [4]. First, in the paper, we describe the triangular B-spline surface of the degree n on the basis of what consequently we present a procedure for calculating and modeling its quadratic form. Finally, we apply the quadratic triangular B-splines on modeling of the human head surface. To achieve smoothness of the modeled human head surface, we propose an algorithm for calculating the control points.

2. Triangular B-Spline Surface

In general, the triangular B-spline surface in 3D space (R₃) of coordinates (h, v, r) is defined over triangulation \mathcal{T} in the plane of coordinates (h, v), which is composed of connected triangles. Points of the surface are calculated as a weighted sum of control points in R₃, which uses triangular B-splines (TBS) [8] as weighting functions. Distribution of the control points in R₃ determines the final form of triangular B-spline surface [9] and those in connected version create a control net. For all that the surface of the degree *n* alone can be composed by surface patches of separate triangles of the triangulation τ , which are determined by their TBS of the same degree. Then a point of a selected patch for a triangle $I = \{t_{00}, t_{10}, t_{20}\}$ is calculated as follows [10]

$$\mathbf{P}(\mathbf{u}) = \sum_{|\boldsymbol{\beta}|=n} \left| \det\left(W_{\boldsymbol{\beta}}^{I} \right) M\left(\mathbf{u} \mid V_{\boldsymbol{\beta}}^{I} \right) \mathbf{c}_{\boldsymbol{\beta}}^{I} \right|$$
(1)

Where $\mathbf{P}(\mathbf{u}) = [P_h(\mathbf{u}), P_v(\mathbf{u}), P_r(\mathbf{u})]$ is given by its coordinates in space R_3 . Each point P(u) depends on the point $\mathbf{u} = (u_h, u_v)$ inside the triangle I in the plane (h,v). Also the control points $\mathbf{c}_{\beta}^{I} = (\mathbf{c}_{\beta}^{I}, \mathbf{c}_{\nu\beta}^{I}, \mathbf{c}_{\nu\beta}^{I})$ are defined in the same space \hat{R}_3 , but not depending on u. In (1) the triangular B-spline $M(\mathbf{u}|V_{\boldsymbol{\beta}})$ of the degree *n* is normalized by the determinant $|\det(W^{I}_{\beta})|$ and this form is used as a weighting function for the control point \mathbf{c}^{I}_{β} . The set of three points $W'_{\beta} = \{t_{0\beta_0}, t_{1\beta_1}, t_{2\beta_2}\}$ determines a triangle whose vertices are given by the last knots with the biggest indices, i.e. $t_{0\beta_0}, t_{1\beta_1}, t_{2\beta_2}$, belonging to the corresponding vertices $\{\mathbf{t}_{00}, \mathbf{t}_{10}, \mathbf{t}_{20}\}$ of the triangle *I*. They are from the set $V_{\boldsymbol{\beta}}^{\boldsymbol{\ell}} = \{ \mathbf{t}_{00}, \dots, \mathbf{t}_{0\boldsymbol{\beta}0}, \mathbf{t}_{10}, \dots, \mathbf{t}_{1\boldsymbol{\beta}1}, \mathbf{t}_{20}, \dots, \mathbf{t}_{2\boldsymbol{\beta}2} \}$ by which a support of the triangular B-spline $M(\mathbf{u}|V_{\beta})$ is defined. Normalized TBS expressed by $|\det(W_{\beta}^{l})| M(\mathbf{u}|V_{\beta}^{l}) \ge 0$ in (1) for the unite control points $\mathbf{c}^{I}_{\beta} = 1$ give

$$\sum_{|\boldsymbol{\beta}|=n} \left| \det \left(W_{\boldsymbol{\beta}}^{I} \right) \right| M \left(\mathbf{u} \middle| V_{\boldsymbol{\beta}}^{I} \right) = 1.$$

This condition of normality is valid inside the whole triangle *I* only for some allocations of knots. The number of the control points $\mathbf{c}_{\boldsymbol{\beta}}^{I}$ in space R_{3} for the surface patch is given by (n + 1)(n + 2) / 2. The value of component β_{i} , i = 0,1,2 of index $\boldsymbol{\beta}$ larger, the closer the corresponding control point to the vertex \mathbf{t}_{i0} of triangle *I* to which then belong more knots. Control points may be anywhere in space R_{3} , but their position determines places where their influence on modeling of the surface patch is the biggest.

The whole surface over the full triangulation τ is composed of surface patches of separate triangles and can be calculated as [11]

$$\mathbf{P}(\mathbf{u}) = \sum_{I \in \tau} \sum_{|\boldsymbol{\beta}|=n} \left| \det\left(W_{\boldsymbol{\beta}}^{I}\right) \right| M\left(\mathbf{u} \left| V_{\boldsymbol{\beta}}^{I} \right) \mathbf{c}_{\boldsymbol{\beta}}^{I}$$
(2)

Note, that nonzero contributions of the particular surface patches are not only in areas of corresponding triangles, but also in the surrounding outside them determined by supports of their triangular B-splines $M(\mathbf{u}|V_{\beta}^{I})$ in (1). It is the basic difference from classical methods of construction of surfaces, for example by using Bézier's surface patches [12]. Just interference of the triangular B-spline surface patches ensures global smoothness of the whole surface without additional limitations of control points positions. Hereby for unite control points in (2) the condition of surface normality over the full triangulation τ stays in validity.

3. Modeling of Quadratic Triangular B-spline Surface

The quadratic triangular B-spline surface [13] is composed of surface patches of separate triangles of the triangulation τ , which are calculated by using TBS of the degree 2 [14]. Then points of the quadratic triangular Bspline patch for a triangle $I = \{\mathbf{t}_{00}, \mathbf{t}_{10}, \mathbf{t}_{20}\}$ are calculated on the basis (1) after substitution n = 2. Consequently by breaking down in such a way arranged equation (1) for calculation its points, from (2) we get

$$\mathbf{P}(\mathbf{u}) = \left| \det \left(W_{110}^{I} \right) \right| M \left(\mathbf{u} \mid V_{110}^{I} \right) \mathbf{c}_{110}^{I} + \\ + \left| \det \left(W_{101}^{I} \right) \right| M \left(\mathbf{u} \mid V_{101}^{I} \right) \mathbf{c}_{101}^{I} + \\ + \left| \det \left(W_{011}^{I} \right) \right| M \left(\mathbf{u} \mid V_{011}^{I} \right) \mathbf{c}_{011}^{I} + \\ + \left| \det \left(W_{200}^{I} \right) \right| M \left(\mathbf{u} \mid V_{200}^{I} \right) \mathbf{c}_{200}^{I} + \\ + \left| \det \left(W_{020}^{I} \right) \right| M \left(\mathbf{u} \mid V_{020}^{I} \right) \mathbf{c}_{020}^{I} + \\ + \left| \det \left(W_{002}^{I} \right) \right| M \left(\mathbf{u} \mid V_{002}^{I} \right) \mathbf{c}_{002}^{I} + \\ + \left| \det \left(W_{002}^{I} \right) \right| M \left(\mathbf{u} \mid V_{002}^{I} \right) \mathbf{c}_{002}^{I} + \\ + \left| \det \left(W_{002}^{I} \right) \right| M \left(\mathbf{u} \mid V_{002}^{I} \right) \mathbf{c}_{002}^{I}$$

where the sets of points V_{β}^{I} determining the supports of separate quadratic TBS in (3) and the corresponding sets of points W_{β}^{I} for calculation of the normalization constants by their determinants with $|\beta| = \beta_0 + \beta_1 + \beta_2 = 2$ are shown in Tab. 1.

V^{I}_{β}	$W^I_{m eta}$
$V_{110}^{I} = \left\{ \mathbf{t}_{00}, \mathbf{t}_{01}, \mathbf{t}_{10}, \mathbf{t}_{11}, \mathbf{t}_{20} \right\}$	$W_{110}^{I} = \left\{ \mathbf{t}_{01}, \mathbf{t}_{11}, \mathbf{t}_{20} \right\}$
$V_{101}^{I} = \left\{ \mathbf{t}_{00}, \mathbf{t}_{01}, \mathbf{t}_{10}, \mathbf{t}_{20}, \mathbf{t}_{21} \right\}$	$W_{101}^{I} = \left\{ \mathbf{t}_{01}, \mathbf{t}_{10}, \mathbf{t}_{21} \right\}$
$V_{011}^{I} = \left\{ \mathbf{t}_{00}, \mathbf{t}_{10}, \mathbf{t}_{11}, \mathbf{t}_{20}, \mathbf{t}_{21} \right\}$	$W_{011}^{I} = \left\{ \mathbf{t}_{00}, \mathbf{t}_{11}, \mathbf{t}_{21} \right\}$
$V_{200}^{I} = \left\{ \mathbf{t}_{00}, \mathbf{t}_{01}, \mathbf{t}_{02}, \mathbf{t}_{10}, \mathbf{t}_{20} \right\}$	$W_{200}^{I} = \left\{ \mathbf{t}_{02}, \mathbf{t}_{10}, \mathbf{t}_{20} \right\}$
$V_{020}^{I} = \left\{ \mathbf{t}_{00}, \mathbf{t}_{10}, \mathbf{t}_{11}, \mathbf{t}_{12}, \mathbf{t}_{20} \right\}$	$W_{020}^{I} = \left\{ \mathbf{t}_{00}, \mathbf{t}_{12}, \mathbf{t}_{20} \right\}$
$V_{002}^{I} = \left\{ \mathbf{t}_{00}, \mathbf{t}_{10}, \mathbf{t}_{20}, \mathbf{t}_{21}, \mathbf{t}_{22} \right\}$	$W_{002}^{I} = \left\{ \mathbf{t}_{00}, \mathbf{t}_{10}, \mathbf{t}_{22} \right\}$

Tab.1. Sets of points V_{β}^{l} and W_{β}^{l} with $|\beta| = \beta_{0} + \beta_{1} + \beta_{2} = 2$.

The number of control points \mathbf{c}_{β}^{I} will be (2+1)(2+2)/2=6, from which 3 are placed near the vertices of triangle *I* after their orthographic projection to the plane (h, v). and the remaining 3 near the centers of its sides after the same projection. Fig. 1 shows the normalized quadratic triangular B-spline patch for unit control points $\mathbf{c}_{\beta}^{I}=1$ with the defined support by the vertices of the triangle *I* and their corresponding knots.

A quadratic triangular B-spline surface over the triangulation τ will be calculated by (2) using (3) for calculation its surface patches. If the triangulation τ has two triangles with the allocation of knots as shown in Fig. 2a, then its normalized form for unit control points is shown in Fig. 2b. From the figure, it is clear that even if the supports of separate surface patches overlap, the condition of normality is always valid over the full triangulation τ , which follows the properties of normalized quadratic TBS. The positions of knots of separate vertices of the triangulation τ will affect the area of normality of the surface. For a triangle, those should lie in areas bounded by its prolonged sides. If the triangulation τ is composed from several triangles, then the areas in which should be knots of common vertices are created by joining of the ones for separate triangles. Even though the

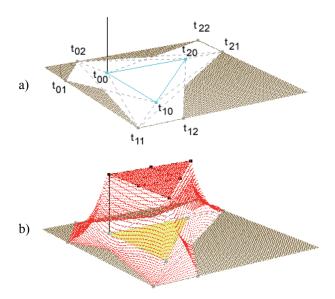


Fig. 1. a) Suport, b) graph of the normalized quadratic triangular B-spline patch.

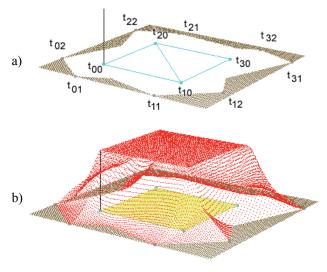


Fig. 2. a) Suport, b) graph of the normalized quadratic triangular B-spline surface over the triangulation τ composed of two triangles.

normality for separate triangles may be disturbed, the whole normality over the full triangulation τ is kept.

Simple modeling [15] of the normalized quadratic triangle B-spline surface can be carried out by changing the position of control points \mathbf{c}_{β}^{I} . For the assumed triangulation τ in Fig. 2a, composed of two triangles with one adjoining side or two common vertices, their number is 12. To the vertices belong not only the same knots, but the control points, too, except for the next two the same control points belong to the center of adjoining side. Possible forms of the modeled quadratic triangular B-spline surface, for the triangulation in Fig. 2a, along with control nets are shown in Fig. 3. From the figure it is clear that linear as well as nonlinear forms (shapes) of the surface can be modeled with keeping the smoothness thank to using the quadratic TBS.

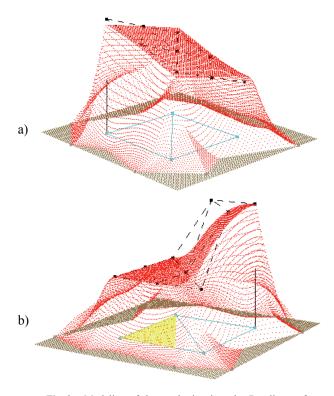


Fig. 3. Modeling of the quadratic triangular B-spline surface with a) linear, b) nonlinear form.

4. Modeling of Human Head Surface

Fundamentals of modeling of the quadratic triangular B-spline surface can be expanded on modeling of the surface of a complex 3D object like the human head. It is based on its 3D polygonal model which is determined by a list of vertices and polygons. Vertices are defined by their coordinates in R₃ and polygons by the ones that create them. Typical polygons are triangles or quadrangles. The main advantage of triangles is that their vertices always lie in the same plane, which leads to simple manipulation such as in graphical means of OpenGL. The density of polygons in a 3D model of the human head depends on the number of details of the separate parts like eyes, mouth, nose, etc. For our purposes we used the freely available 3D polygonal model Candide 3-1-6 [16] (see Fig. 4), which contains 113 vertices and 184 polygons (triangles) and represents a coarse approximation of the human head surface.

The triangulation τ , over which the quadratic triangular B-spline human head surface will be calculated. is given by the orthographic projection of the model Candide from R₃ to the plane of coordinates (h, v), i.e. by its front view in Fig. 4a. Then two knots are allocated to each of its vertices in such a way to keep valid the condition of normality over the full triangulation τ . However, some non accepted positions have to be excluded according to Fig. 5.

In preprocessing of the input triangulation τ the limitations for positions of knots are established and then

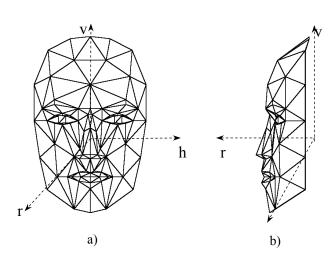


Fig. 4. 3D polygonal model Candide 3-1-6 : a) front view, b) profile.

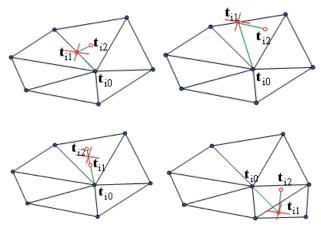


Fig. 5. Non-accepted positions of knots \mathbf{t}_{i1} and \mathbf{t}_{i2} of the vertex \mathbf{t}_{i0} in a triangulation.

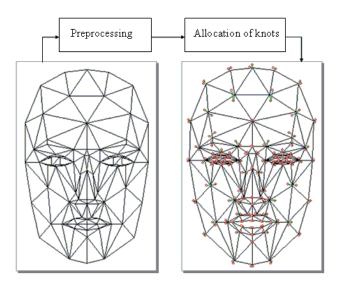


Fig. 6. Allocated knots to vertices of the triangulation t of the model Candide 3-1-6.

their assigning to separate vertices is carried out by the method of Monte Carlo [17]. The method has one degree of freedom, i.e. the distance of knots from the vertex \mathbf{t}_{i0} which they belong to. It is determined for symmetric objects like the human head where the left and right sides are the same. The results of allocation of the knots to vertices of the triangulation τ by the method are illustrated in Fig. 6.

Knots have an influence on the modeled quadratic triangular B-spline human head surface in area of intersection of supports of the separate quadratic TBS with vertices to which the ones are assigned. The biggest influence of the knots on the modeled surface is near their surroundings.

Assuming a regular distribution of the knots when the condition of normality is valid over the full tri-angulation τ , the next modeling of the human head surface is possible by the control points \mathbf{c}_{β}^{I} in space \mathbf{R}_{3} with coordinates (h, v, r) as it is seen in Fig. 4. We recognize vertex ($\mathbf{c}_{200}^{I}, \mathbf{c}_{020}^{I}, \mathbf{c}_{002}^{I}$) and side ($\mathbf{c}_{110}^{I}, \mathbf{c}_{011}^{I}, \mathbf{c}_{101}^{I}$) control points which are shown in Fig. 7 for a triangle $I = {\mathbf{t}_{00}, \mathbf{t}_{10}, \mathbf{t}_{20}}.$ While vertex control points are identical with vertices of the model Candide in R₃, the side control points only lie close to centers of sides of its triangles and their exact positions have to be calculated. The separate control points will mostly influence the modeled human head surface in their surroundings. Forasmuch as a triangle has 6 control points, the full triangulation will have together 6 times the number of its triangles of control points. For all that, if two or more triangles have a common vertex or side, the control points belonging to them have to have the same positions.

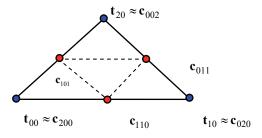


Fig. 7. Vertex and side control points of a triangle.

At the calculation of side control points, we continue in such a way that for each side of triangles of the model Candide in R₃ the parametric equation of a line p on which its control point will lie is determined. The line passes through the center **S** of the side and is perpendicular to it in the plane (v, r). Hereby it is identical with the axis of an angle α in the plane (h, r) where orthographical projections of flowlines of the center **S** and remaining vertices of triangles with this common side contain it. For a part of 3D model in Fig. 8, the properties of line p are illustrated for the side given by vertices (**t**₃₀, **t**₈₀) in Fig.9.

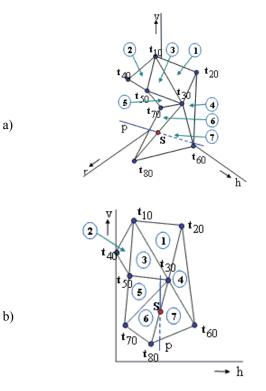


Fig. 8. a) A part of 3D model in space R3, b) its orthographical projection to the plane (h, v).

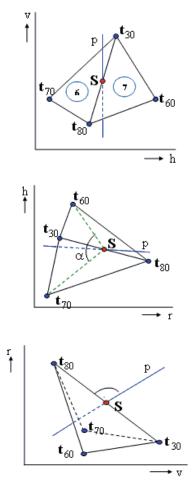


Fig. 9. The properties of line p in the center **S** of side $(\mathbf{t}_{30}, \mathbf{t}_{80})$.

In general the line is in space R₃, but for sides of 3D model only with one triangle corresponding to border of the triangulation τ it is determined in the plane (h, v). Its parametric expression $p(\mathbf{S}, \mathbf{g}) = \mathbf{S} + m\mathbf{g}$ is given by the center $\mathbf{S} = (\mathbf{S}_{h}, \mathbf{S}_{v}, \mathbf{S}_{r})^{T}$ determined by an arithmetic average of vertices of corresponding side and the directional vector $\mathbf{g} = (g_{h}, g_{v}, g_{r})^{T}$ whose components follow out from the required properties.

Next, we calculate the distances of the orthographical projections of each side of 3D model triangles to the planes (v, r), (h, r), (h, v) and they will be the components of the vector $\mathbf{D} = (d_{vr}, d_{hr}, d_{hv})^{\mathrm{T}}$. Also, for the side we determine a set J of all triangles in which there is at least one of two vertices of the side as it is seen in Fig. 8, where for the side $(\mathbf{t}_{30}, \mathbf{t}_{80})$, the set is $J = \{1, 3, 4, 5, 6, 7\}$. Then for each of them we make up an equation of this plane $\mathbf{H}_i \mathbf{V} + h_i = 0$, i = 1, ..., |J|, where the vector $\mathbf{V}=(h, v, r)^{\mathrm{T}}$ represents a point of the one and the vector of constants $\mathbf{H}_i = (a_i, b_i, c_i)$ as well as the single constant h_i are given by vertices of the triangle. After substitution of the points of line p for $\mathbf{V} = \mathbf{S} + m\mathbf{g}$ in the equations, it is possible to calculate its intersections with the planes as follows

$$\mathbf{H}_{i}\left(\mathbf{S}+m_{i}\mathbf{g}\right)+h_{i}=0\tag{4}$$

where m_i is the calculated parameter of intersection with the plane of i-th triangle from set J. Finally, the resultant side control point belonging to the center **S** of the corresponding side of a triangle is calculated as

$$\mathbf{s}_{\boldsymbol{\beta}}^{I} = \mathbf{S} + \frac{\mathbf{k}_{1} \mathbf{D}}{\left| J \right|} \sum_{i \in M} \tanh\left(\mathbf{k}_{2} m_{i} \mathbf{g}\right)$$
(5)

where |J| is the number of triangles of the set J and k_1 , k_2 are constants by which we can affect its position in space R_3 . Using an experiment for the triangulation τ of the model Candide, the values of the constants $k_1 = \pi / 10$, $k_2 = 3$ have been obtained for side control points inside the triangulation and $k_1 = 2\pi / 30$, $k_2 = 5$ for the ones at its border. Next, the calculated side control points can be manually removed for correction purpose of the modeled human head surface. Positions of side control points have relevant influence on the modeled surface mainly near their surroundings. A block scheme of the complete process of modeling the human head surface by using TBS is shown in Fig.10.

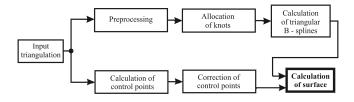


Fig. 10. Block scheme of modeling of the human head surface by using the triangular B-splines.

The human head surface can be calculated continually in each point inside the input triangulation or over its finer structure that we used for calculation of the resultant surface in Fig. 11. After its texturing by texture of a selected avatar, the modeled human head is in Fig.12.

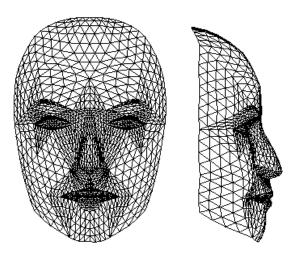


Fig. 11. The quadratic triangular B-spline human head surface.

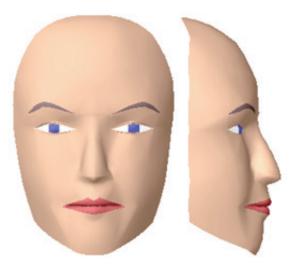


Fig. 12. The modeled human head of a selected avatar by the quadratic triangular B-spline surface.

5. Conclusion

Moding of the human head surface by using the triangular B-splines enables making its smoothness form from an edginess 3D model composed of the triangular net. The surface is created by surface patches calculated using the quadratic triangular B-splines for separate triangles of the net. The shape of the surface is given by the distribution of the vertex and side control points, enabling its global and local modeling. The relationship between the surface and the control net is invariant toward transformations like scaling, rotation and translation. The idea of modeling using the triangular B-splines is universal and can be applied on modeling of any 3D object surface.

MPEG-4 SNHC specifies for 3D polygonal mod-els of the human head their neutral (initial) state as well as feature points. These are arranged in groups referring to certain parts of the human head as mouth, nose, eyes, etc. By using the feature points from a real human head it is possible to form a selected universal polygonal 3D model in such a way to bring its feature points in coincidence with those ones from the real human head. Coordinates of the feature points of the real human head represent facial definition parameters (FDP). Also, in the simplest case, they can be directly obtained from the output of a 3D scanner. Because 3D scanners are less accessible and more frequently the real human head is scanned by using a camera, FDPs are calculated from its two orthogonal views, i.e. frontal and profile of the head. Then on the basis of knowledge of FDP, using the proposed method of modelling with the triangular B-splines, it is possible to create a surface with the shape of a real human head which is a very important part of its cloning.

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