Multicriteria Optimization of Antennas in Time-Domain

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Abstract. An original approach to the time-domain multicriteria optimization of antennas is presented. For a given excitation pulse, the time-domain objective function takes the "time-domain impedance matching", distortion of responses at the feeding point and in a desired radiating direction (with respect to the excitation pulse), and the radiated energy in the desired direction into account. The objective function is tested on the optimization of a bow-tie antenna using the particle swarm optimization. The proposed approach is suitable for the design of broadband antennas.

Keywords

Multicriteria optimization, particle swarm optimization (PSO), time domain integral equation (TDIE).

1. Introduction

The time-domain integral equation (TDIE) method has become a popular tool applied to the numerical analysis of electromagnetic radiation and scattering [1]-[4]. Essentially, if broadband information is desired, the timedomain solution of electromagnetic problems is more efficient than the frequency-domain one. For the analysis of antennas in the transmitting mode, the structure is excited by a desired voltage pulse at the feeding point of the antenna in order to find the transient response of the current. In case of antennas in the receiving mode, the goal is the same, but the whole structure is excited by the incident wave.

Due to the frequency-domain nature of antenna parameters, time responses of computed quantities have to be converted to the frequency-domain, where the objective function is formulated. However, in order to avoid the Fourier transformation of the time response at each step of an optimization procedure, the objective function is more conveniently defined in the time-domain. Following this principle, the objective function is formulated in [5] in the time-domain, taking only matching of an antenna to the desired excitation pulse into account. Other important phenomena (such as the influence of a feeding line of an antenna, the antenna radiation), are not considered.

In this paper, the multicriteria objective function for the optimization of antennas directly in the time-domain is discussed. The proposed approach is suitable for the design of broadband antennas.

2. Time-Domain Parameters

In case of broadband or pulse radiation antennas, an antenna should be matched to the feeding line, radiate a waveform similar to the excitation pulse, and most energy should be radiated in a direction where the pulse is of the desired shape. For the proper optimization in the time-domain, all this facts should be considered.

An antenna is matched to the feeding line if no energy is reflected back from its feeding point. In the timedomain, energy propagating forward and backward along the feeding transmission line can hardly be distinguished. Moreover, an antenna is usually analyzed without the feeding transmission line.

The time-domain condition that no energy is reflected from the feeding point of the antenna (in case the antenna is analyzed without the feeding line) can be accomplished if:

- 1. The shape of the excitation voltage pulse U(t) at the feeding point is the same as the current response I(t). Then, the antenna is able to accept all the energy of the excitation pulse. According to the systems theory [6], the transfer function of such kind of systems is a constant, at least for the most important part of the spectrum of the excitation signal. In the case of the antenna, the transfer function is equal to the input admittance of the antenna. Since the location of the admittance is then real. However, the input admittance of the antenna can be different from the admittance of the feeding line.
- 2. The input admittance of the antenna at its feeding point is the same as the admittance of the feeding line.

For checking the similarity between the excitation voltage pulse U(t) and the computed current response I(t) at the feeding point, the normalized cross-correlation function ρ_{UI} can be used. In [7], this quantity is referred to as *fidelity*, and is defined as the maximum of the normalized cross-correlation function ρ_{UI} between two pulses U(t) and I(t)

$$FF_{\max} = \rho_{UI} = \left\lfloor \frac{\left| \rho_{UI}(t) \right|}{\sqrt{\rho_{UU}(0)\rho_{II}(0)}} \right\rfloor_{\max}$$
(1)

where $\rho_{UU}(0)$ and $\rho_{II}(0)$ are the auto-correlation functions of the pulses U(t) and I(t), respectively. In our case, the fidelity should not be the maximum of the cross-correlation function ρ_{UI} , but the normalized cross-correlation function for t = 0 s, since both pulses are computed at the same location. Therefore, this quantity is denoted to as *the fidelity factor* at the feeding point FF_0 . Substituting t = 0 s in (1), we arrive at

$$FF_{0} = \rho_{UI} = \left[\frac{\rho_{UI}(0)}{\sqrt{\rho_{UU}(0)\rho_{II}(0)}}\right].$$
 (2)

If FF_0 is 1, then the shapes of both pulses are the same, and the antenna is "matched" to the excitation pulse. Otherwise, the value FF_0 is smaller than 1. In addition, we removed the absolute value applied on the cross-correlation function ρ_{UI} , since the real part of the input admittance of the antenna is positive.

The next step is represented by the computation of the input admittance. Upon going back to the frequency-domain, the following idea is accounted for: as long as the important components of the excitation pulse are within the frequency range limited by the frequencies ω_1 and ω_2 , the spectrum of the current response is also in this range. For $FF_0 = 1$, the transfer function Y (in this case, the input admittance) is constant and real and, consequently, the current in the frequency-domain can be evaluated as

$$I(\omega) = YU(\omega), \text{ for } \omega \in (\omega_1, \omega_2)$$

$$I(\omega) \approx 0, \text{ otherwise}$$
(3)

where $U(\omega)$ is the spectrum of the excitation signal U(t) and $I(\omega)$ is the spectrum of the computed current response I(t). Applying inverse Fourier transform to (3), the input admittance follows and it reads

$$Y = \frac{I(t)}{U(t)}.$$
 (4)

Equation (4) is valid for $FF_0 = 1$ only, and for instants *t* where $U(t) \neq 0$ and $I(t) \neq 0$. During optimization, FF_0 does not equal 1 and the input admittance of the antenna is not constant and real, implying that (4) is not valid. For these reasons, we introduce a time-domain average input admittance Y_{avrg} , which is defined as an average of *N* ratios of the current and the voltage values at the time t_n

$$Y_{avrg} = \frac{1}{N} \sum_{n=1}^{N} \frac{I(t_n)}{U(t_n)} \,.$$
(5)

At points t_n the relative error of the shape of pulses is

$$\varepsilon(t) = \frac{I_{norm}(t) - U_{norm}(t)}{U_{norm}(t)}.100$$
(6)

where

$$I_{norm}(t) = \frac{I(t)}{\max\left[I(t)\right]},\tag{7a}$$

$$U_{norm}(t) = \frac{U(t)}{\max\left[U(t)\right]}$$
(7b)

is smaller than the desired relative error ε_d . Thus, the number of the current and the voltage values at the time t_n , N, depends on the desired relative error ε_d of the shape of pulses. An acceptable value for this error is about 5 %.

The closer the fidelity factor FF_0 is to 1, the closer the time-domain average input admittance Y_{avrg} is to the real and constant input admittance Y. Thus, the antenna time-domain average input admittance is an auxiliary quantity used in the optimization procedure only.

In order to match the antenna to the feeding line, the input admittance of the line Y_W has to be equal to the input admittance of the antenna Y_{avrg} (or, when FF_0 is approximately 1, to Y). To compare their similarity, the time-domain matching factor is introduced as

$$MY = 1 - \frac{\left|Y_{avrg} - Y_{w}\right|}{\max[Y_{avrg}; Y_{w}]}.$$
(8)

Again, the closer this factor is to 1, the better the antenna matching to the feeding line will be.

Apart from the antenna matching, the radiated waveform should be similar in shape to the excitation pulse and, moreover, most of the energy should be radiated in the direction where the pulse has the desired shape. Consequently, we now turn to the similarity between the excitation voltage pulse U(t) and the radiated pulse $|\mathbf{E}(\mathcal{G}, \varphi, t)|$, the intensity of the radiated pulse being evaluated in the far-zone of the antenna. In the desired direction, defined in terms of the elevation angle \mathcal{G}_d and the azimuth angle φ_d , the fidelity factor and the normalized cross-correlation function ρ_{UE} can be used in the same sense as in [7] or, alternatively, according to (1) because the pulse's location in time depends on the distance r between the antenna and the observation point according to the expression

$$FE_{\max} = \rho_{UE} = \left[\frac{\left|\rho_{UE}(t)\right|}{\sqrt{\rho_{UU}(0)\rho_{EE}(0)}}\right]_{\max}.$$
 (9)

The meaning of the symbols in (9) is similar to that of those used in (1). Note that (9) indicates that FE_{max} gets

closer to 1 as the similarity of the pulses U(t) and $|\mathbf{E}(\mathcal{G}_{d}, \varphi_{d}, t)|$ increases.

The energy radiated in the desired direction is the last quantity we are interested in for the time-domain optimization of antennas. The energy of the transient pulse in the direction defined by the angles (\mathcal{G}, φ) can be evaluated as [6]

$$E_{E}(\vartheta,\varphi) = \int_{-\infty}^{\infty} \left| \mathbf{E}(\vartheta,\varphi,t) \right|^{2} dt .$$
 (10)

Evaluating (10) for all directions (in discrete sense) and normalizing the result by its maximal value $E_{\text{EMax}}=\max[E_{\text{E}}(\vartheta, \varphi)]$, we obtain the mean-value directivity pattern of the antenna

$$E_{Enorm}\left(\vartheta,\varphi\right) = \frac{E_{E}(\vartheta,\varphi)}{E_{EMax}}.$$
 (11)

The maximum radiation appears in the direction where the value of $E_{\text{Enorm}}(\vartheta, \varphi)$ is 1.

3. Time-Domain Multicriteria Objective Function

In the previous section, the time-domain antenna parameters were described and discussed. The introduced parameters are now exploited for proper optimization of broadband and pulse radiation antennas in the timedomain. Recall that the optimum value of all parameters defined in the equations (2), (8), (9) and (11) is 1.

The multicriteria objective function can be defined in a straightforward way as

$$OF = \left(\left(1 - FF_0 \right)^2 + \left(1 - MY \right)^2 + \left(1 - FE_{\max} \left(\mathcal{G}_d, \varphi_d \right) \right)^2 + \left(1 - FE_{\max} \left(\mathcal{G}_d, \varphi_d \right) \right)^2 \right)^{\frac{1}{2}}$$
(12)
+ $\left(1 - E_{Enorm} \left(\mathcal{G}_d, \varphi_d \right) \right)^2 \right)^{\frac{1}{2}}$

with the angles \mathcal{G}_d and φ_d corresponding to the desired direction in which most of the energy needs to be radiated. With respect to the function in (12) it is firstly noted that the first and the second term account for the matching of the antenna to the feeding line. Furthermore, the third term ensures that the transmitted pulse is not distorted for the desired direction of radiation and \mathcal{G}_d and φ_d and the fourth term ensures that most of the energy is radiated in that direction. All requirements are met if the objective function (12) is zero, which is the absolute minimum of this function.

4. Numerical Example

The proposed time-domain, multicriteria function is now used for the optimization of the simple bow-tie

antenna [8] presented in Fig. 1. Its main design parameters are the length of the dipole *L*, the width of the feeding strip *w*, and the arm angle of the bow-tie α that will be taken to be the state variables in the optimization task. The objective is to design an antenna that is matched to a feeding line with the admittance $Y_W = 10$ mS in the frequency range from 2 to 4 GHz, which radiates the energy uniformly within the given frequency range in the direction $\varphi_d = 0^\circ$, and $\vartheta_d = 0^\circ$ (perpendicularly to the plane of the drawing in Fig. 1), and also ensures maximum radiation is in that direction. These demands can be accomplished by minimizing the objective function (12). Note that the demand that the antenna radiates energy uniformly within the given frequency translates in the relevant band-limited transmitted pulse not being distorted.



Fig. 1. Typical bow-tie antenna and the definition of its parameters.

The antenna is analyzed in the time-domain by applying TDIE. The body of the analyzed structure is modeled by triangular patches. Rao-Wilton-Glisson (RWG) expansion functions [11] are used as spatial basis and testing functions. Weighted Laguerre polynomials are employed as temporal basis and weighting functions. Thus, the marching-on-in-order scheme [2], [3] is utilized. The feeding edge model [9] is used for the excitation of the antenna. The particle swarm optimization (PSO) algorithm [10] uses the numerical model for evaluating the objective function.

For optimization, the harmonic signal is modulated by the Gaussian pulse

$$U(t) = U_0 \frac{4}{cT\sqrt{\pi}} e^{-\left[\frac{4}{T}(t-t_0)\right]^2} \cos\left(2\pi f_0(t-t_0)\right)$$
(13)

where *T* is the width of the Gaussian pulse, *c* is the velocity of the electromagnetic wave in vacuum, f_0 is the frequency of the harmonic signal and t_0 is the time delay of the pulse. For the given frequency range, the pulse has the following parameters: $U_0 = 10\pi$ V, T = 2.4 ns, $t_0 = 2.3$ ns, and $f_0 = 3 \cdot 10^9$ Hz. The pulse and its spectrum are depicted in Fig. 2.



Fig. 2. Harmonic signal modulated by Gaussian pulse a), the spectrum b).

The state variables can vary in the following limits:

- $L \in <100 \text{ mm}; 190 \text{ mm}>,$
- $w \in \langle 5 \text{ mm}; 15 \text{ mm} \rangle$,
- $\alpha \in \langle 30^{\circ}; 70^{\circ} \rangle$.

The desired relative error of the shape of pulses \mathcal{E}_d is set to 5 %.

PSO is used in its conventional form [10]. A swarm consists of 15 agents, and the optimization runs for 70 iterations. The inertial weight is linearly decreasing from the value 0.9 in the initial iteration to the value 0.4 in the last one. Both the personal scaling factor and the global one are set to 1.49. The space of variables in the state vector is surrounded by absorbing walls.



Fig. 3. Evolution of objective function.

The evolution of the objective function is depicted in Fig. 3. For the final iteration, the objective function reaches the value OF=0.0535, with the partial criterions amounting to $|1-FF_0| = 0.04498$, |1-MY| = 0.0094 ($Y_{\text{avrg}} = 9.91$ mS for $\varepsilon_{\text{d}} = 5\%$), $|1-FE_{\text{max}}(\varphi_{\text{d}}, \vartheta_{\text{d}})| = 0.0274$ and $|1-E_{\text{Enorm}}(\varphi_{\text{d}}, \vartheta_{\text{d}})| = 0$. Correspondingly, the entries in

the optimized state variables vector are: L = 112.48 mm, w = 13.17 mm and $\alpha = 64.27^{\circ}$. Obviously, most of the energy is radiated in the desired direction (the last criterion being zero).

The transient responses of the current at the feeding point of the antenna and the radiated pulse in the desired direction, both normalized to the square root of their autocorrelation functions at t = 0 s, are shown in Figs. 4 and 5, respectively. To facilitate comparisons, the excitation pulse is normalized in the same way as the responses. Note that the radiated pulse is shifted in time to the instant when the maximum fidelity factor FE_{max} between this response and the excitation pulse occurs. The excitation pulse, the current response and the radiated pulse are very similar, but not the same.



Fig. 4. Normalized excitation pulse and current response of the optimized antenna.



Fig. 5. Normalized excitation and radiated pulses of the optimized antenna.

The computed return loss parameter S_{11} [8] for the excitation pulse and the current response is depicted in Fig. 6 (denoted by TD) after mapping the results to the frequency-domain. Overall, the optimized antenna is very well matched to the feeding line with the desired admittance $Y_{\rm W} = 10$ mS, with a return loss (well) under

-10 dB over the complete frequency range from 2 to 4 GHz, except for a reduced region between 2.27 and 2.55 GHz, peaking at f = 2.42 GHz, where $S_{11} = -8.9$ dB.

For the verification of the antenna radiation in the desired direction, the magnitude of the following transfer function is computed

$$K(f) = \frac{FFT\{\left|\mathbf{E}\left(\varphi_{d}, \mathcal{G}_{d}, t\right)\right\}}{FFT\{U(t)\}}$$
(14)

that is normalized according to the expression

$$K_{norm}(f) = \frac{K(f)}{\max[K(f)]}.$$
 (15)

The normalized transfer function K_{norm} is depicted in Fig. 7 (denoted by TD). It is apparent that the minimum radiation corresponds to the frequency of 4 GHz, where the normalized transfer function is 0.28.



For verification, the optimized bow-tie antenna was analyzed by the method of moments in the frequency domain [11]. The computed characteristics are denoted in Figs. 6 and 7 by FD. The agreement of both solutions is good. Note that the transfer function computed from the data of the frequency domain analysis is normalized to the maximum value of the transfer function K(f) (14).

Based on the results, it can be stated that, overall, the designed bow-tie antenna offers a good tradeoff between good radiation properties and the impedance (admittance) matching in the desired frequency range.

5. Conclusion

In the paper, the multicriteria optimization of antennas is performed directly in the time-domain. The proposed objective function takes into account, for a given excitation pulse, the "time-domain impedance matching", a distortion of responses at the feeding point and in a desired radiating direction (with respect to the excitation pulse), and the radiated energy in the desired direction. The objective function was used for the optimization of a bowtie antenna by means of the particle swarm optimization. The optimized antenna exhibits favorable characteristics.

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