New Channel Coding Methods for Satellite Communication

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Abstract. This paper deals with the new progressive channel coding methods for short message transmission via satellite transponder using predetermined length of frame. The key benefits of this contribution are modification and implementation of a new turbo code and utilization of unique features with applications of methods for bit error rate estimation and algorithm for output message reconstruction. The mentioned methods allow an error free communication with very low E_b/N_0 ratio and they have been adopted for satellite communication, however they can be applied for other systems working with very low E_b/N_0 ratio.

Keywords

Turbo code, coding, interleaving, MAP, decoding, bit error rate, bit error rate estimation, satellite communication.

1. Introduction

The signal transmitted via satellite transponder is affected by many factors. Satellite links are usually designed with extremely low margin of link budget. Therefore low E_b/N_0 ratios at the point of receiver are typical and define the basic feature of satellite communications. There are some ways with successful and reliable data message transmission in spite of these adverse conditions. The fundamental method for achievement of reliable message decoding from the signals with very low E_b/N_0 ratio is effective channel coding methods. The turbo codes represent an optimal technique due to their unique properties.

2. Turbo Codes

The turbo codes were introduced first in 1993 [4]. At the present time, they are implemented in many modern communication systems above all thanks to their excellent efficiency or more precisely to their high code gain [8]. Therefore they can reach low bit error rates (BER) in the course of extremely low E_b/N_0 ratios. The crucial advantage is the fact that turbo codes do not have any marked limit for bit error rate reducing. The principle of an iteration decoding is applied and increasing number of iteration steps allows practically unlimited reduction of bit error rate.

In spite of many advantageous properties of turbo codes, they have a few of disadvantages that complicate their implementation. The fundamental handicap of turbo codes is presented by their high latency due to iteration decoding process and relatively complicated decoding algorithm SISO (Soft Input Soft Output). Moreover a setting of optimal parameters for encoder and decoder is practically insolvable. The project of proper parameters is indeed difficult because the enormous number of combinations of parameters for turbo code design can be applied and it depends on the implementation of given turbo code and objective requirements. Till this time no mathematical tool for turbo code design has been found.

A number of algorithms and architectures for turbo codes have been proposed for specific applications [1], [3]. In consequence this, the compromise between turbo code complexity and efficiency is the basic task of turbo code designing with the optimal features for desirable application.



Fig. 1. Fundamental architecture of turbo coder [1].

In case of time-uncritical application (off-line systems) or utilization of efficient computing system, the error free detection of message can be reached even for channel with strong noise and jamming. This idea is very important for the implementation of system for short data message service (SMS) that is under consideration. Because SMS system does not need to work in real time and certain latency is sufferable, the turbo code can be used like a primary component for free error detection of signals affected by very low E_b/N_0 ratio.

2.1 Decoding of Turbo Codes

The turbo codes provide code gains that reach only several tithes of dB from theoretical maximum channel capacity but with higher encoder complexity. Thus, classical algorithms cannot be used for turbo codes decoding. Therefore a new scheme has been developed with two SISO decoders connected through an iteration loop.

The main reason of iterative decoder utilization is *a posteriori* probability $P(u_k|\mathbf{y})$ estimation, where u_k is *k*-th data bit and \mathbf{y} is received sequence (code word). The decoder estimates *a posteriori* ratio for every *k*-th bit that is determined by

$$\frac{P(u_k = +1|\mathbf{y})}{P(u_k = -1|\mathbf{y})}.$$
(1)

If the ratio is higher than 1, decoder decides that k-th bit equals to '1'. Vice versa, if the ratio is lower than 1, decoder evaluates k-th bit like '0'.

If the logarithmic *a posteriori* ratio $L(u_k)$ will be used the decoding task will be more simplified

$$L(u_k) = \log \frac{P(u_k = +1|\mathbf{y})}{P(u_k = -1|\mathbf{y})}.$$
(2)

In case of the probabilities of data bits are the same, the decision based on a posteriori ratio is equivalent like decision based on LLR – Log-Likelihood Ratio. The decoder does decision based on LLR

$$u_k = sign[L(u_k)]. \tag{3}$$

Thus sign of u_k is determined by HARD decision and value $|L(u_k)|$ designates reliability of this decision.

3. Turbo Code Modification

Various requirements on turbo code properties are demanded for given application of communication system. The telemetry turbo code has been chosen for short data message transmission like a base code.

The telemetry turbo code is determined for short data blocks transmission and it can work with very low E_b/N_0 [1]. Therefore it is possible to suppose that a modification of this code is suitable for the development of a proper code for short data message service via satellite transponder. The modification consists in the implementation of an optimum interleaver with appropriate data block length *k* and code ratio *R*. The basic operation of the interleaver is a generation of sequences with the smallest mutual correlations. An efficiency of interleaver is given by its proposed parameters. The interleaver quality is pivotal because this element affects in principle the efficiency of turbo code and the approaching to Shannon limit [5].



Fig. 2. Resulting BER for various types of turbo codes with data block lengths 3568 and code rate R = 1/4.

Three types of interleavers (telemetry [1], pseudorandom and edge effect elimination interleaver [5]) have been implemented in the telemetry turbo code with code ratios R = 1/2, R = 1/3, R = 1/4 and R = 1/6. These standard code ratios are used also in other communication systems. The turbo code has been tested with different data block lengths k =1024, 1784, 3568, 4096, 7136, 8464, and 9216 bits, in dependence on interleaver type. The turbo code for special edge effect elimination interleaver, code ratio R = 1/2 and block length k = 4096 bits reached the best results. This modified turbo code attains the smallest bit error rate in comparison with other modifications, e.g. $BER = 1.9 \cdot 10^{-4}$ for $E_b/N_0 = 0.8$ dB (Fig. 2). The codes with R = 1/6 achieve even lower bit error rate but they produce too high redundancy. Therefore the mentioned variant (R = 1/4, k = 4096) of turbo code has been chosen for the proposed system.

The results for BER = $1 \cdot 10^{-2} \div 1 \cdot 10^{-3}$ have been taken as a primary aspect for turbo codes evaluation (see Fig. 3, Fig. 4).



Fig. 3. Demanding E_b/N_0 to achieving of BER 10⁻² and 10⁻³ for various types interleaver (R = 1/4).



Fig. 4. BER value comparison depending on implemented type of ineterleaver.

4. BER Estimation

The BER cannot be simply determined in real systems without auxiliary pilot signal or training message. However the knowledge of BER is needful for a control of the iteration decoding process and an effective BER estimation method for unknown received message is prerequisite. BER estimation methods presented in the next sections are based on applied methods in publications [12], [13]. These methods have been modified for applications using turbo decoders and they result from the classical turbo code decoding theory.

4.1 The BER Estimation Based on APP

The first method for the BER estimation is based on a posteriori probability (APP). Probabilities $APP_i(+A)$ and $APP_i(-A)$ can be determined in the output of the turbo decoder for each transmission bit x_i . Than the BER estimation can be described by the equation

$$BER = \frac{1}{N} \cdot \sum_{i=1}^{N} \min(APP_i(-A), APP_i(+A))$$
(4)

where the probability of each term is defined by formulas

$$APP(+A) = \frac{1}{1 + e^{-LLR}}, \qquad (5)$$

$$APP(-A) = \frac{1}{1 + e^{+LLR}}.$$
 (6)

The resulting BER estimation can be expressed by the equation

$$BER = \frac{1}{N} \cdot \sum_{i=1}^{N} \frac{1}{1 + e^{|LLR_i|}}.$$
 (7)



Fig. 5. Errors of BER estimations based on APP for various lengths of blocks.

The BER estimation method based on *a posteriori* probability (APP) gives relatively correct results. Comparison of BER estimation errors by APP method for block length k = 1024, 4096 and 8464 bits is presented in Fig. 5. The error of BER estimation is not too high but the highest error is for block length k = 4096 bits, which has been chosen for short data message transmission system.

4.2 The Modified BER Estimation Method Based on LLR

This method is developed on the basis of theory in [13]. The probability can be calculated for each bit by equation (8), where $\Lambda(n)$ is a logarithmic likelihood ratio (LLR)

$$P(n) = \frac{e^{\Lambda(n)}}{e^{\Lambda(n)} + 1}.$$
(8)

This method has been tested by many simulations, but it does not bring good results for the BER estimation of the developed turbo code and this method has been modified.

The novel method displayed in Fig. 6 is based on the principle described in the previous text, but total probability as a sum of each bit probability contributions is not calculated. The first step of this method is a decision about correct or incorrect detection of each bit by a set of decision rules.



Fig. 6. Block scheme of modified BER estimation method.

HARD value (x _{Hi})	SOFT value (x _{si})	Output bit (y _i)
$x_{Hi} > 0$	$x_{Si} \ge -t_1$	1
$x_{Hi} > 0$	$x_{Si} < -t_1$	0
$x_{Hi} \leq 0$	$x_{Si} \leq t_1$	0
$x_{Hi} \leq 0$	$x_{Si} > t_1$	1
$x_{Hi} > -t_2 \& x_{Hi} < t_2$	$x_{Si} > -t_2 \& x_{Si} < t_2$	error
t_1 – threshold 1, t_2 – threshold 2		

Tab. 1. Set of decision rules.

The principle of the BER estimation is relatively simple. However, the thresholds (t_1 and t_2) have to be determined correctly. The threshold value ($t_1 = 2.70805$) is specified by formula (8). The threshold t_1 is a limiting value, which indicates a high probability when the bit is decoded correctly or not. In case of the threshold t_2 , the determination of limit is more complicated. If values x_{Hi} and x_{Si} are in the interval between $-t_2$ and $+t_2$, it means very high probability of error decision. The resulting limit is established by experimental BER measurements for various turbo codes.



Fig. 7. BER estimation based on LLR for turbo code with special interleaver, R = 1/4, k = 4096 b.

The evaluation of the proposed method has been made for block lengths k = 1024, 4096 and 8464 bits. The BER estimation is inaccurate for threshold value $t_2 = 0.5$ and $t_2 = 2$ and the resulting error obtains the values up to 80%.



Fig. 8. Errors of BER estimations based on LLR with k = 4096 b.

On the other hand, the thresholds $t_2 = 1$ and $t_2 = 1.5$ give satisfactory results and the BER estimation approaches closely to the true curve.

The method reaches the best results for threshold $t_2 = 1$ and designed turbo code with special edge effect elimination interleaver (R = 1/4, k = 4096 b). The estimation error is not higher than 35% (see Fig. 7, Fig. 8).

4.3 Comparison of BER Estimation Methods

Two BER estimation methods have been applied. The first method described in [12] uses a posteriori probability (APP), the second one, which is based on theory in [13], applies LLR values.

Both methods yield very good results and they are applicable for BER estimation. However, the modified method is more accurate for the threshold $t_2 = 1$. Maximal deviation is 34.6% only for the residual error rate value BER = $8.3 \cdot 10^{-5}$. The deviation for BER in range ($10^{-2} \div 10^{-3}$) is max. 17.5% (Fig. 9).



Fig. 9. Comparison of BER estimation methods.

The result of the BER estimation is depending on threshold value t_2 as mentioned above. The accuracy of parameter t_2 is not critical and for the high-precise BER estimation can be set in the interval $1 \le t_2 \le 1.5$.

5. Symbol Value Determination Methodology of Sequence for Nonzero BER

5.1 Probability Method for Symbol Value Calculating of Data Sequence

The method for symbol value calculating (Fig. 10) uses value x_{Hi} , which inputs to the decoder, and SOFT value x_{Si} , which outputs from decoder. Bit sequences (packet of message) are divided into four parts (sender address, receiver address, message length, message data). Resulting bit values of the message data are determined by the following steps:

Value transformation (x_{Ti}) – transformation is executed on the basis of bit values of a received sequence in a decoder input.

$$transformation_{1}:\begin{cases} x_{Hi} > 0 \Longrightarrow x_{Ti} = +1 \\ x_{Hi} < 0 \Longrightarrow x_{Ti} = -1 \end{cases}$$
(9)

Probability calculation (p_i) – it is based on the equations (10) and (11) [13]. The bit values in the decoder input are related to the probability calculation. If the term $x_{Hi} > 0$ is valid, the probability is calculated according to (10). If the term $x_{Hi} < 0$ is valid, the probability is calculated according to (11).

$$P(1)_i = \frac{e^{\Lambda(x_{Si})}}{e^{\Lambda(x_{Si})} + 1},$$
(10)

$$P(0)_{i} = 1 - \frac{e^{\Lambda(x_{Si})}}{e^{\Lambda(x_{Si})} + 1}.$$
 (11)



Fig. 10. Flow chart of probability method for symbol value calculating.

Probability value transformation (p_{Ti}) – the transformation (12) is made for results from the previous step.

transformation₂: $p_i: \langle 0,1 \rangle \Rightarrow p_{Ti}: \langle -1,+1 \rangle$. (12)

Calculating of final bit value (y_j) – the final value is calculated according to (13). The value *k* expresses the number of repetition of data message part.

$$y_{i} = \frac{\sum_{i=1}^{k} x_{T_{i}} \cdot p_{T_{i}}}{k}$$
(13)

5.2 The Simplified Method for Bit Value Calculating

The simplified method for symbol determination used SOFT values from decoder output. The flowchart of this algorithm is shown in Fig. 11.

The received packet of message is split in to four parts (sender address, receiver address, message length, data message). The sender and receiver address are divided to 6×2 B. The part containing information about message length is divided to 10×1 B. The arithmetic mean is calculated for each bit of separated parts. The bit value is determined in basis of the arithmetic mean following by the operation of HARD decision.

The sender and receiver address processing is finished by this step. The part of packet specified as a data message carries an information message, which can be divided on n blocks with the same content repeated more than 4 times. The maximum length of message is 160 characters, i.e. 160 B. The arithmetic mean value is calculated for each bit of repetitions and the final bit value is determined by HARD decision. The advantage of this method is a utilization of shorter messages than 160 B resulting in more reliable decoding.



Fig. 11. Algorithm of simplified method for symbol value calculating.

5.3 The Results of Methods for Symbol Value Determining

The relations of the BER_{sms} and number of iteration are shown in Fig. 11, 12 and 13. The graphs display the BER_{sms} characteristics for $E_b/N_0 = 0.6$ dB, $E_b/N_0 = 0.8$ dB and $E_b/N_0 = 1$ dB. The smaller BER_{sms} is achieved by using both methods but for higher redundancy. The bit error rates obtained without application of the methods for BER_{sms} reduction are displayed by blue bars. The green bars represent BER_{sms} values for the probability method and red bars for the simplified method.

The zero BER is nearly always achieved for probability method with 10 iterations for $E_b/N_0 = 0.6$ dB. On the other hand, the simplified method does not ever allow to get the zero BER for $E_b/N_0 = 0.6$ dB. For both methods, the zero BER is nearly always achieved for $E_b/N_0 = 0.8$ dB. The probability method gives better results because the BER diminishes to zero after 8 iterations. Also, the probability method is better for $E_b/N_0 = 1$ dB because BER is zero after 4 iterations only.







Fig. 13. BER of the final data message $(E_b/N_0 = 0.8 \text{ dB})$.



Fig. 14. Achieved BER of the final data message for $E_b/N_0 = 1$ dB.

5.4 The Symbol Determination Methods Evaluation

Both methods are used for symbol determination from repeating parts of data message. The first probability method applies the probability calculation and value transformation. The second simplified method is based on the arithmetic mean calculation of SOFT values in the decoder output. Both methods bring very good results and the final BER falls to very small values. However, the probability method achieved better results as Fig. 12, Fig. 13 and Fig. 14 show.

6. Conclusion

The new progressive channel coding methods for short message transmission via satellite transponder have been developed. The key benefit of this contribution is a design and implementation of new modified turbo code with unique features resulting from application of estimation methods for bit error rate and final forming of the output message. The mentioned methods allow error free communication for very low E_b/N_0 ratios and they are suitable for satellite communication however also for other systems working with limiting E_b/N_0 ratios.

Acknowledgements

The presented work has been financially supported by the Czech Science Foundation project no. 102/07/P514 "Research of Digital Detection Methods for Low Energy Signals", by the Czech Science Foundation project no. 102/08/H027, by the project of the Ministry of Education of the Czech Republic no. OC09016 "Components of Advanced Radio Communication Systems", and by the research program of the Ministry of Education of the Czech Republic no. MSM0021630513 "Advanced Electronic Communication Systems and Technologies" (ELCOM).

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