Wave Propagation in Lossy and Superconducting Circular Waveguides

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Abstract. *We present an accurate approach to compute the attenuation of waves, propagating in circular waveguides with lossy and superconducting walls. A set of transcendental equation is developed by matching the fields at the surface of the wall with the electrical properties of the wall material. The propagation constant k_z is found by numerically solving for the root of the equation. The complex conductivity of the superconductor is obtained from the Mattis-Bardeen equations. We have compared the loss of TE_{11} mode computed using our technique with that using the perturbation and Stratton's methods. The results from the three methods agree very well at a reasonable range of frequencies above the cutoff. The curves, however, deviate below cutoff and at millimeter wave frequencies. We attribute the discrepancies to the dispersive effect and the presence of the longitudinal fields in a lossy waveguide. At frequencies below the gap, the superconducting waveguide exhibits lossless transmission behavior. Above the gap frequency, Cooper-pair breaking becomes dominant and the loss increases significantly.*

Keywords

Circular waveguides, superconductor, propagation constant, dispersive effect, Cooper-pair.

1. Introduction

The rigorous formulation developed by Stratton [1] has been widely used to analyze the propagation of waves in circular waveguides [2] – [6]. In Stratton's approach, a circular cylinder of radius a is assumed to be embedded in an infinite homogeneous medium. Matching the tangential fields of the two mediums at the boundary of a yields a transcendental equation which allows exact computation of the complex propagation constant of the waveguide. Nevertheless, due to the difficulty in matching the boundary conditions in Cartesian coordinates, this approach fails to be implemented in the case of rectangular waveguides [7], [8].

Due to its simplicity and analytical solution, the approximate perturbation method, has generally been employed to analyze wave propagation in imperfectly conducting [9] – [12] and superconducting [13] waveguides. In this method, the fields' expressions are derived by assuming the wall to be of infinite conductivity. This allows the solution to be separated into pure TE and TM modes [12]. For a waveguide with finite loss, however, a superposition of both TE and TM modes is necessary to satisfy the boundary conditions [14]. To calculate the attenuation, ohmic losses at the walls are assumed due to small fields' penetration into the wall surfaces. As shown in [14], when the operating frequency f approaches cutoff f_c, the attenuation obtained using such method diverges to infinity. This phenomenon which only exists in lossless waveguides is clearly inadequate for surfaces with finite conductivity and superconductivity. This is because, in contrast with a perfect conductor, field penetration occurs at both lossy and superconducting walls.

In order to account for the field penetration, an alternative boundary condition based on the penetration depth of the Meissner effect has been suggested to study the wave properties for superconducting waveguides [15] – [19]. In the work of these authors, the boundary condition for the longitudinal magnetic field H_z of a TE mode is given by,

$$\frac{\partial H_z}{\partial a_n} - \frac{1}{\lambda_L} H_z = 0 \quad (1)$$

where a_n is a normal unit vector and λ_L, known as the London penetration depth, is a measure of the distance of magnetic field penetration into the superconductor. An important implication of this theoretical study is that the dominant mode for a rectangular waveguide is found to have switched from TE_{10} to TE_{11}; while that for a circular waveguide has switched from TE_{11} to TE_{01}. Yassin et al. has performed an experimental validation on the above theory using a superconducting circular waveguide [20]. The experimental result, however, shows that the work in [15] – [19] turned out to be invalid. The mode order in a superconducting waveguide remains the same as those found in a perfectly conducting waveguide.

Circular and rectangular waveguides have been widely applied in receivers of radio telescope [21] – [24]. In [25], we have developed and discussed a novel technique to compute the propagation constant of waves in rectangular waveguides. Here, we shall extend further the approach in [25] to the case of lossy and superconducting circular waveguides. In our method, the solution for the attenuation constant is found by solving the transcendental equation derived from using the electrical properties of the wall material expressed as surface impedance. In our results, we will compare and discuss the loss obtained using our method with those using the perturbation and Stratton's methods.

2. Fields in a Circular Waveguide

The longitudinal electric and magnetic fields E_z and H_z, respectively, propagating in a circular waveguide, as shown in Fig. 1, can be derived by solving Helmholtz homogeneous equation. Using the method of separation of variables [11], we obtain the following set of field equations:

$$H_z = C_n' J_n(hr)\sin n\phi, \tag{2}$$

$$E_z = C_n J_n(hr)\cos n\phi \tag{3}$$

where C_n and C_n' denote the coefficients of the longitudinal fields, $h = \sqrt{k^2 - k_z^2}$, k is the wavenumber in free space, k_z the propagation constant, r the radial distance, $J_n(hr)$ is called the Bessel function of the first kind, $J_n'(hr)$ is its derivative, and n is the order of the Bessel function. All field components consist of the wave factor in the form of $\exp[j(\omega t - k_z z)]$, where t represents the time and ω the angular frequency. The wave factor is, thus, omitted in the following derivations.

The propagation constant k_z is a complex variable which constitutes a phase constant β_z and an attenuation constant α_z, as shown in (4) below:

$$k_z = \beta_z - j\alpha_z. \tag{4}$$

Substituting (2) and (3) into Maxwell's source-free curl equations and expressing the transverse field components in terms of E_z and H_z [11], we obtain:

$$E_\phi = \frac{1}{h^2}\left[\frac{jnk_z}{r} C_n J_n(hr)\sin n\phi + j\omega\mu h C_n' J_n'(hr)\cos n\phi\right] \tag{5}$$

$$H_\phi = -\frac{1}{h^2}\left[\frac{jnk_z}{r} C_n' J_n(hr)\cos n\phi + j\omega\varepsilon h C_n J_n'(hr)\cos n\phi\right] \tag{6}$$

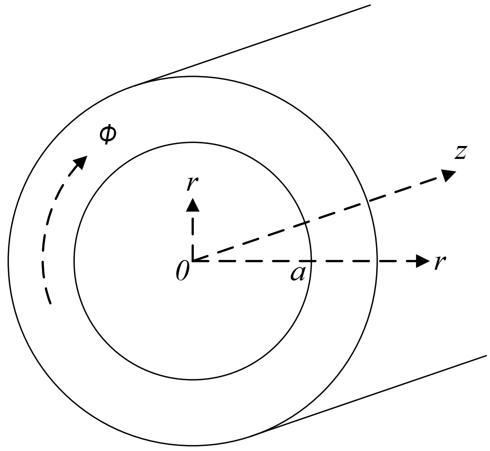

Fig. 1. A circular waveguide.

3. Constitutive Relations for TE and TM modes

At the wall, the tangential electric and magnetic fields, i.e. $\boldsymbol{E_t}$ and $\boldsymbol{H_t}$, respectively, are related through a surface impedance Z_s by [11], [26]:

$$\boldsymbol{E_t} = -Z_s(\boldsymbol{a_n} \times \boldsymbol{H_t}). \tag{7}$$

Z_s can be expressed in terms of the electrical properties of the wall material:

$$Z_s = \sqrt{\frac{\mu_w}{\varepsilon_w}} \tag{8}$$

where μ_w and ε_w are the permeability and permittivity of the wall material, respectively. ε_w is complex and is given as [11]:

$$\varepsilon_w = \varepsilon - j\frac{\sigma}{\omega} \tag{9}$$

where σ is the conductivity of the wall. Due to the existence of the energy gap $2\Delta(T)$ for a superconductor, σ is complex and frequency dependent:

$$\sigma = \sigma_1 - j\sigma_2. \tag{10}$$

The equations for the complex conductivity have been developed by Mattis and Bardeen from the microscopic analysis of Bardeen-Cooper-Schrieffer (BCS) theory [27], [28], [33]:

$$\frac{\sigma_1}{\sigma_n} = \frac{2}{\hbar\omega}\int_\Delta^\infty [f(E) - f(E+\hbar\omega)]\frac{E^2+\Delta^2+\hbar\omega E}{(E^2-\Delta^2)^{1/2}\left[(E+\hbar\omega)^2-\Delta^2\right]^{1/2}}dE + \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega}^{-\Delta}[1-2f(E+\hbar\omega)]\frac{E^2+\Delta^2+\hbar\omega E}{(E^2-\Delta^2)^{1/2}\left[(E+\hbar\omega)^2-\Delta^2\right]^{1/2}}dE \tag{11}$$

$$\frac{\sigma_2}{\sigma_n} = \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega,-\Delta}^{\Delta}[1-2f(E+\hbar\omega)]\frac{E^2+\Delta^2+\hbar\omega E}{(\Delta^2-E^2)^{1/2}\left[(E+\hbar\omega)^2-\Delta^2\right]^{1/2}}dE \tag{12}$$

where $\hbar$ is the reduced Planck's constant, σ_n the normal conductivity, and $\Delta = \Delta(T)$ the energy-gap parameter. The function,

$$f(E) = \frac{1}{1+\exp(E/kT)} \tag{13}$$

gives the Fermi-Dirac statistics and k is the Boltzmann's constant. The first integral in (11) describes the effect of the thermally excited quasiparticles. The second integral denotes the generation of quasiparticles by fields with frequencies f corresponding to energies above the gap energy. Thus, the second integral is zero for $\hbar\omega < 2\Delta$. Since σ_2 indicates the contribution due to the Cooper pairs, the lower integration limit in (12) becomes $-\Delta$ when $\hbar\omega > 2\Delta$. Δ depends on temperature and is obtained from the relation [28]:

$$\ln\left(\tilde{\Delta}\right) = -2\int_0^\infty \left(E^2+\tilde{\Delta}^2\right)^{-1/2}\left\{1+\exp\left[\left(\pi/\gamma_E\tilde{T}\right)\left(E^2+\tilde{\Delta}^2\right)^{1/2}\right]\right\}^{-1} dE\,. \tag{14}$$

where $\tilde{\Delta} = \Delta(T)/\Delta(0)$, $\tilde{T} = T/T_c$, and $\gamma_E = 1.781$ is the Euler's constant.

At the boundary of the wall with radius $r = a$, $Z_s = \frac{E_\phi}{H_z} = -\frac{E_z}{H_\phi} = \sqrt{\frac{\mu_w}{\varepsilon_w}}$. Substituting (2), (3), (5), and (6) into (7), we obtain:

$$\left[\frac{jnk_z}{h^2 a}\right]C_n + \left[\frac{j\omega\mu J_n'(ha)}{hJ_n(ha)} - \sqrt{\frac{\mu_w}{\varepsilon_w}}\right]C_n' = 0\,, \tag{15}$$

$$\left[\frac{j\omega\varepsilon J_n'(ha)}{hJ_n(ha)} - \sqrt{\frac{\varepsilon_w}{\mu_w}}\right]C_n + \left[\frac{jnk_z}{h^2 a}\right]C_n' = 0\,. \tag{16}$$

Solving the determinants of the coefficients C_n and C_n' in (15) and (16) results in the following transcendental equations:

$$\left[jh^2\sqrt{\frac{\mu_w}{\varepsilon_w}} + \omega\mu h\frac{J_n'(ha)}{J_n(ha)}\right]\left[jh^2\sqrt{\frac{\varepsilon_w}{\mu_w}} + \omega\varepsilon h\frac{J_n'(ha)}{J_n(ha)}\right] = \left[\frac{nk_z}{a}\right]^2. \tag{17a}$$

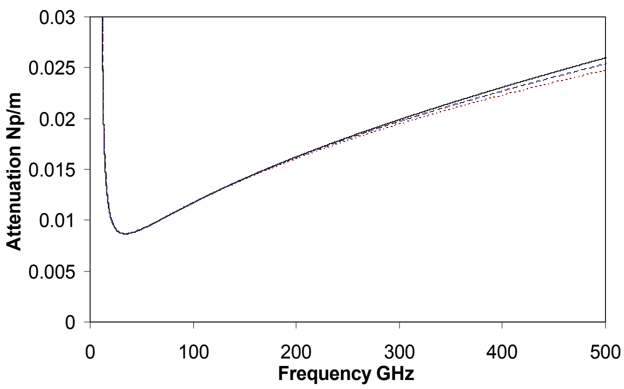

Fig. 2. Attenuation of TE_{11} mode above cutoff. The solid line was calculated using our method, dotted line using Stratton's method, and the dashed line using the perturbation method.

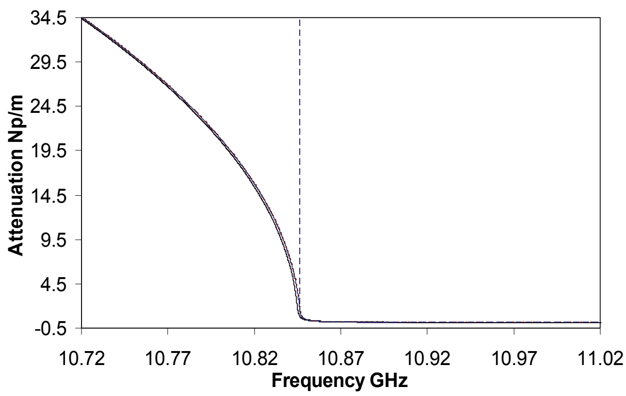

Fig. 3. Attenuation of TE_{11} mode at the vicinity of cutoff. The solid line was calculated using our method, dotted line using Stratton's method, and the dashed line using the perturbation method.

The propagation constant k_z can be computed by applying a root-searching algorithm such as the Powell Hybrid algorithm in [29] to solve for the root of (17a) for TE modes. The attenuation constant α_z can then be obtained from k_z by extracting the imaginary part of (4). It is to be noted that, since TE and TM modes are determined by the roots of $J_n(ha)'/J_n(ha) = 0$ and $J_n(ha)/J_n(ha)' = 0$, respectively [1], an alternate form of the equation is required for TM modes:

$$\left[jh^2\sqrt{\frac{\mu_c}{\varepsilon_c}}\frac{J_n(u)}{J_n(u)'} + \omega\mu h\right]\left[jh^2\sqrt{\frac{\varepsilon_c}{\mu_c}}\frac{J_n(u)}{J_n(u)'} + \omega\varepsilon h\right] = \left[\frac{nk_z}{a}\frac{J_n(u)}{J_n(u)'}\right]^2 \tag{17b}$$

4. Results and Discussion

The attenuation constant for the dominant TE_{11} wave propagating in a copper circular waveguide with $a = 8.1$ mm are shown in Figs. 2 to 4. We can see from Fig. 2 that the attenuation curves plotted using our method agrees closely with those from the perturbation and Stratton's method at a reasonable range of frequencies f above the cutoff f_c. There are, however, two regions where our curves are found to differ significantly with that obtained using the other two methods. The curves deviate at frequencies immediately below f_c and above millimeter wave frequencies. As can be seen in Fig. 3, when f approaches f_c, the attenuation given by the perturbation method diverges to infinity, with a singularity at $f = f_c$. On the other hand, as f decreases below f_c, the attenuation computed using our method and Stratton's method are in close agreement. Both curves diverge continuously to a more highly attenuating mode. As the frequency decreases further, the attenuation rises to such high values that signals propagation become almost impossible. Clearly, it is more realistic to expect losses to be high but finite at frequencies below cutoff, rather than diverging to infinity. Abe and Yamaguchi [6] had performed an experimental validation for the loss of waves below cutoff. The attenuation curves shown in Abe-Yamaguchi's measurement were high but finite as well, which confirms the validity of Stratton's and our results. The inaccuracy in the perturbation method at cutoff is due to the fact that the fields' equations are assumed to be the same as those in a lossless waveguide.

As depicted in Figs. 2 and 4, when f increases above approximately 250 GHz, the attenuation obtained using our method increases beyond that predicted by the perturbation method. We attribute the differences to the fact that at extremely high frequencies, the field in a lossy waveguide can no longer be approximated to those derived from a perfectly conducting waveguide. At such high frequencies, the wave propagating in the waveguide is a hybrid of TE and TM modes and the presence of E_z can no longer be neglected. According to the theoretical and experimental validation carried out by Imbriale et al. in [34], the cross product terms between the different modes which co-exist at the same time, give rise to additional dissipation of loss. The mode coupling effect results in higher loss than the propagation of a single mode alone. It is interesting to see that, in this range, the attenuation computed using Stratton's equation is even lower than that from the perturbation method. It is worthwhile noting that, Yassin et al. had computed the loss using Stratton's equation in [2] and had obtained a similar result as in Fig. 4 as well. For the case of dielectric or lossy conducting wall, the fields penetrating into the wall must be evanescent. Stratton had applied Hankel function of the first kind and its derivative – i.e. $H_n(ha)$ and $H_n(ha)'$, to represent the elementary fields penetrating into the wall material. When performing a numerical computation for Stratton's equation using a Fortran compiler, the problem that we encountered very often was that $H_n(ha)$ and $H_n(ha)'$ dropped below the minimum permissible value for floating point variables in the compiler (the size of a double precision is about 8 bytes). This results in failure during code compilation. To solve this problem, we have thus approximated $H_n(ha)'/H_n(ha) \approx 1$ when both $H_n(ha)$ and $H_n(ha)'$ decrease below 1.0×10^{-300}. Hence, we attribute the discrepancies between Stratton's and our methods at frequencies below cutoff and above millimeter frequencies as due to the approximation imposed to $H_n(ha)'/H_n(ha)$ when numerically solving Stratton's equation.

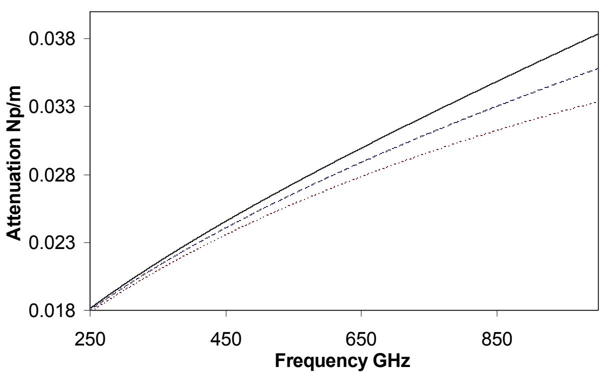

Fig. 4. Attenuation of TE_{11} mode at millimetre wave frequencies. The solid line was calculated using our method, dotted line using Stratton's method, and the dashed line using the perturbation method.

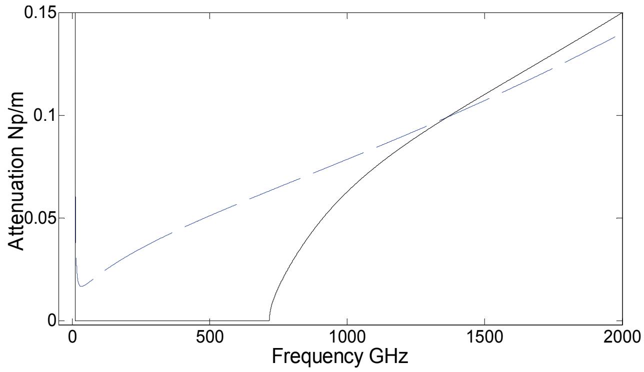

Fig. 5. Attenuation for TE_{11} mode in a Nb circular waveguide at T = 4.2 K (solid lines) and room temperature (dashed lines).

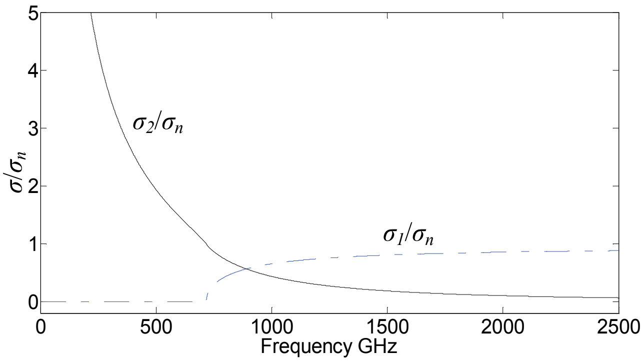

Fig. 6. The normalized complex conductivity of niobium at 4.2 K, computed using Mattis and Bardeen equation.

Since quasiparticle is generated as the magnitude $\hbar\omega$ of the quantum energy exceeds the energy gap 2Δ, a superconductor loses its superconducting behavior as the operating frequency f exceeds its gap frequency f_g of $\Delta/(\hbar\pi)$. Due to its high gap frequency of about 700 GHz at 4.2 K, Niobium (Nb) has generally been employed as the superconducting material for the detection of millimeter and sub-millimeter waves in superconductor-insulator-superconductor (SIS) receivers [30] – [32]. In our analysis of superconducting waveguides, we have, thus computed the attenuation of TE_{11} waves in a Nb circular waveguide with a = 8.1 mm. Fig. 5 shows a comparison of attenuation in Nb waveguide during normal state at room temperature and superconducting state at T = 4.2 K. Below the gap frequency f_g, the superconducting waveguide behaves exactly like a perfectly conducting waveguide. The attenuation diverges to infinity at frequency $f = f_c$. Above cutoff, the superconducting waveguide exhibits lossless attenuation. To explain this phenomenon, we have computed the complex conductivity of the superconducting Nb at 4.2 K using Mattis-Bardeen equation in (11) and (12). As can be seen in Fig. 6, σ_1 which indicates the effect of the quasiparticles is negligible at frequencies below f_g. As the frequency exceeds f_g, σ_2 decreases gradually toward zero while σ_1 approaches the value of σ_n, implying that Cooper-pair breaking takes place above f_g. With the increase of quasiparticles, we can thus, expect the random collision of quasiparticles with the lattice structure becomes more frequent, resulting in higher conduction loss at frequencies above f_g.

In fact, we can observe from Fig. 5 that the loss of the superconducting waveguide operating above f_g increases gradually and eventually surpasses the loss of the waveguide operating at room temperature. Duzer-Turner in [33] has derived the surface resistance of a superconductor using the two-fluid model, as given in (18) below:

$$R_s = \frac{\omega^2 \mu^2 \lambda_L{}^3 n_n \sigma_n}{2}. \tag{18}$$

where n_n is the number density of the quasiparticles. As compared to the value of R_s for normal conductors which could be simplified from (8) and (9) as $\sqrt{\frac{\omega\mu}{2\sigma}}$, we observe that the surface resistance R_s for superconductors increases as the square of the frequency, while R_s for normal conductors only increases proportional to the square root. Since power loss is directly proportional to the surface resistance R_s of the wall, we can, thus, attribute the higher loss above f_g as due to the fast increase of R_s in superconducting waveguides.

5. Conclusion

As a conclusion, we have presented an analysis on wave propagation in both normal and superconducting circular waveguides. The complex conductivity of a superconductor is computed using Mattis and Bardeen equation, developed from the BCS theory. A set of transcendental equation is derived to compute the propagation constant of circular waveguides by matching the tangential fields with the surface impedance at the boundary of the walls.

We have compared the loss of a lossy circular waveguide computed using Stratton's exact equation, the approximate perturbation method, and our method with the experimental measurement. At frequencies f below cutoff f_c, the attenuation curves obtained from both our equation and Stratton's equation tally with the S_{21} parameter from the measurement. As f increases at a reasonable range above f_c, the attenuation computed using the three methods agree very well. Nevertheless, as f approaches millimeter wave frequencies, we observe that the loss obtained using our method is higher than those from the perturbation method. Since our method takes into account the co-existence of both TE and TM modes, we attribute the higher loss as due to the presence of both longitudinal electric and magnetic and electric fields. We also find that at such high frequencies, the loss obtained using Stratton's equation actually turns out to be lower than the perturbation method. We attribute such discrepancies as due to the approximation made to avoid reaching the limitation of the floating point value in the compiler when computing $H_n(ha)$ and $H_n(ha)$'.

By incorporating the values of the complex conductivities into the transcendental equations, we obtained results which indicate that the superconducting waveguide is lossless below the gap frequency f_g. Above f_g, however, Cooper-pairs are broken into quasiparticles resulting in an increase of ohmic losses in the waveguide.

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