Fine-Tuning on the Effective Patch Radius Expression of the Circular Microstrip Patch Antennas

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Abstract. *In this study, the effective patch radius expression for the circular microstrip antennas is improved by means of several manipulations. Departing from previously proposed equations in the literature, one of the most accurate equations is picked up, and this equation is fine-tuned by means of Particle Swarm Optimization technique. Throughout the study, impacts of other parameters (such as the definition of the fitness/objective function, the degree-of-freedom in the proposed effective patch radius expression, the number of measured resonant frequency values) are observed in a controlled manner. Finally, about 3% additional improvement is achieved over a very accurate formula, which was proposed earlier.*

Keywords

Circular microstrip patch antenna, closed-form expression, effective patch radius, particle swarm optimization, resonant frequency.

1. Introduction

Microstrip antennas (MSAs) have found great acceptance among the electromagnetic and microwave theory practitioners due to their numerous advantages. Hence, considerable amount of studies have been devoted to the characterization of these structures with different geometries.

Particularly, a circular microstrip patch resonator can be used either as a separate antenna, or as a component of oscillators and filters in MWICs. It is quite important to develop accurate expressions for the calculation of these resonant frequencies (and hence, the prediction of relevant parameters of the structure), since the bandwidth of MSAs around their operating resonant frequencies is very narrow. Eventually, obtaining simple models for performance analysis of MSAs is an increasing need for practical applications.

Previously, there have been a considerable number of attempts (such as [1–10]) in order to develop simple closed-form expressions for the effective patch radius of a circular MSA. In these studies, the usual approach is to incorporate the influence of the fringing field at the edges and the dielectric inhomogeneity via a parameter called the 'effective patch radius, a_{eff}', which is slightly larger than the physical patch radius a as seen in Fig. 1. It is evident from the literature that a_{eff} of a circular MSA is determined by the relative dielectric constant of the substrate (ε_r), the physical patch radius (a), and the thickness of the substrate (h). In the studies existing in the literature, the resonant frequency of circular MSAs for fundamental mode is calculated approximately by means of these effective radius expressions.

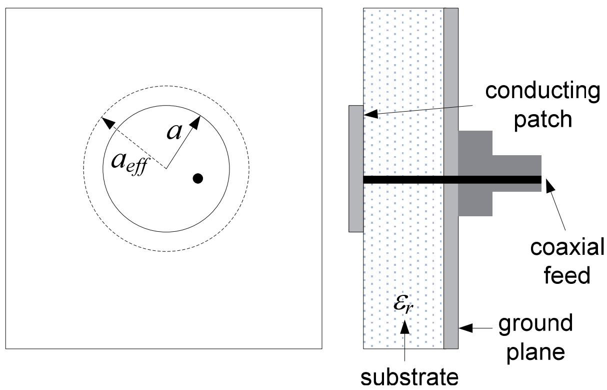

Fig. 1. Circular microstrip patch antenna geometry.

As seen in Fig. 1, the circular MSA consists of a patch of radius a, which is parallel to the ground plane; and this patch is separated from the ground plane by a dielectric substrate with relative permittivity ε_r, and thickness h. For this geometry, the resonant frequencies of the TM_{nm} modes can be computed as:

$$f_{nm} = \frac{\alpha_{nm}}{2\pi a\sqrt{\mu_0 \varepsilon}} = \frac{\alpha_{nm} c}{2\pi a\sqrt{\varepsilon_r}} \tag{1}$$

where α_{nm} is the mth zero of the derivative of the Bessel function of order n, and c is the velocity of light in free space. The value of α_{nm} (i.e. for $n = m = 1$) is 1.84118 for the dominant mode of the circular patch, which is TM_{11}.

In [10], Akdagli and Guney constructed a closed-form model for the effective patch radius depending on a, h, and ε_r; and computed the relevant coefficients in their assumed model by using the corresponding experimental data available in literature [1, 3, 5, 11-14] via the Genetic Algorithm

(GA). The extraordinariness of the expression obtained by Akdagli and Guney is as follows: Most of the other expressions available in the literature are valid for either electrically thin (normally of the order of $h/\lambda_d = 0.02$, where λ_d is the wavelength inside the substrate) or electrically thick circular MSAs. However, the expression of Akdagli and Guney yields very accurate resonant frequency estimates for a wide range of electrical thickness.

In this paper, our main aim is to apply several other approaches and to try to increase the accuracy of the expression obtained by Akdagli and Guney. For this purpose, first, we will apply the Particle Swarm Optimization (PSO) rather than GA in order to obtain the coefficients of the closed-form expression. Second, we will perform some fine tunings on the objective function that is used for the computation of the coefficients. Third, we will increase the degree-of-freedom (i.e. the number of coefficients) in the closed-form expression. Finally, we will increase the number of experimental data used for the computation of the coefficients by incorporating additional values measured in [15-18]. At each step, we will try to observe and comment on the impacts of our manipulations on the accuracy of the closed-form expressions.

The outline of the paper is as follows: After this introductory section; in Section 2, we will remind the GA and PSO; together with brief descriptions and qualitative comparisons. In Section 3, step-by-step as mentioned in the previous paragraph, we will perform the fine tunings on the closed-form expression, present the results and try to conduct relevant discussions. Section 4 will include concluding remarks.

2. Genetic Algorithm and Particle Swarm Optimization

GA and PSO are two of the major algorithms belonging to the class of nature-inspired (or bio-inspired) optimization algorithms. The basic idea lying beneath the nature-inspired algorithms is the imitation some mechanisms existing in the nature in order to solve the optimization problems. In the following subsections, after the general descriptions of GA and PSO, a qualitative comparison of these algorithms will be presented.

2.1 Genetic Algorithm (GA)

GA is a class of adaptive heuristic search and optimization algorithm based on the “survival of the best (fittest)” principle of natural selection. It is an iterative optimization procedure, and it maintains a population of probable solutions within a search space (which is usually discrete) over many simulated generations. The population members (called as phenotypes, which are usually vectors with bounded real number components) are represented by means of artificial chromosomes (called as genotypes, which are again vectors of higher dimensions with binary number components). At each iteration (or generation), three basic genetic operations “selection, crossover, and mutation” are performed. The basic concepts of GA were primarily developed by Holland, in [19]. After this work, numerous researchers have contributed to the development of this field.

2.2 Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) is a method proposed by Eberhart and Kennedy [20] after getting influenced by the behaviors of the animals living as colonies/swarms. Mimicking the swarms searching for nutrition sources in 3-dimensional space, the method depends on motions of swarm members (so called ‘particles’) searching for the global best in an n-dimensional continuous space. The position of each particle is a solution candidate; every time the fitness of these solutions is recomputed. Each particle has a cognitive behavior (i.e. remembering its own good memories and having tendency to return there); as well as a social behavior (i.e. observing the rest of the swarm and having tendency to go where most other particles go), in addition to its exploration capability (i.e. the tendency for random search throughout the domain). The balance of all these tendencies is the key of the success and power of the method.

In this study, the formulation given in [21] (the most common PSO formulation) is implemented and applied.

2.3 Comparison of GA and PSO

From their procedures, it can be observed that PSO and GA share many common points (as being nature-inspired, stochastic, population-based, systematizing the trial-error approach, etc.). However, due to their definitions, certain differences exist between the two methods. For inverse problems in which the solution is to be chosen among the members of disjoints sets (such as picking the most appropriate element from a database), GA might be the right choice. On the other hand if the selection is going to be made from continuous sets, PSO is more suitable due to its nature.

Compared with GA, PSO has a very simple implementation consisting of only a few lines of code in any language. Additionally, PSO has been shown in certain instances to outperform to GA in single- and multi-objective optimization problems. The success of PSO over GA is in terms of its better convergence and speed; and in most cases better accuracy. Those are the main reasons for the usage of PSO in this work.

3. Material and Method

In this section the details of the solution setups, which have been constructed for the fine-tuning of the effective patch radius expression, are given. For all the setups, the

PSO parameters seen in Tab. 1 are used. The inertial weight is decreased linearly from 0.95 down to 0.4 as suggested in [22]. In order to keep the particles inside the search space, reflecting boundary conditions (as defined in [23]) are applied.

Swarm Size	25
Number of Iterations	100
Cognitive tendency, c_1	1.494
Social tendency, c_2	1.494
Inertial weight, w	0.95 linearly down to 0.4
Search space	$0 \leq \beta_i$

Tab. 1. PSO parameters for all solution setups.

3.1 Proposals of Alternative Effective Patch Radius and Objective Function Expressions

As in [10], the effective patch radius is assumed to be in the form

$$a_{eff\_1} = a + h\left[\beta_1 + \beta_2\left(\left(\frac{h}{a}\right)^{\beta_3} + \left(\frac{1}{\varepsilon_r}\right)^{\beta_4}\right)\right] \tag{2}$$

where β_1, β_2, β_3 and β_4 are the values to be determined by means of an optimization method. This expression can be rewritten as:

$$a_{eff\_1} = a + h\left[\beta_1 + \beta_2\left(\frac{h}{a}\right)^{\beta_3} + \beta_2\left(\frac{1}{\varepsilon_r}\right)^{\beta_4}\right] \tag{3}$$

As can be seen in (3), the multiplicand of the h/a term and $1/\varepsilon_r$ term is same, namely β_2. In fact, it is possible to increase the accuracy of the expression by forcing these multiplicands different. In other words, the effective patch radius expression can be considered in the following alternative form by introducing a new parameter β_5:

$$a_{eff\_2} = a + h\left[\beta_1 + \beta_2\left(\frac{h}{a}\right)^{\beta_3} + \beta_5\left(\frac{1}{\varepsilon_r}\right)^{\beta_4}\right]. \tag{4}$$

Throughout the optimization, the following objective function

$$f_1 = \sum_{j=1}^{N}\left|f_{measured}(j) - f_{computed}(j)\right| \tag{5}$$

with the unit of MHz can be defined and used. Here, N is the number of antenna configurations, of which the measured resonance frequencies are used as reference. This function depends on the absolute error (but not the percentage error). The objective function might be re-defined in accordance with the desired performance. In other words, an alternative objective function:

$$f_2 = \sum_{j=1}^{N}\frac{\left|f_{measured}(j) - f_{computed}(j)\right|}{f_{measured}(j)} \tag{6}$$

which is normalized and hence unitless, might as well be defined and used. With these definitions, a function which measures the percentage error in the resonant frequency (considering the difference between the measured value and computed value via the closed form expression), can be defined as follows:

$$e = \frac{1}{N}\sum_{j=1}^{N}\frac{\left|f_{measured}(j) - f_{computed}(j)\right|}{f_{measured}(j)} \tag{7}$$

3.2 Solution Setups

In [10], Akdagli and Guney solved the same problem with GA by considering the 21 antenna configurations with "√" signs on the 7th column in Tab. 3. They assumed the effective patch radius expression as in the form a_{eff_1}. Throughout the solution, they have used f_1 as their objective function.

The main aim of the first solution setup is to observe the impact of the usage of PSO instead of GA. As in [10], a_{eff_1} and f_1 are used, and N is taken to be 21. As expected, PSO outperforms to GA; 0.02% improvement is achieved (with respect to that of [10]) in the solution error.

In the second solution setup, the aim is to observe the impact of changing the objective function. For this purpose, only the objective function is changed to f_2, and all other control parameters are kept as in the first setup. Consequently, 0.1% more improvement is achieved (with respect to the first setup) in the solution error.

In the third solution setup, observing the impact of the degree-of-freedom (D.O.F., i.e. the number of β parameters in the expression) constitutes the main aim. For this purpose, the effective patch radius expression is changed to a_{eff_2}, and all other control parameters are kept as in the second setup. As a result, 0.06% additional improvement is achieved (with respect to the second setup) in the solution error.

Finally, the main aim of the fourth solution setup is to observe the impact of the number of measured resonant frequency values and antenna configurations considered throughout the optimization process. For this purpose, additional measured values were found via a literature survey; and they were incorporated in the optimization routine (yielding a total of 36 configurations instead of 21). All other control parameters are kept as in the third setup. Consequently, 2.75% additional improvement is achieved (with respect to the third setup) in the solution error.

Tab. 2 lists the parameter values in each setup together with the obtained results and general observations.

		Obj. Func.	a_{eff}	D.O.F. in a_{eff}	N	β_1	β_2	β_3	β_4	β_5	Notes
[10]		f_1	Eq. (2)	4	21	0.247	610.731	8.690	8.152	-	Used as the main ref.
This Study (Setup #)	1	f_1	Eq. (2)	4	21	0.24700	610.72517	8.69108	8.15123	-	0.02% improvement with respect to [10]
	2	f_2	Eq. (2)	4	21	0.24698	610.73566	8.69048	8.15487	-	0.1% improvement with respect to Setup 1
	3	f_2	Eq. (4)	5	21	0.24698	610.73566	8.69048	8.15487	609.26200	0.06% improvement with respect to Setup 2
	4	f_2	Eq. (4)	5	36	0.24877	610.74245	9.20410	8.15488	609.04877	2.75% improvement with respect to Setup 3

Tab. 2. The general outline of each solution setup (D.O.F, standing for degree-of-freedom, is the number of β parameters in the proposed expression; and N is the number of resonant frequency measurements considered throughout the optimization).

4. Discussions and Conclusion

In this study, departing from the closed form expression(s) derived earlier by other researchers in previous studies, a more accurate effective patch radius expression is investigated.

Compared to other expressions existing in the open literature, Akdagli and Guney's [10] expression was the one giving more accurate results for a very wide range of substrate thickness. Step by step, by changing one control parameter in the solution procedure, the impact of the parameter change on the solution accuracy is observed; where the general outline corresponding to each step is summarized in Tab. 2.

	Achieved Error (%)	
	When the first 21 configurations are considered	When the whole 36 configurations are considered
Akdagli and Guney [10]	≈ 0.506	≈ 0.811
Setup 1	≈ 0.506	≈ 0.811
Setup 2	≈ 0.505	≈ 0.810
Setup 3	≈ 0.505	≈ 0.810
Setup 4	≈ 0.519	≈ 0.788

Tab. 3. Achieved percentage errors for all solution setups with comparison to [10].

The following major remarks and conclusions can be made in light of this study: The percentage error in Akdagli and Guney's expression [10] by using the measured values of 21 antenna configurations is 0.506% as seen in Tab. 3 (Akdagli and Guney have computed this value as 0.48% in their own paper; the difference in the error terms is probably due to the difference in arithmetic precision in the studies). On the other hand, when the other antenna configurations are also considered, it is observed that the percentage error of Akdagli and Guney's expression is much more, which is about 0.811%.

Setups 1 to 3 do not seem to have dramatic impact on the error terms as seen in Tab. 2 and Tab. 3 (regardless of the number of antenna configurations in the study). The improvement in the percentage error achieved by Setup 1 is just 0.02%; this value is about 0.1% for Setup 2; and 0.06% for Setup 3.

The most critical impact is observed in Setup 4, i.e. when the number of antenna configurations considered during the optimization is increased. For this setup, from Tab. 3, one gets the indication that the error seems to increase (from 0.506% to 0.519%) when only the results for the first 21 antennas are considered. However, when the whole set with 36 antennas are considered, it is observed that the results of this setup are much more accurate than those obtained in the other setups (0.788% instead of 0.811%). As a matter of fact, the overall achievement in the percentage error is 3% throughout the 4 setups, since $(1 - 2\times10^{-4}) \times (1 - 10^{-3}) \times (1 - 6\times10^{-4}) \times (1 - 6\times10^{-4}) = 0.97 = (1 - 3\times10^{-2})$. In other words, the a_{eff} expression given in (4) with 5 β coefficients given in Tab. 2 shall better be used instead of the a_{eff} expression of Akdagli and Guney; since it yields 3% more accurate estimates for the resonant frequency.

Tab. 4 lists all antenna configurations used throughout this study together with measured values in the literature and computed values via different patch radius expressions together with the relevant percentage errors achieved for each setup.

As a final remark, it should be noted that PSO is a powerful tool for the solution of complex multidimensional optimization problems both in continuous and discrete domains. The results of this study, once more demonstrate that the method is also applicable for determination of unknown parameters in closed form expressions, once an appropriate formulation could be defined.

In the case of the effective patch radius expression of the circular microstrip patch antennas, certainly, credit should be given to those who are cited in this study for proposing some alternative expressions and improving them by the time.

Physical and Electrical Parameters				Resonant Frequency f_r (MHz)												
a (cm)	h (cm)	ε_r	h / λ_d $\times 10^{-2}$	Measured		[10] ? (*)	[10]		Setup 1		Setup 2		Setup 3		Setup 4	
				Value	in		Obtained Value	Error (%)	Obtained Value	Error (%)	Obtained Value	Error (%)	Obtained Value	Error (%)	Obtained Value	Error (%)
1.1500	0.1588	2.65	3.8118	4425	[11]	√	4413.452	0.261	4413.360	0.263	4413.809	0.253	4414.107	0.246	4413.134	0.268
1.0700	0.1588	2.65	4.0685	4723	[11]	√	4722.183	0.017	4722.078	0.020	4722.592	0.009	4722.933	0.001	4721.827	0.025
0.9600	0.1588	2.65	4.5001	5224	[11]	√	5224.692	0.013	5224.564	0.011	5225.192	0.023	5225.610	0.031	5224.284	0.005
0.7400	0.1588	2.65	5.7147	6634	[11]	√	6636.360	0.036	6636.155	0.032	6637.168	0.048	6637.843	0.058	6636.296	0.035
0.8200	0.1588	2.65	5.2323	6074	[11]	√	6042.928	0.512	6042.757	0.514	6043.598	0.501	6044.158	0.491	6042.556	0.518
3.4930	0.1588	2.50	1.3140	1570	[1]	√	1549.791	1.287	1549.775	1.288	1549.856	1.283	1549.913	1.279	1549.799	1.287
13.894	12.700	2.70	2.6294	378	[1]	√	370.402	2.010	374.997	0.794	375.000	0.794	375.000	0.794	370.378	2.016
1.2700	0.0794	2.59	1.7336	4070	[1]	√	4168.747	2.426	4168.699	2.425	4168.931	2.431	4169.09	2.435	4168.661	2.424
3.4930	0.3175	2.50	2.5268	1510	[1]	√	1510.032	0.002	1510.000	0.000	1510.154	0.010	1510.263	0.017	1510.046	0.003
3.8000	0.1524	2.49	1.1567	1443	[3]	√	1431.240	0.815	1431.226	0.816	1431.293	0.811	1431.342	0.808	1431.249	0.814
6.8000	0.0800	2.32	0.3392	835	[12]	√	839.991	0.598	839.987	0.597	840.001	0.599	840.021	0.601	840.001	0.599
6.8000	0.1590	2.32	0.6692	829	[12]	√	831.508	0.303	831.500	0.302	831.538	0.306	831.567	0.310	831.537	0.306
6.8000	0.3180	2.32	1.3159	815	[12]	√	814.944	0.007	814.929	0.009	815.000	0.000	815.057	0.007	815.000	0.000
5.0000	0.1590	2.32	0.9106	1128	[13]	√	1122.636	0.476	1122.622	0.477	1122.690	0.471	1122.744	0.466	1122.69	0.471
4.9500	0.2350	4.55	1.3785	825	[5]	√	822.829	0.263	822.829	0.263	822.831	0.263	822.231	0.336	822.762	0.271
3.9750	0.2350	4.55	1.7210	1030	[5]	√	1021.720	0.804	1021.720	0.804	1021.722	0.804	1021.723	0.804	1021.616	0.814
2.9900	0.2350	4.55	2.2724	1360	[5]	√	1351.830	0.601	1351.830	0.601	1351.833	0.601	1351.834	0.600	1351.647	0.614
2.0000	0.2350	4.55	3.3468	2003	[5]	√	2001.916	0.054	2001.915	0.054	2001.923	0.054	2001.925	0.054	2001.516	0.074
1.0400	0.2350	4.55	6.2659	3750	[5]	√	3749.974	0.001	3749.974	0.001	3750.000	0.000	3750.000	0.000	3749.207	0.021
0.7700	0.2350	4.55	8.2626	4945	[5]	√	4944.968	0.001	4945.000	0.000	4945.028	0.001	4945.036	0.001	4945.455	0.009
4.8500	0.3180	2.52	1.8493	1099	[14]	√	1100.448	0.132	1100.432	0.130	1100.510	0.137	1100.564	0.142	1100.448	0.132
0.1970	0.04900	2.43	6.5181	25600	[18]	×	24436.556	4.545	24435.047	4.551	24442.452	4.522	24447.937	4.500	24448.533	4.498
0.3959	0.04900	2.43	3.3354	13100	[18]	×	13129.876	0.228	13129.433	0.225	13131.575	0.241	13133.158	0.253	13130.718	0.234
0.5890	0.04900	2.43	2.2813	8960	[18]	×	9057.707	1.090	9057.496	1.088	9058.515	1.099	9059.268	1.108	9058.104	1.095
0.8002	0.04900	2.43	1.7339	6810	[18]	×	6763.434	0.684	6763.317	0.686	6763.885	0.677	6764.305	0.671	6763.656	0.681
0.9962	0.04900	2.43	1.3927	5470	[18]	×	5476.175	0.113	5476.098	0.111	5476.470	0.118	5476.746	0.123	5476.632	0.121
0.4775	0.1194	10.0	6.8644	5455	[18]	×	5478.599	0.433	5478.606	0.433	5478.628	0.433	5478.628	0.433	5478.674	0.434
0.7163	0.1194	10.0	4.5931	3650	[18]	×	3727.455	2.122	3727.455	2.122	3727.467	2.122	3727.467	2.122	3726.436	2.094
1.8900	0.0350	2.47	0.5290	2885	[15]	×	2925.366	1.399	2925.352	1.399	2925.420	1.401	2925.470	1.403	2925.381	1.400
1.8900	0.0750	2.47	1.1237	2860	[15]	×	2887.240	0.952	2887.211	0.951	2887.353	0.956	2887.457	0.960	2887.272	0.954
1.8900	0.1600	2.47	2.3553	2810	[15]	×	2809.438	0.020	2809.374	0.022	2809.662	0.012	2809.871	0.005	2809.498	0.018
4.1910	0.1588	2.50	1.0994	1314	[15]	×	1297.371	1.266	1297.359	1.266	1297.416	1.262	1297.456	1.259	1297.376	1.265
4.1910	0.3175	2.50	2.1520	1286	[16]	×	1269.391	1.292	1269.369	1.293	1269.477	1.285	1269.554	1.279	1269.400	1.291
1.4100	0.1600	2.62	3.0560	3540	[17]	×	3651.061	3.137	3650.992	3.135	3651.325	3.145	3651.550	3.151	3650.881	3.132
1.3500	0.3200	2.62	6.2156	3600	[17]	×	3606.791	0.189	3606.661	0.185	3607.309	0.203	3607.748	0.215	3607.343	0.204
1.3000	0.4700	2.62	8.8756	3500	[17]	×	3460.858	1.118	3460.779	1.121	3461.601	1.097	3462.194	1.080	3498.075	0.055

Tab. 4. Antenna configurations and f_r values obtained with alternative effective patch radius expressions (*: The √ and × signs seen in the 7th column indicates whether that antenna configuration was considered in Akdagli and Guney [10] or not).

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