BER Performance Simulation of Generalized MC DS-CDMA System with Time-Limited Blackman Chip Waveform

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Abstract. Multiple access interference encountered in multicarrier direct sequence-code division multiple access (MC DS-CDMA) is the most important difficulty that depends mainly on the correlation properties of the spreading sequences as well as the shape of the chip waveforms employed. In this paper, bit error rate (BER) performance of the generalized MC DS-CDMA system that employs time-limited Blackman chip waveform is presented for Nakagami-m fading channels. Simulation results show that the use of Blackman chip waveform can improve the BER performance of the generalized MC DS-CDMA system, as compared to the performances achieved by using time-limited chip waveforms in the literature.

Keywords
BER performance simulation, generalized MC DS-CDMA system, Blackman chip waveform.

1. Introduction

Multicarrier direct sequence-code division multiple access (MC DS-CDMA) is a novel radio access scheme that is preferred widespread for wireless personal communication systems [5, 6, 8, 21, 24]. Despite advantages, the performance of the MC DS-CDMA system is affected significantly by the interference among users which is called as multiple access interference (MAI). This interference is basically dependent on the correlation properties of the spreading sequences used and the chip waveform preferred. Bit error rate (BER) and service quality of MC DS-CDMA systems depend on what proportion this interference can be removed.

The error performance in a single user communication realized over an additive white Gaussian noise (AWGN) channel depends mainly on signal energy instead of a special chip waveform. In contrast, the appropriate design of a chip waveform plays an important role on the performance of a DS-CDMA system and by this way the performance can be improved. The reason for this improvement is that the statistical properties of MAI depend on the real shape of the chip waveform used. Most of the studies carried out in the field of MC DS-CDMA communication systems have focused on the use of rectangular chip waveform [4, 14, 18, 19, 20]. However, a number of works with the aim of finding new and different chip waveforms instead of rectangular chip waveform have been presented in the literature [1, 2, 3, 7, 11, 12, 13]. In general, a chip waveform is either band-limited or time-limited. Even though the chip waveforms are band-limited in practice, time-limited chip waveforms that are defined in one chip duration have been usually considered in the subject of MC DS-CDMA systems.

In a recent paper [22], Yang and Hanzo have proposed a class of generalized MC DS-CDMA schemes with rectangular chip waveform, and evaluated its performance over multipath Nakagami-m fading channels. In [25], the same authors have extended their performance investigation of the generalized MC DS-CDMA by considering two additional types of chip waveforms, namely, the half-sine and the raised-cosine chip waveforms, in addition to the previously used rectangular chip waveform. In their work, they have concluded that for a given subcarrier spacing between two adjacent subcarriers, there exist a corresponding best choice of the chip waveform. The impact of time-limited chip waveforms on the performance of MC DS-CDMA systems has been presented in [17]. The authors have demonstrated that the BER performance of MC DS-CDMA systems can be improved by using the half-sine, the raised-cosine or the Blackman chip waveforms for both an AWGN channel and a Rayleigh fading channel.

In this paper, we extend the BER performance evaluation presented in [25], by considering the time-limited Blackman chip waveform for transmissions over multipath Nakagami-m fading channels. Simulation results show that the use of Blackman chip waveform can improve the BER performance of the generalized MC DS-CDMA system, as compared to the performances achieved by using the rectangular, the half-sine and the raised-cosine waveforms [9, 10].
2. Description of the MC DS-CDMA System

In this section, a brief description on the performance expressions proposed in [22] for the generalized MC DS-CDMA system is presented. At the transmitting point of the generalized MC DS-CDMA system, the binary data stream having bit duration of $T_b$ is serial-to-parallel converted to $U$ number of parallel substreams. Therefore, the symbol duration is $T_S = UT_b$. After serial-to-parallel conversion, the $u$th substream modulates a subcarrier frequency $f_u$ using binary phase shift keying (BPSK) for $u = 1, 2, ..., U$. Then, the $U$ subcarrier-modulated substreams are added in order to produce the complex modulated signal. Finally, spectral spreading is applied on the complex signal by the multiplication of this signal with a spreading sequence. The processing gain, $N_c$, of the subcarrier signal can be written as [22, Eq. (6)]

$$N_c = UN_i - \frac{(U-1)\lambda}{2}$$

where $U$ is the number of subcarriers; $N_i$ is the spreading gain of a corresponding single-carrier DS-CDMA system and $\lambda$ is the normalized subcarrier spacing. In this study, it is assumed that the channel between the $4$th transmitter and the corresponding receiver is a multipath Nakagami-$m$ fading channel [15]. The total number of diversity paths, $L_p$, is given as [22, Eq. (8)]

$$L_p \approx \left[ \frac{2N_c(L_i - 1)}{2N_c + (U-1)\lambda} \right] + 1$$

where $L_i$ is the number of resolvable paths of the corresponding single-carrier DS-CDMA system. The average bit error rate (BER) for the generalized MC DS-CDMA system is written as [22, Eq. (47)]

$$P_b = \frac{1}{\pi} \int_{0}^{\pi/2} \int_{0}^{\pi} \left( \frac{m \sin^2 \theta}{\gamma_c e^{-\theta} + m \sin^2 \theta} \right)^m d\theta$$

where $l = 0, 1, ..., L_i - 1$, $L_i$ ($1 \leq L \leq L_p$) is the number of diversity branches used by receiver; $m$ is the Nakagami-$m$ fading parameter and $\eta$ is the rate of average power decay. $\gamma_c$ in (3) can be obtained as [22, Eq. (41)]

$$\gamma_c = \left( \frac{\Omega_0 L_c}{\alpha_0^2} \right)^{-1} + \frac{2(KL_i - 1)(1 - e^{-\eta})}{L_i(1 - e^{-\eta})} \left( \frac{T_b}{U} + (U - 1)I_u \right)$$

where $K$ is the total number of the asynchronous users in the system, $\Omega_0$ is the average signal strength corresponding to the first resolvable path and $I_u$ is the interference variance that should be defined for a specific chip waveform employed. $I_u$ in (4) is the average of the MAI term $I_u^{(k)}$ due to path $p_k$.

3. Outline of the Generalized Derivation

This section presents a brief outline for the derivation of the $I_M$ and the $I_0$ expressions in (4) when a Blackman function is considered as a chip waveform. In [25], it has been shown that for a given $a_n$, value, $I_u^{(k)}$ can be approximated as a Gaussian random variable having zero mean and variance given by

$$\text{Var}[I_u^{(k)}] = \Omega_0^2 \alpha_0^2$$

where $\Omega_0^2 = E \left[ \alpha_0^2 \right]$, $\alpha_0^2$ represents the Nakagami-$m$ distributed channel fading amplitude, and [25, Eq. (6)]

$$\zeta_2^{(k)} = \frac{1}{T_s^2} \left[ E_{\alpha_0} \left[ R_k^2(\tau, \varphi, u, v) \right] + E_{\alpha_0} \left[ R_k^2(\tau, \varphi, u, v) \right] \right]$$

where $T_s$ shows the symbol duration of the MC DS-CDMA signal. $\tau$ and $\varphi$ represent the delay time and phase angle, respectively. $E_{\alpha_0} \left[ R_k^2(\tau, \varphi, u, v) \right]$ and $E_{\alpha_0} \left[ R_k^2(\tau, \varphi, u, v) \right]$ represent the second central moments of the extended partial autocorrelation functions $R_k(\tau, \varphi, u, v)$ and $R_k(\tau, \varphi, u, v)$ of the chip waveforms. The mathematical expression of the Blackman chip waveform is given as [16]:

$$\psi(t) = c \left[ k_1 - k_2 \cos \left( \frac{2\pi}{T_c} \right) + k_3 \cos \left( \frac{4\pi}{T_c} \right) \right] p_{\tau}(t)$$

where

$$p_{\tau}(t) = \begin{cases} 1, & 0 \leq t \leq T_c \\ 0, & \text{otherwise} \end{cases}$$

The values of $k_n$, $n \in \{1, 2, 3\}$, employed are $k_1 = 0.42$, $k_2 = 0.5$, and $k_3 = 0.08$. Using these values that were obtained from expansion of Bessel function, $c$ can be written as

$$c = \left( \frac{k_1^2 + \frac{k_2^2}{2} + \frac{k_3^2}{2} - k_1^2}{2} \right)^{-1}$$

In this step, we must provide the expressions of $E_{\alpha_0} \left[ R_k^2(\tau, \varphi, u, v) \right]$ and $E_{\alpha_0} \left[ R_k^2(\tau, \varphi, u, v) \right]$ for the Blackman chip waveform considered in this paper. With the help of MathCAD software’s symbolic mathematics, the following results are obtained

$$E_{\alpha_0} \left[ R_k^2(\tau, \varphi, u, v) \right] = E_{\alpha_0} \left[ R_k^2(\tau, \varphi, u, v) \right] = \frac{T_s^2 c^4 k_1^4}{64\pi^2} \times \chi(u, v)$$

where
\[
\begin{align*}
\zeta(u,v) &= \left[ \frac{9}{\zeta^2} + \frac{5.68}{[N_r + \zeta]^2} + \frac{5.68}{[N_r - \zeta]^2} + \frac{1}{[2N_r + \zeta]^2} + \frac{1}{[2N_r - \zeta]^2} \right] \\
&\quad + \frac{1}{[2N_r - \zeta]} - \frac{119}{[N_r + \zeta]^2} - \frac{119}{[N_r - \zeta]^2} + 1.42 \\
&\quad - \frac{142}{2N_r - \zeta} - \frac{3.36}{N_r^2 - \zeta^2} - \frac{284}{[N_r - \zeta]^2} + \frac{284}{[N_r + \zeta]^2} - \frac{284}{2N_r + \zeta} \\
&\quad - \frac{119}{N_r + \zeta} + \frac{119}{N_r - \zeta} - \frac{142}{2N_r + \zeta} + \frac{142}{2N_r - \zeta} - \frac{142}{[2N_r + \zeta]} \\
&\quad - \frac{2}{4N_r^2 - \zeta^2} + \frac{2}{N_r + \zeta} + \frac{4}{N_r - \zeta} + \frac{5.68}{N_r} + \frac{119}{N_r} \\
&\quad + \frac{3.36}{N_r + \zeta} + \frac{3.36}{N_r - \zeta} - \frac{3.36}{2N_r + \zeta} + \frac{3.36}{2N_r - \zeta} \\
&\quad - \frac{3.36}{[N_r + \zeta]^2} - \frac{3.36}{[N_r - \zeta]^2} - \frac{3.36}{2N_r + \zeta} - \frac{3.36}{2N_r - \zeta} \\
&\quad \times \sin\left(\frac{2\pi N_r}{N_r^2 - \zeta^2}\right) \sin\left(\frac{2\pi N_r}{N_r + \zeta}\right) \\
&\quad \times \sin\left(\frac{2\pi N_r}{N_r - \zeta}\right) \\
&\quad \left(\frac{1}{2N_r + \zeta} + \frac{1}{2N_r - \zeta} + \frac{1}{4N_r - \zeta} \right) \\
&\quad \times \left(\frac{284}{2N_r + \zeta} + \frac{284}{2N_r - \zeta} + \frac{284}{4N_r - \zeta} \right) \\
&\quad - \frac{3.32}{[N_r - \zeta][2N_r - \zeta]} \\
&\quad \times \sin\left(\frac{2\pi N_r}{N_r^2 - \zeta^2}\right).
\end{align*}
\]

(11)

\(\zeta\) in (11) is \(\zeta = \lambda(u - v)\) and it represents the normalized spacing between \(u\)th and \(v\)th subcarriers. Now, \(\zeta_m^{(i)}\) given by (6) can be obtained by using the above defined expressions. Based on the relationship between \(I_m\) and \(\zeta_m^{(i)}\) [23, Eq. (27)], \(\bar{I}_m\) for the Blackman chip waveform is derived as

\[
I_m = \frac{0.34N_r}{32\pi^2} \chi(u,v). 
\]

(12)

If \(\zeta = 0\), then \(I_0\) can be written as

\[
I_0 = \frac{3.43}{24N_r} + \frac{118.27}{64N_r^2\pi^2}. 
\]

(13)

### 4. Simulation Results

The BER performance of the generalized MC DS-CDMA system using time-limited Blackman chip waveform for transmission over Nakagami-\(m\) fading channels is presented in this section. The performance results belonging to the rectangular, the half-sine and the raised-cosine chip waveforms are also included into the simulation studies to illustrate the improvement carried out by using the Blackman chip waveform. It is well known that the MATLAB is powerful software that one could do numerical calculations and implement simulations. Therefore, the BER performance of the system employing time-limited Blackman chip waveform is simulated in the MATLAB environment. The system parameters kept constant in the simulations are: The number of resolvable paths of the corresponding single-carrier DS-CDMA system is \(L_1 = 32\) and the spreading gain is \(N_1 = 128\). The number of simultaneous users is \(K = 10\). The signal to noise ratio per bit is set to \(E_b/N_0 = 15\) dB. The rate of average power decay is considered as \(\eta = 0.2\). Two different number of subcarriers are used: \(U = 8\) and \(U = 32\). Finally, it is assumed that the receiver combines all of the resolvable paths of the channel \((L = L_p)\).

The BER performance of the generalized MC DS-CDMA system is shown in Fig. 1 as a function of the normalized subcarrier spacing, when the Blackman chip waveform is employed. It can be seen that for both Nakagami-\(m\) fading parameter \((m = 1\) and \(m = 3\)), the Blackman chip waveform performs the best performance especially for the low values of \(\lambda\), while the BER performances achieved by the other chip waveforms are nearly the same.
Fig. 2 depicts the BER performance of the generalized MC DS-CDMA system with Blackman chip waveform versus the normalized subcarrier spacing for $U = 32$. As in the case of Fig. 1, it can be seen that the BER performance produced by using the Blackman chip waveform is better than the other performances obtained by using the rectangular, the half-sine, and the raised-cosine chip waveforms, for both $m = 1$ and $m = 3$.

Because of the importance of the optimum spacing $\lambda_{opt}$, which enable to minimize the MAI inflicted upon each of the subcarrier signals; it will be useful to give a comparison on the BERs at this specific spacing. Tab. 1 shows the BER values obtained for each chip waveform examined in this study about the optimum spacing when $m = 3$. As shown in Tab. 1, the lowest BER at $\lambda_{opt}$ is obtained with the use of Blackman chip waveform [9, 10].

### References


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