Probabilistic Model for Free-Space Optical Links Under Continental Fog Conditions

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Abstract. *The error characteristics of a free-space optical (FSO) channel are significantly different from the fiber based optical links and thus require a deep physical understanding of the propagation channel. In particular different fog conditions greatly influence the optical transmissions and thus a channel model is required to estimate the detrimental fog effects. In this paper we shall present the probabilistic model for radiation fog from the measured data over a 80 m FSO link installed at Graz, Austria. The fog events are classified into thick fog, moderate fog, light fog and general fog based on the international code of visibility range. We applied some probability distribution functions (PDFs) such as Kumaraswamy, Johnson S_B and Logistic distribution, to the actual measured optical attenuations. The performance of each distribution is evaluated by Q-Q and P-P plots. It is found that Kumaraswamy distribution is the best fit for general fog, while Logistic distribution is the optimum choice for thick fog. On the other hand, Johnson S_B distribution best fits the moderate and light fog related measured attenuation data. The difference in these probabilistic models and the resultant variation in the received signal strength under different fog types needs to be considered in designing an efficient FSO system.*

Keywords

Free Space Optics (FSO), optical attenuations, Mie scattering, Probability Density Function (PDF), visibility, channel modeling.

1. Introduction

Free-space optical (FSO) links are of prime importance in order to meet the need for future terrestrial and ground-space communication applications [1]. The optical carrier frequencies in the range of 20-300 THz makes FSO links as important technology for future bandwidth hungry communication applications [2]. FSO links can potentially be used to bridge the last mile access network gap, to provide broadband internet access to rural areas and to link mobile base station. Some of the possible applications are electronic commerce, streaming audio and video, teleconferencing, real-time medical imaging transfer, enterprise networking, work sharing and high speed interplanetary links [3].

Atmospheric attenuators like fog, rain, snow, mist and haze severly degrade the system performance. Absorption and scattering of radiation from fog, clouds, dust, snow and smoke cause significant attenuation of a laser beam propagating through the atmosphere. Fog and clouds are typical dominating factors causing atmospheric attenuation over a considerable period of the time. However other factors like rain and snow are generally less significant. Turbulence-induced atmospheric scintillation causes severe fluctuation in received signal power [4]. Scintillation effects, for terrestrial FSO links, can be mitigated by increasing transmission power, wavelength diversity [5], multiple transmit beam [6] and multiple receiver [7]. The proposed techniques can only be a way to reduce the effect of scintillation but often not efficient against aerosol absorption or scattering [8]. The main impairing factor for terrestrial FSO links is fog [4] and thus the primary focus of this paper.

Statistical characterization and modeling of fog can provide important inputs for better system design of FSO links. The FSO links are impaired by various atmospheric attenuators and this paper provides a through statistical study of the received signal strength under a foggy channel.

Probabilistic models have been proposed for fog drop size distribution (DSD). Shettle [9] used modified Gamma distribution to model fog and clouds' DSD. Muhammad et al. [10], recently presented the PDF estimates of two selected continental fog events. They proposed the Lognormal and Gamma PDF as the closest fit for the continental fog. Wakeby distribution was proposed in [12] for dense continental fog conditions.

In this paper we shall present a probabilistic model based on six months long measurements of optical attenuation data under continental fog conditions. Different PDFs have been compared on the basis of their best fit (from Q-Q and P-P function plots) for measured data and their CDF are being presented. The organization of this paper is as follows: fog effects on FSO are discussed in Section 2. Fog attenuation measurements and attenuation analysis have been studied in Section 3. In Section 4, different PDFs that fit best to the measured data are discussed. Section 5 provides results whereas Section 6 concludes the paper.

2. Fog Effects on FSO

Fog is characterized by several physical parameters such as liquid water content, particle size distribution, temperature and humidity. Since the size of fog particles is comparable to the transmission wavelength of optical and near infrared waves, Mie scattering applies and results in high attenuation [13]. Fog comprises of fine water droplets, ice crystals or smoke particles suspended in the atmosphere. Fog near the earth surface reduces the visibility and can cause degradation of an optical link performance. In general, the probability of occurrence of fog is much higher in winter than in summer for continental environments when temperature approaches 0° C and relative humidity rises above 80% [11].

There are different kinds of fog; among them continental and maritime are prominent. According to international code of visibility any kind of fog can further be categorized into four distinct types. The four types of fog depending upon the visibility range are given in Tab. 1.

Visibility range (m)	Description	Sp. Attenuation (dB/km)
40 - 70	Dense fog	250-143
70 - 250	Thick fog	143-40
250 - 500	Moderate Fog	40-20
500 - 1000	Light fog	20-9.3

Tab. 1. International code of visibility range.

Fog reduces the availability and reliability of FSO links significantly. The attenuation due to fog reaches up-to 480 dB/km in maritime environment [11] and 236 dB/km for continental fog conditions. The visible and near-infrared wavelengths are highly attenuated due to fog. However, the mid and long-wave infrared spectral regions are not as sensitive to fog. The probabilistic models for thick, moderate and light fog have been discussed in the later sections of this article and also results are being presented which could fit generic fog conditions. By general fog we mean the fog which consists of whole range of visibilities and corresponding attenuation ranges.

3. Fog Attenuation Measurements

The research group OptiKom at TU Graz developed the optical link for transmission measurement at infrared wavelengths. This measurement system basically consists of an optical transmitter and receiver system, each equipped in a waterproof housing mounted on a tripod with mechanical options for alignment. The technical specifications for the system are given in Tab. 2.

Parameters	**850 nm FSO Link**	**950 nm FSO Link**
Tx Wavelength	850 nm	950 nm
Tx Technology	LED	LED
Rx Technology	Si-APD	Si-APD
Tx avg. optical power	8 mW	1mW per diode
Avg. radiated power	3.5 mW	4 mW
Tx aperture diameter	1 X 25 mm convex lens	4 X 25 mm convex lens
Rx aperture diameter	98mm	98 mm
Tx divergence angle	2.4 degree	0.8 degree
Rx acceptance angle	1.7 degree	1.7 degree
Rx sensitivity	Min -35 dBm	Min -35 dBm
Specific link margin	224 dB/Km	224 dB/Km
Link distance	79.8m	79.8m

Tab. 2. FSO system specification.

The link distance was selected carefully as a compromise between accuracy and allowable attenuation range, depending on the expected maximum fog attenuation. The system provided a dynamic range of 25 dB at each wavelength. The link distance of 79.8 m allowed to measure specific attenuation up to 310 dB/km. The measured data was processed and evaluated in MATLAB. Specific attenuation in dB/km is used for analysis of the data as a standard.

Various measurement campaigns were conducted to study the fog effects on FSO links. The optical attenuations has been measured reaching up to 120 dB/km averaged on a minute scale over a link distance of 79.8 m. The maximum specific attenuation that was recorded reached up to 236 dB/km. One of the most successful measurement campaign was started on September 27, 2005 and continued till March 1, 2006 for 156 days. During the whole measurement campaign we observed 18 major fog events, the details of which are tabulated in Tab. 3.

It is important to mention that the minimum duration of these 18 fog events is two and half hour. The optical signal attenuations (specific attenuation) were computed from the received signal power using an appropriate model (implemented in LabView) based on the hardware specification of the self-developed FSO system. Upon analyzing these 18 fog events, we noticed that the last four fog events belong to dense fog type having optical attenuations higher than 143 dB/km. The remaining fog events were of thick fog types. The 9th column (last column) in Tab. 3 shows the maximum value of specific attenuation reached for respective fog event.

ID	Start date	End date	Start time	End time	Duration	50%	90%	Max.
1	29.9.2005	29.9.2005	02:00:00	07:29:59	5:30	2.392972005	37.63699285	83.9315
2	25.10.2005	25.10.2005	03:00:00	10:59:59	08:00	38.29357457	93.19137303	112.1198
3	26.10.2005	26.10.2005	01:30:00	03:59:59	02:30	7.0810	110.8667	118.0525
4	10.11.2005	11.11.2005	16:00:00	09:59:59	18:00	12.9737	29.3020	71.4139
5	11.11.2005	12.11.2005	14:00:00	11:00:00	21:00	10.5382	33.8819	63.3891
6	22.11.2005	22.11.2005	00:00:00	05:59:59	06:00	5.9392	100.7586	113.8157
7	25.11.2005	28.11.2005	19:00:00	09:59:59	63:00	8.0926	19.6037	66.8660
8	28.11.2005	29.11.2005	22:00:00	05:59:59	08:00	9.2515	113.8157	124.6511
9	30.11.2005	30.11.2005	06:00:00	11:59:59	06:00	7.3776	103.2164	116.2736
10	02.12.2005	02.12.2005	01:00:00	17:24:59	16:25	0.9405	4.5956	96.4585
11	13.12.2005	14.12.2005	21:00:00	11:59:59	15:00	9.7898	62.4800	97.3173
12	25.12.2005	28.12.2005	19:32:00	13:30:59	65:59	6.4271	13.5516	42.8042
13	09.01.2006	09.01.2006	05:00:00	09:59:59	05:00	72.1849	109.5157	116.2022
14	09.01.2006	10.01.2006	22:00:00	04:59:59	07:00	1.0790	100.7586	121.5819
15	30.01.2006	31.01.2006	21:00:00	09:59:59	13:00	20.8970	81.7411	165.0155
16	31.01.2006	01.02.2006	15:00:00	08:59:59	18:00	46.6857	154.7429	225.7779
17	01.02.2006	02.02.2006	20:00:00	10:19:59	14:20	87.3802	168.3753	217.0663
18	02.02.2006	03.02.2006	21:00:00	07:59:59	11:00	28.9490	159.9085	236.0736

Tab. 3. Statistics of fog attenuation measurement campaign at Graz, Austria.

In order to study the influence of different fog types and to suggest distribution model of every kind of fog for terrestrial FSO links, we categorized the whole set of attenuation data recorded over the period of six months into four distinct classes of fog types based on the international code of visibility as mentioned in Tab. 1. By this way, all the attenuation values higher than 143 dB/km were taken in the bin of dense fog, between 40-143 dB/km into thick fog, 20-40 dB/km into moderate fog and 9.3-20 dB/km into light fog. The main focus of this research is to find the best fit probabilistic model for free-space optical links under different kind of continental fog conditions using the data of all 18 fog events.

4. Probabilistic Models

Probabilistic modeling of received signal strength helps acquire a prior estimate of attenuation PDF. We compare all statistical distributions to find the best fit distribution through the Q-Q plot and the P-P plot. Kolmogorov-Smirnov test is generally used to measure the goodness of fit. However it requires that the measurement and the fit distribution function should be statistically independent which is not the case for our analysis; as the distribution parameters are being calculated directly from the measured data. Therefore we will use the Q-Q plot and the P-P plot to select the best fit. The results of the Q-Q plot and the P-P plot suggest that a Kumaraswamy distribution describes well the characteristics of the received signal strength on the FSO channel under foggy conditions. The Wakeby distribution was suggested as the best fit for dense continental fog conditions [12]. A Logistic distribution best fits the thick fog and Johnson S_B distribution best fits the moderate and light fog, respectively.

Kumaraswamy distribution is a two-parameter family of distributions which has many similarities to the beta distribution and a number of advantages in terms of tractability [15]. This distribution is mainly used for hydrological processes. The distribution function of Kumaraswamy distribution is

$$f(x) = \frac{a_1 a_2 z^{a_1-1}[1-z^{a_1}]^{a_2-1}}{b-a} \tag{1}$$

where $z \equiv (x-a)/(b-a)$, a_1 and a_2 are shape parameter with (a_1 and $a_2 > 0$), a, b is boundary parameters ($a < b$) with domain $a \leq x \leq b$.

The logistic distribution is similar to normal distribution but this distribution is quicker to calculate than the normal [16]. Another advantage over the normal distribution is that it has a closed form CDF. But it has longer tails and a higher kurtosis than the normal. The distribution function of logistic function is

$$f(x) = \frac{\alpha}{\beta}[\frac{x-\gamma}{\beta}]^{\alpha-1}[1+[\frac{x-\gamma}{\beta}]^{\alpha}]^{-2} \tag{2}$$

where α is shape parameter ($\alpha > 0$), β is scale parameter ($\beta > 0$) and γ is location parameter. The domain is $\gamma \leq x < +\infty$.

The Johnson S_B distribution, or alternatively the 4-parameter lognormal model, is appealing on theoretical grounds as a candidate probability distribution function for ratios, or variates constrained by extremes [17]. It has found application in a variety of fields including ambient air pollution, rainfall distribution and forestry. The distribution function of Johnson S_B distribution function is

$$f(x) = \frac{\delta}{\lambda\sqrt{2\pi}z[1-z]}\exp[-\frac{1}{2}[\gamma+\delta\ln[\frac{z}{1-z}]]^2] \tag{3}$$

where $z \equiv x - \xi/\lambda$, γ and $\delta(\delta > 0)$ are shape parameters, $\lambda(\lambda > 0)$ is scale parameter and ξ is location parameter with domain $\xi \leq x \leq \xi + \lambda$.

5. Results

We applied distribution fitting techniques on measured data to get the most appropriate probabilistic model for terrestrial free-space optical communication links under different continental fog conditions (different types of fog given in Tab. 1). We fitted all the probability density functions on the actual measured data by visualizing their PDF and the CDF. The Q-Q plot and the P-P plot are used to select the two best fit distribution models for each kind of fog.

The analysis of the measured data resulted in the calculation of some basic parameters for different types of fog i.e. dense, thick, moderate and light fog, along with general fog conditions. In Tab. 4, the summary of the statistics of the measured optical attenuation data is provided.

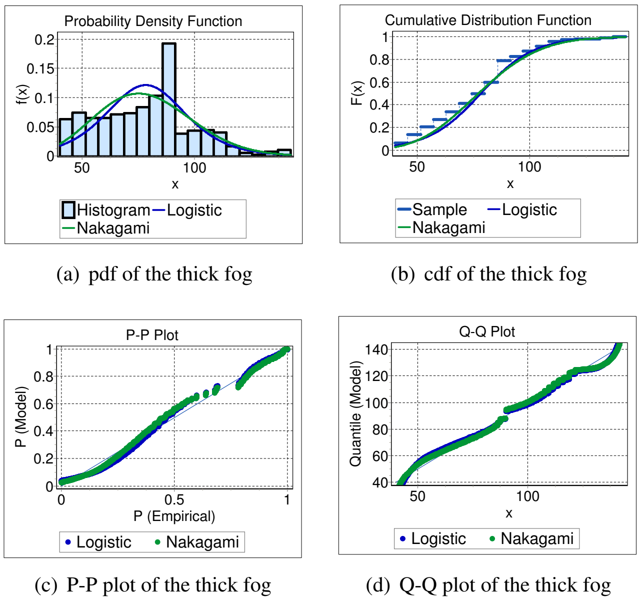
Description	Dense fog	Thick fog	Moderate fog	Light fog	General fog
Sample Size	25451	183290	86355	225259	520355
Mean	171.82	78.318	27.364	13.338	46.306
Variance	376.3	457.33	30.706	8.660	1849.9
Std. Error	0.121	0.049	0.018	0.0062	0.059
Skewness	0.791	0.253	0.564	0.53469	1.439
CV x 100	11.289	27.31	20.25	22.06	92.88

Tab. 4. Descriptive statistics of all fog events.

We have used coefficient of variation (CV) as a standard quantity to describe the variation in different types of fog. The CV is a ratio of the standard deviation to the mean and it is a useful measure for comparing the data sets with unequal sample sizes. The results shows that dense fog is the most stable fog (having the least variation) whereas general fog has much higher variation. The main reason being that rapid changes in attenuation are recorded during the fog formation and dissipation. The skewness for all kind of fog is positive which indicates that the data is skewed right.

5.1 Thick Fog

Thick fog is characterized when the visibility range is 40 - 70 m and attenuation level is between 40 and 143 dB/km. In order to evaluate statistically the attenuation data for thick fog, and to observe its characteristics, we selected fog attenuation values exceeding from 40 dB/km and less than 143 dB/km. The descriptive statistics of the selected attenuation data set are presented in Tab. 4. We applied distribution fitting techniques on the sampled attenuation data by comparing different probability density functions. The Q-Q plot and the P-P plot show that the two best fit distributions are the Logistic distribution and the Nakagami distribution for thick fog. The PDF over the histogram of the measured data for two best fit distribution is given in Fig. 1(a). We have also showed the CDF for the sampled data in Fig. 1(b). The P-P plot and the Q-Q plot for attenuations corresponding to thick fog are shown in Figs. 1(c) and 1(d) respectively.



(a) pdf of the thick fog

(b) cdf of the thick fog

(c) P-P plot of the thick fog

(d) Q-Q plot of the thick fog

Fig. 1. Statistical analysis of the thick fog.

Description	**Distribution**	**Parameters**
Thick fog	Logistic*	σ =11.79, μ =78.318
	Nakagami	m =3.5126,Ω =6591.0
Moderate fog	Johnson S_B *	γ=0.53036, δ =0.62405,λ =20.894 , ξ =19.886
	Wakeby	α =11.123, β =0.44992, γ =0, δ=0, ξ =19.693
Light fog	Johnson S_B *	γ=0.51767,δ =0.65162, λ =11.315 , ξ =9.2184
	Wakeby	α =6.1241 , β =0.48097, γ =0 δ=0, ξ =9.2025
General fog	Kumaraswamy*	α_1 =0.45233, α_2=1.6528, a=9.3395, b=236.09
	Gen. Gamma (4P)	k=1.0277, α =0.58616, β =62.564 ,γ =9.3395

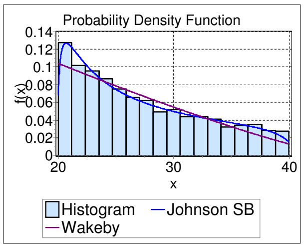
Tab. 5. Optimum parameters for selected Distribution.

In Fig. 1(a) probability density plot shows that the Logistic density function has better fit than the Nakagami distribution for thick fog data. It is also evident from the CDF plot in Fig. 1(b) that the Logistic distribution has better fit. The P-P plot (Fig. 1(c)) and Q-Q plot (Fig. 1(d)) are not continuous for thick fog data, which shows some unusual peaks in the histogram. These breaks represent that during fog formation or fog dissipation the channel is changing abruptly. For thick fog it is very difficult to decide which distribution is better on the basis of the P-P plot and the Q-Q plot. Hence, the CDF can be used to find the best fit. In Fig. 1(b) the CDF for the logistic distribution is more linear as compared to the Nakagami distribution which suggest that the logistic density function fits better for thick fog.

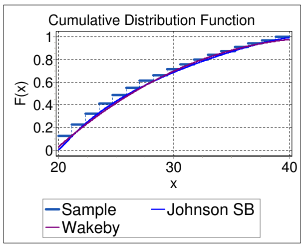
We have calculated the optimum parameters for selected distributions for every kind of fog and they are provided in Table 5. Note that * in Tab. 5 indicates the best fit distribution according to the results of the Q-Q plot and the P-P plot. Tab. 5 mentions the two best fit distributions on our data for all kinds of fog. The optimum parameters for the logistic and the Nakagami distribution for thick fog conditions are also provided.

5.2 Moderate Fog

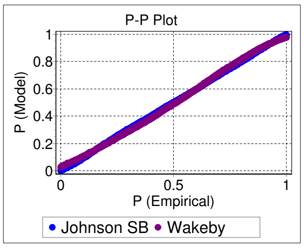
We have selected fog attenuation values in the range of 40-20 dB/km measured on a second scale on 80 m FSO link for the statistical evaluation of the attenuation data for moderate fog and to observe its characteristics. The descriptive statistics of the selected attenuation data are given in Tab. 4. The distribution fitting has been applied on the sampled attenuation data by comparing different probability density functions. Our findings suggest that the Johnson S_B and the Wakeby distribution model fit better than other PDFs. For moderate fog, the histogram over the PDF and the CDF along with the P-P plot and the Q-Q plot are given in Figs. 2(a), 2(b), 2(c), 2(d) respectively.



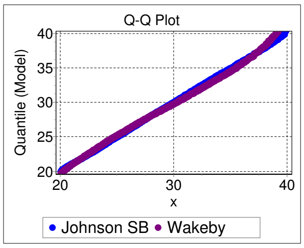
(a) pdf of the moderate fog



(b) cdf of the moderate fog



(c) P-P plot of the moderate fog



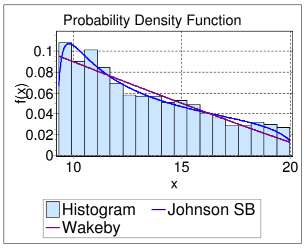
(d) Q-Q plot of the moderate fog

Fig. 2. Statistical analysis of the moderate fog.

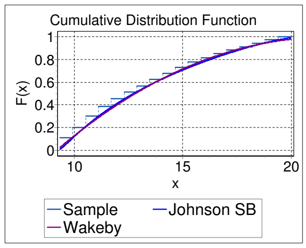
It is evident from Fig. 2(a) that the probability density plot of the Johnson S_B density function has better fit for moderate fog attenuation data. The CDF of the Johnson S_B and the Wakeby distribution is shown in Fig. 2(b) for moderate continental fog. In Fig. 2(d), the Q-Q plot for the Johnson S_B distribution is converging at the end while for the Wakeby distribution is diverging, which indicates that the Johnson S_B distribution is a better model for moderate fog data. The optimum parameters for the Wakeby and the Johnson S_B are given in Tab. 5.

5.3 Light Fog

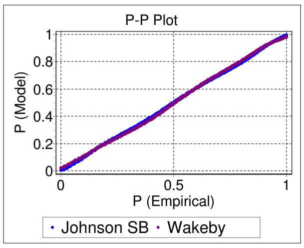
We have taken attenuation data in the range of 20-9.3 dB/km to find the probabilistic model for light fog conditions. The descriptive statistics of the selected attenuation data set are given in Tab. 4. For light fog, the PDF over histogram, the CDF, the P-P plot and the Q-Q plot are given in Figs. 3(a), 3(b), 3(c), 3(d) respectively.



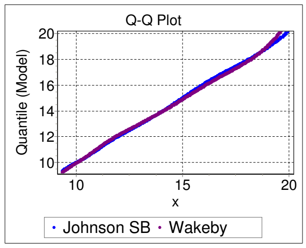
(a) pdf of the light fog



(b) cdf of the light fog



(c) P-P plot of the light fog



(d) Q-Q plot of the light fog

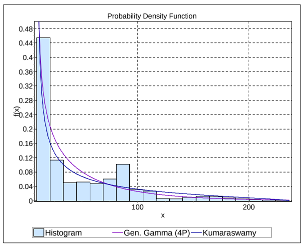
Fig. 3. Statistical analysis of the light fog data.

The probability density plot in Fig. 3(a) shows that the Johnson S_B density function has better fit for light fog attenuation data. The P-P plot in Fig. 3(c) shows that both distribution models are close to the reference line but in Fig. 3(d), and the Q-Q plot for the Johnson S_B distribution is converging at the end while for the Wakeby distribution is diverging, which proves that the Johnson S_B distribution is a better model for light fog data. The calculated parameters for the Wakeby and the Johnson S_B are given in Tab. 5.

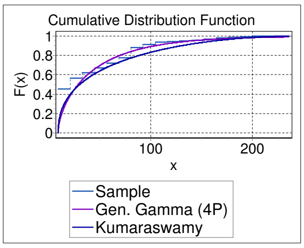
5.4 General Fog

For the statistical evaluation of the attenuation data for general continental fog, and to observe its characteristics we selected the whole set of attenuation data measured on a second scale on the 80 m FSO link. The descriptive statistics of the selected attenuation data set are presented in Tab. 4. For general fog, the PDF over the histogram and the CDF along with the Q-Q plot and the P-P plot are given in Figs. 4(a), 4(b), 4(c), 4(d) respectively.

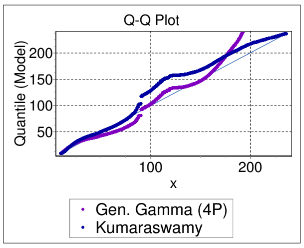
In Fig. 4(a) the probability density plot shows that the Kumaraswamy density function has better fit for the general fog data. The cumulative distribution function of the Kumaraswamy and the Gamma distribution are shown in Fig. 4(b). It is visible in Fig. 4(b) that the CDF of the Kumaraswamy distribution fits better than the Gamma distribution. From Fig. 4(d), it is obvious that the P-P plot for the Kumaraswamy distribution is much closer to a reference line than the Gamma distribution. The Q-Q plot in Fig. 4(c) confirms that the Kumaraswamy density function has better fit for general fog data. In the Q-Q plot, it is obvious that for higher attenuation the Q-Q plot for the Kumaraswamy distribution is converging whereas for the Gamma distribution is diverging. These results show that the Kumaraswamy distribution is a reasonable choice for predicting received signal strength under general continental fog. The calculated parameters for the Kumaraswamy and the Gamma distribution are given in Tab. 5.



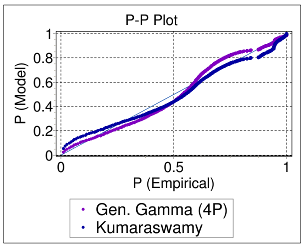
(a) pdf of the general fog



(b) cdf of the general fog



(c) Q-Q plot of the general fog



(d) P-P plot of the general fog

Fig. 4. Statistical analysis of the general fog data.

6. Conclusions

Adverse atmospheric weather conditions limit the performance of terrestrial optical wireless links in terms of availability, reliability and link distance. Fog being the foremost reason leads to very high attenuations of the optical signal transmitted in terrestrial free-space. The high variability of attenuation can cause link outages that could last up to several hours. The empirical models like the famous Kim and Kruse models provides reasonable estimates for optical attenuation under fog. However, they shed no light towards the distribution of the received signal strength. The statistical models presented in this article shall yield better understanding of the spatial and temporal variations of the fog attenuation. The logistic distribution has been found as appropriate for thick fog, Johnson S_B for moderate and light fog and Kumaraswamy distribution for general continental fog. These results suggest that these distributions can provide suitable estimates for received signal strength under thick, moderate, light and general continental fog conditions.

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