## **Optimum Detection Location-Based Cooperative Spectrum Sensing in Cognitive Radio**

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Abstract. Cognitive radio arises as a hot research issue in wireless communications recently, attributed to its capability of enhancing spectral efficiency and catering for the growing demand for bandwidth. As a good embodiment of cognitive radio's unique feature, i.e. making use of every bit spectral resource, spectrum sensing plays a vital role in the implementation of cognitive radio. To alleviate negative effect on cooperative spectrum sensing brought by bit errors, we introduce a novel concept, i.e. Optimum Detection Location (ODL) and present two algorithms of different computational complexity for locating ODL, together with an ODL-Based cooperative spectrum sensing scheme, with the motivation to exploit the gain derived from geographic advantages and multiuser diversity. Numerical and simulation results both demonstrate that our proposed spectrum sensing scheme can significantly improve the sensing performance in the case of reporting channel with bit errors.

#### Keywords

Cognitive radio, spectrum sensing, Wireless Region Area Network (WRAN), Optimum Detection Location (ODL).

#### 1. Introduction

Recently, the misconception that there is scarcity of spectrum for wireless communication is provoked mostly by the intense competition for spectrum usage. However, studies from the Federal Communication Commission (FCC) show that the utilization of licensed spectrum only ranges from 15% to 85% [1]. In order to use this spectrum (white space) to the largest extent, IEEE 802.22 Wireless Region Area Network (WRAN) Group is chartered with the development of a CR-based standard for wireless regional area networks (WRAN) [2].

In cognitive radio systems, secondary users can use the licensed spectrum as long as the primary user is absent. Therefore spectrum sensing plays an important role in the implementation of cognitive radio. To improve the sensing reliability, cooperative spectrum sensing has been proposed to exploit multiuser diversity in sensing process [3]. It is usually performed in two successive stages: sensing and reporting. In the sensing stage, every cognitive user performs spectrum sensing individually. In the reporting stage, all the local sensing observations are reported to a common receiver and the latter will make a final decision on the absence or the presence of the primary user.

To the best of our knowledge, most works concerning cooperative spectrum sensing are based on the assumption of ideal reporting channels [4]-[6]. In practice, bit errors are unavoidable due to adverse factors introduced along transmission path, and they will cause negative impact to the performance of cooperative spectrum sensing. In this paper, we will take bit errors in reporting channels into account and propose an optimum detection location-based cooperative spectrum sensing scheme to improve the sensing performance. Our proposed cooperative spectrum sensing scheme is based on a novel concept, i.e. Optimum Detection Location (ODL) brought up in this paper, which represents the location where the secondary user has the best sensing performance. Besides, we provide two algorithms for fixing on ODL.

The rest of the paper is organized as follows. In Section 2, system model, energy detection and cooperative spectrum sensing with reporting are studied. In Section 3, we propose two algorithms to find the optimum detection location. In Section 4, the ODL-based cooperative spectrum sensing scheme is proposed and OR fusion rule scheme is analyzed theoretically. Simulation results are shown and discussed in Section 5. Finally, we draw our conclusions in Section 6.

## 2. Cooperative Spectrum Sensing

#### 2.1 System Model

In this paper, we adopt the system model based on the IEEE 802. 22 WRAN deployment scenario, as is illustrated

in Fig. 1. The system model includes a TV broadcast station as the primary user, a WRAN base station (BS) as the secondary base station and customer-premises equipments (CPEs) as the secondary users. The primary user and the secondary BS are far apart and the secondary users are uniformly randomly distributed within the coverage radius of the secondary BS.



Fig. 1. The topology of WRAN.

For simplicity, we assume that all channels only experience path loss. Thus the primary user's signal to the noise ratio received at each secondary user is computed by,

$$\gamma_{1,i} = \frac{P_u \beta}{\sigma^2 d_{1,i}} \quad \text{for } i = 1, 2, ..., M$$
(1)

where M is the total number of CPEs in the network. Likewise, *i*-th secondary user's signal to the noise ratio received at BS is computed by

$$\gamma_{2,i} = \frac{P_s \beta}{\sigma^2 d_{2,i}} \quad \text{for } i = 1, 2, ..., M$$
(2)

where  $d_{1,i}$  is the distance between PU and *i*-th secondary user, and  $d_{2,i}$  is the distance between BS and *i*-th secondary user.  $P_u$  and  $P_s$  is the signal power of PU and secondary user respectively. Noise power  $\sigma^2$  is identical in WRAN system, and  $\alpha$  is the path loss exponent factor and  $\beta$  is a scalar.

#### 2.2 Energy Detection

Energy detection is suboptimal but is simple to implement. For implementation simplicity, we restrict ourselves to energy detection in the spectrum sensing. The test statistic for energy detector is given by

$$T(y) = \frac{1}{N} \sum_{n=1}^{N} |y(n)|^{2}$$
(3)

where y(n) denotes the sampled received signal at secondary user. Under H<sub>0</sub>, i.e. the absence of PU, y(n) equals noise u(n), which is Gaussian iid random process with zero mean and variance  $\sigma^2$ . For a large *N*, PDF of *T*(*y*) can be approximated by a Gaussian distribution using central limit theorem. For a chosen threshold, the probability of false alarm is given by

$$P_{f}(\varepsilon) = P_{r}(T(y) > \varepsilon \mid H_{0}) = Q\left[\left(\frac{\varepsilon}{\sigma^{2}} - 1\right)\sqrt{N}\right].$$
 (4)

Under H<sub>1</sub>, i.e. the presence of PU, y(n) is the sum of PU's signal s(n) and noise u(n). Assume s(n) is a complex PSK modulated signal and we defined  $\gamma_1 = \sigma_s^2/\sigma^2$  as the primary user's signal power to noise ratio received at the secondary user. For a chosen threshold, the probability of detection is given by

$$P_d(\varepsilon) = P_r(T(y) > \varepsilon \mid H_1) = Q\left[\left(\frac{\varepsilon}{\sigma^2} - \gamma_1 - 1\right)\sqrt{\frac{N}{2\gamma_1 + 1}}\right].$$
 (5)

Note that PU's signal power is especially weak at secondary users in cognitive radio networks [7], so  $2\gamma_1+1$  can be neglected in (5), and (5) can be rewritten as

$$P_d(\varepsilon) = Q \left[ \left(\frac{\varepsilon}{\sigma^2} - \gamma_1 - 1\right) \sqrt{\frac{N}{2\gamma_1 + 1}} \right] \approx Q \left[ \left(\frac{\varepsilon}{\sigma^2} - \gamma_1 - 1\right) \sqrt{N} \right].$$
(6)

#### 2.3 Cooperative Spectrum Sensing with Reporting Errors

Cooperative spectrum sensing is usually employed in cognitive radio networks to overcome hidden node problem and improve sensing reliability. To minimize transmission overhead of sensing data, we assume that CPEs perform energy detection independently and then forward their one-bit decision to BS. In practice, the reporting channels may suffer from bit errors which will deteriorate the performance of cooperative spectrum sensing. For brevity, we will start to deal with this problem in the case when there is only one CPE in the WRAN. Assuming BPSK, the bit-error rate  $P_b$  is given by

$$P_b(\gamma_2) = Q(\sqrt{2\gamma_2}) \tag{7}$$

where Q(.) is the Q-function.  $\gamma_2$  stands for the secondary user's signal to the noise ratio received at BS, which is expressed as

$$\gamma_2 = \frac{P_s \beta}{\sigma^2 d_2^{\ \alpha}} \,. \tag{8}$$

Therefore, we can derive the probability of detection  $P_d^s$  in WRAN as

$$P_{d}^{s} = (1 - P_{d})P_{b} + P_{d}(1 - P_{b}) = P_{d} + P_{b} - 2P_{d}P_{b}.$$
 (9)

Similarly, the probability of false alarm  $P_f^s$  is given by

$$P_f^s = P_f + P_b - 2P_f P_b \,. \tag{10}$$

Combining  $P_f$ ,  $P_d$  in (4), (6) respectively, we have

$$P_{f} = Q \left[ \left( \frac{\varepsilon}{\sigma^{2}} - 1 \right) \cdot \sqrt{N} \right] + Q(\sqrt{2\gamma_{2}})$$
$$-2Q \left[ \left( \frac{\varepsilon}{\sigma^{2}} - 1 \right) \cdot \sqrt{N} \right] \cdot Q(\sqrt{2\gamma_{2}})$$
(11)

$$P_{d} = Q \left[ \left( \frac{\varepsilon}{\sigma^{2}} - \gamma_{1} - 1 \right) \cdot \sqrt{N} \right] + Q(\sqrt{2\gamma_{2}}) -2Q \left[ \left( \frac{\varepsilon}{\sigma^{2}} - \gamma_{1} - 1 \right) \cdot \sqrt{N} \right] \cdot Q(\sqrt{2\gamma_{2}})$$
(12)

#### **3.** Optimum Detection Location

According to the two stages in cooperative spectrum sensing, system sensing performance in a WRAN is determined by two factors: one is the performance of single node spectrum sensing; the other is the bit-error rate of reporting channels. Since both of the two factors are impacted by signal to noise ratio, hence the veracity of every CPE's sensing result received by BS varies according to their deployment location in the network. Here, we define the location where CPE has the best performance as ODL.

#### 3.1 ODL Modeling

*Lemma*. Given that ODL exists in the network, it must situate in the line which connects PU and BS.

Proof: Convert 
$$P_d^s$$
 in (9) to  
 $P_d^s = P_d + P_b - 2P_d P_b = P_b + P_d \cdot (1 - 2P_b)$ 

with  $d_2$  fixed,  $\gamma_2$  and  $P_b$  is constant, and

$$\therefore P_b \le 0.5 \quad \forall \gamma_2 \ge 0$$
$$\therefore \quad (1 - 2P_b) \ge 0$$

Since Q(x) is a decreasing function of x,  $P_d$  is monotonous increasing function of  $\gamma_1$ , and a monotonous decreasing function of  $d_1$ . For a specific  $d_2$ , it is obvious that we have the minimum  $d_1$  when CPE lies in the straight line connecting PU and BS.

Thus, the searching region for ODL can be limited in the straight line mentioned above. From the secondary users' perspective, a guaranteed low  $P_f$  is desired so that they will have a high capacity. Thus we fix the  $P_f$  of the network at their desired value and then maximize  $P_d$  as much as possible.

Under this constant false alarm rate (CFAR) requirement, the optimization problem for locating ODL is expressed as

$$\max P_{d} = Q \left[ \left( \frac{\varepsilon}{\sigma^{2}} - \gamma_{1} - 1 \right) \cdot \sqrt{N} \right] + Q(\sqrt{2\gamma_{2}})$$

$$- 2Q \left[ \left( \frac{\varepsilon}{\sigma^{2}} - \gamma_{1} - 1 \right) \cdot \sqrt{N} \right] \cdot Q(\sqrt{2\gamma_{2}})$$
(13)

*s.t* .

$$P_{f} = Q \left[ \left( \frac{\mathcal{E}}{\sigma^{2}} - 1 \right) \cdot \sqrt{N} \right] + Q(\sqrt{2\gamma_{2}})$$

$$- 2Q \left[ \left( \frac{\mathcal{E}}{\sigma^{2}} - 1 \right) \cdot \sqrt{N} \right] \cdot Q(\sqrt{2\gamma_{2}}) = \overline{P}_{f}$$
(14)

$$d_1 + d_2 = D$$
,  $0 \le d_2 \le R$  (15)

$$\gamma_1 = \frac{P_u \beta}{d_1^{\alpha}},\tag{16}$$

$$\gamma_2 = \frac{P_s \beta}{d_2^{\alpha}} \,. \tag{17}$$

#### 3.2 An Optimum Algorithm for Solving ODL

The classical method for the above constrained optimization problem is the standard Lagrange multiplier algorithm. We have the Lagrange function for the problem as

$$\Omega = P_d - \rho_1 \cdot (P_f - \bar{P}_f) - \rho_2 (d_1 + d_2 - D)$$
(18)

where  $\rho_1$  and  $\rho_2$  are multipliers. Then, we differentiate  $\Omega$  with respect to  $\gamma_1$ ,  $\gamma_2$  and  $\varepsilon$ , and can obtain equations (19)-(21)

$$\frac{\partial\Omega}{\partial\gamma_1} = \sqrt{\frac{N}{2\pi}} \cdot EXP(-\frac{N(\frac{\varepsilon}{\sigma^2} - \gamma_1 - 1)^2}{2}) \cdot \left[1 - 2Q(\sqrt{2\gamma_2})\right] -\rho_2 \cdot \frac{(P_u\beta)^{1/\alpha}}{\alpha} \cdot \gamma_1^{-(\frac{1}{\alpha} + 1)}$$
(19)

$$\frac{\partial \Omega}{\partial \gamma_2} = \sqrt{\frac{1}{\gamma_2 \pi}} \cdot EXP(-\gamma_2) \cdot \left\{ \mathcal{Q}[(\frac{\varepsilon}{\sigma^2} - \gamma_1 - 1) \cdot \sqrt{N}] -\rho_1 \cdot \mathcal{Q}\left[(\frac{\varepsilon}{\sigma^2} - 1) \cdot \sqrt{N}\right] + \frac{\rho_1 - 1}{2} \right\} - \rho_2 \cdot \frac{(P_s \beta)^{1/\alpha}}{\alpha} \cdot \gamma_2^{-(\frac{1}{\alpha} + 1)}$$
(20)

$$\frac{\partial\Omega}{\partial\varepsilon} = \frac{1}{\sigma^2} \sqrt{\frac{N}{2\pi}} \cdot \left[ 1 - 2Q(\sqrt{2\gamma_2}) \right] \cdot \left[ EXP(-\frac{N(\frac{\varepsilon}{\sigma^2} - \gamma_1 - 1)^2}{2}) - \rho_1 \cdot EXP(-\frac{N(\frac{\varepsilon}{\sigma^2} - 1)^2}{2}) \right]$$
(21)

Forcing  $\partial \Omega / \partial \gamma_1$ ,  $\partial \Omega / \partial \gamma_2$ ,  $\partial \Omega / \partial \varepsilon$  to be zero, with constraints (14)-(17), we can convert the optimization problem to

$$\frac{\partial\Omega}{\partial\gamma_1} = 0, \quad \frac{\partial\Omega}{\partial\gamma_2} = 0, \quad \frac{\partial\Omega}{\partial\varepsilon} = 0,$$
 (22)

s.t. 
$$P_f = \overline{P}_f$$
, (23)

$$d_1 + d_2 = D$$
,  $0 \le d_2 \le R$ , (24)

$$\gamma_1 = \frac{P_u \beta}{d_1^{\alpha}} , \qquad (25)$$

$$\gamma_2 = \frac{P_s \beta}{d_2^{\ \alpha}}.$$
 (26)

First, we will start with the range of  $\gamma_2$ , which is greatly associated with the objective of our optimization  $d_2$ . From  $P_f = \overline{P}_f$ , we have

$$\sqrt{N}\left(\frac{\varepsilon}{\sigma^2} - 1\right) = Q^{-1}\left(\frac{\overline{P}_f - Q(\sqrt{2\gamma_2})}{1 - 2Q(\sqrt{2\gamma_2})}\right) .$$
(27)

Since the range of x in  $Q^{-1}(x)$  is (0, 1), then

$$\frac{\overline{P}_{f} - \mathcal{Q}(\sqrt{2\gamma_{2}})}{1 - 2\mathcal{Q}(\sqrt{2\gamma_{2}})} \in (0,1) \implies \gamma_{2} \in \left(\frac{\left(\mathcal{Q}^{-1}(P_{f})\right)^{2}}{2}, +\infty\right).$$
(28)

Equation (28) implies that the constraint  $P_f = \overline{P}_f$  requires  $\gamma_2$  to be large enough, i.e. CPE should not locate far apart from BS. On the other hand, the topology limit of WRAN, i.e.  $0 \le d_2 \le R$ , is expressed by

$$\gamma_2 \in \left(\frac{P_s \beta}{R^{\alpha}}, +\infty\right). \tag{29}$$

Taking these two aspects in (28)-(29) into account, the range of  $\gamma_2$  can be denoted as  $\gamma_2 \in \Upsilon$ , where

$$\Upsilon = \begin{cases} \left(\frac{\left(Q^{-1}(P_f)\right)^2}{2}, +\infty\right), & \text{if } \frac{\left(Q^{-1}(P_f)\right)^2}{2} \ge \frac{P_s \beta}{R^{\alpha}} \\ \left(\frac{P_s \beta}{R^{\alpha}}, +\infty\right) & \text{otherwise} \end{cases}$$
(30)

For notation brevity, we convert variables  $\varepsilon$ ,  $\gamma_1$ ,  $\rho_1$ ,  $\rho_2$ into the function of  $F_1(\gamma_2)$ ,  $F_2(\gamma_2)$ ,  $F_3(\gamma_2)$ , and construct a function from (19) and (20) as

$$G(\gamma_{2}) = \gamma_{2}^{-(\frac{1}{\alpha}+1)} \cdot P_{s}^{1/\alpha} \cdot \frac{\partial \Omega}{\partial \gamma_{2}} - \gamma_{1}^{-(\frac{1}{\alpha}+1)} \cdot P_{u}^{1/\alpha} \cdot \frac{\partial \Omega}{\partial \gamma_{1}}$$

$$= \gamma_{2}^{\frac{1}{\alpha}+1} \cdot P_{s}^{1/\alpha} \sqrt{\frac{1}{\gamma_{2}\pi}} \cdot EXP(-\gamma_{2}) \cdot \left\{ \mathcal{Q}[F_{1}(\gamma_{2}) - \sqrt{N}F_{2}(\gamma_{2})] \right\}$$

$$-\frac{1}{2} + F_{3}(\gamma_{2}) \cdot \left\{ \frac{1}{2} - \mathcal{Q}[F_{1}(\gamma_{2})] \right\} - P_{u}^{1/\alpha} \cdot [F_{2}(\gamma_{2})]^{\frac{1}{\alpha}+1} \sqrt{\frac{N}{2\pi}}$$

$$\cdot EXP(-\frac{\left(F_{1}(\gamma_{2}) - \sqrt{N} \cdot F_{2}(\gamma_{2})\right)^{2}}{2}) \cdot \left[1 - 2\mathcal{Q}(\sqrt{2\gamma_{2}})\right] \quad (31)$$
where

 $\rho_{l}$ 

$$\begin{split} \sqrt{N} \left(\frac{\varepsilon}{\sigma^2} - 1\right) &= F_1(\gamma_2) = Q^{-1} \left(\frac{P_f - Q(\sqrt{2\gamma_2})}{1 - 2Q(\sqrt{2\gamma_2})}\right), \\ \gamma_1 &= F_2(\gamma_2) = \left(\frac{D}{(P_u\beta)^{1/\alpha}} - \left(\frac{P_s}{P_u \cdot \gamma_2}\right)^{1/\alpha}\right)^{-\alpha}, \\ &= F_3(\gamma_2) = EXP(-\frac{N(\frac{\varepsilon}{\sigma^2} - \gamma_1 - 1)^2}{2}) / EXP(-\frac{N(\frac{\varepsilon}{\sigma^2} - 1)^2}{2}) \\ &= EXP\left[\frac{\sqrt{N} \cdot F_2(\gamma_2)}{2} \cdot (2F_1(\gamma_2) - \sqrt{N} \cdot F_2(\gamma_2))\right], \end{split}$$

$$\rho_{2} = F_{4}(\gamma_{2}) = \gamma_{2}^{\frac{1}{\alpha}+1} \sqrt{\frac{1}{\gamma_{2}\pi}} \cdot EXP(-\gamma_{2})$$
$$\cdot \left\{ \mathcal{Q}[F_{1}(\gamma_{2}) - \sqrt{N}F_{2}(\gamma_{2})] - \frac{1}{2} + F_{3}(\gamma_{2}) \cdot \left\{ \frac{1}{2} - \mathcal{Q}[F_{1}(\gamma_{2})] \right\} \right\}$$

The discussion of solving  $\gamma_2$  from (31) can be divided into three circumstances as follows:

i. If (19)-(20) are satisfied in the range of  $\gamma_2$ , that is

$$G(\gamma_2) = \gamma_2^{-(\frac{1}{\alpha}+1)} \cdot P_s^{1/\alpha} \cdot \frac{\partial \Omega}{\partial \gamma_2} - \gamma_1^{-(\frac{1}{\alpha}+1)} \cdot P_u^{1/\alpha} \cdot \frac{\partial \Omega}{\partial \gamma_1} = 0.$$
 (32)

We will have  $(\gamma_2^{1}, \gamma_2^{2}, ..., \gamma_2^{n}) = G^{-1}(0)$ , where  $\gamma_2^{i}$ , (i = 1, 2, ..., n) is the solution to  $G(\gamma_2) = 0$ . Because the extremum of  $\gamma_2$  derived from Lagrange multiplier algorithm must contain the one leading to the maximum of  $P_d$ , thus it is feasible to find out  $\hat{\gamma}_2$  corresponding to  $P_{d \max}^1$  by substituting  $\gamma_2^{i}$ , (i = 1, 2, ..., n) into (12) and searching for the one that gets the maximum value of  $P_d$ . The comparison process can be presented by

$$\hat{\gamma}_{2} = \arg\max_{i=1,2,\dots,n} \left\{ \mathcal{Q} \Big[ F_{1} \Big( \gamma_{2}^{i} \Big) - \sqrt{N} F_{2} \Big( \gamma_{2}^{i} \Big) \Big] + \mathcal{Q} \Big( \sqrt{2\gamma_{2}^{i}} \Big) -2 \cdot \mathcal{Q} \Big( \sqrt{2\gamma_{2}^{i}} \Big) \cdot \mathcal{Q} \Big[ F_{1} \Big( \gamma_{2}^{i} \Big) - \sqrt{N} F_{2} \Big( \gamma_{2}^{i} \Big) \Big] \right\}$$
(33)

and

$$P_{d \max}^{1} = Q \Big[ F_{1} \left( \hat{\gamma}_{2} \right) - \sqrt{N} F_{2} \left( \hat{\gamma}_{2} \right) \Big] + Q \Big( \sqrt{2 \hat{\gamma}_{2}} \Big) \\ -2 \cdot Q \Big( \sqrt{2 \hat{\gamma}_{2}} \Big) \cdot Q \Big[ F_{1} \left( \hat{\gamma}_{2} \right) - \sqrt{N} F_{2} \left( \hat{\gamma}_{2} \right) \Big]$$
(34)

The  $\hat{d}_2$  corresponding to ODL is

$$\hat{d}_2 = \left(\frac{P_s \beta}{\hat{\gamma}_2}\right)^{1/\alpha}.$$
(35)

$$\mathbf{ii.} \quad \text{If } G(\gamma_2) = \gamma_2^{-(\frac{1}{\alpha}+1)} \cdot P_s^{1/\alpha} \cdot \frac{\partial \Omega}{\partial \gamma_2} - \gamma_1^{-(\frac{1}{\alpha}+1)} \cdot P_u^{1/\alpha} \cdot \frac{\partial \Omega}{\partial \gamma_1} > 0 , \forall \gamma_2 \in \Upsilon,$$

it is obvious that  $\partial \Omega / \partial \gamma_2 > 0$ ,  $\partial \Omega / \partial \gamma_1 < 0$ , implying that the targeted function  $\Omega$  is a monotonous increasing function of  $\gamma_2$ . The  $P_d$  is also a monotonous increasing function of  $\gamma_2$  in the fact that  $\Omega = P_d$ . Hence  $P_d$  will approach its maximum value when  $\gamma_2 \rightarrow +\infty$ ,

$$P_{d \max}^{2} \to Q(Q^{-1}(P_{f}) - P_{u}\beta D^{-\alpha}).$$
(36)

In this case,  $\hat{d}_2$  corresponding to ODL is denoted by

$$\hat{d}_2 \to 0. \tag{37}$$

**iii**. If 
$$G(\gamma_2) = \gamma_2^{-(\frac{1}{\alpha}+1)} \cdot P_s^{1/\alpha} \cdot \frac{\partial \Omega}{\partial \gamma_2} - \gamma_1^{-(\frac{1}{\alpha}+1)} \cdot P_u^{1/\alpha} \cdot \frac{\partial \Omega}{\partial \gamma_1} < 0, \forall \gamma_2 \in \Upsilon$$
,

we can easily obtain that  $P_d$  is a monotonous decreasing function of  $\gamma_2$ .

$$P_{d \max}^{3} \rightarrow \begin{cases} \mathcal{Q}\left[\frac{\left(\mathcal{Q}^{-1}(P_{f})\right)^{2}}{2}\right], & \text{if } \frac{\left(\mathcal{Q}^{-1}(P_{f})\right)^{2}}{2} \geq \frac{P_{s}\beta}{R^{\alpha}} \\ \Theta, & \text{otherwise} \end{cases}$$
(38)

where

$$\Theta = \mathcal{Q} \left[ \mathcal{Q}^{-1} \left( \frac{P_f - \mathcal{Q}(\sqrt{2P_s\beta/R^{\alpha}})}{1 - 2\mathcal{Q}(\sqrt{2P_s\beta/R^{\alpha}})} \right) - \sqrt{N} \cdot \left( \frac{D}{(P_u\beta)^{1/\alpha}} - \left( \frac{R^{\alpha}}{P_u \cdot \beta} \right)^{1/\alpha} \right)^{-\alpha} \right] + \mathcal{Q}(\sqrt{2\frac{P_s\beta}{R^{\alpha}}}) - 2 \cdot \mathcal{Q} \left[ \mathcal{Q}^{-1} \left( \frac{P_f - \mathcal{Q}(\sqrt{2P_s\beta/R^{\alpha}})}{1 - 2\mathcal{Q}(\sqrt{2P_s\beta/R^{\alpha}})} \right) - \sqrt{N} \cdot \left( \frac{D}{(P_u\beta)^{1/\alpha}} - \left( \frac{R^{\alpha}}{P_u \cdot \beta} \right)^{1/\alpha} \right)^{-\alpha} \right] \cdot \mathcal{Q}(\sqrt{2\frac{P_s\beta}{R^{\alpha}}})$$

In this case,  $\hat{d}_2$  corresponding to ODL is denoted as

$$\hat{d}_{2} \rightarrow \begin{cases} \left(\frac{2P_{s}\beta}{\left(Q^{-1}(P_{f})\right)^{2}}\right)^{1/\alpha}, & \text{if } \frac{\left(Q^{-1}(P_{f})\right)^{2}}{2} \ge \frac{P_{s}\beta}{R^{\alpha}} \\ R, & \text{otherwise} \end{cases}$$
(39)

It is worth noting that the ODL obtained in Case ii and Case iii is infinitely close to BS and the edge of WRAN district respectively. In practice, we can select somewhere near the BS or the edge of WRAN district as ODL, as long as the error produced by such approximation can be tolerated.

# 3.3 A Full Searching Algorithm for Locating ODL

Although the standard Lagrange multiplier algorithm can obtain the optimum solution, it is of high computational complexity and is difficult to be implemented. Herein, full searching algorithm which has relatively low complexity is a promising candidate for locating ODL. For simplicity, we neglect the edge of  $d_2$ 's range, and constrain the searching space of  $d_2$  to  $(0, d_2$ \_Range), where

$$d_2$$
\_Range=min $\left(R, \left(\frac{2P_s\beta}{\left(Q^{-1}(P_f)\right)^2}\right)^{1/\alpha}\right)$ .

The scouting interval is set to  $\Delta d$ , and the error of searching result won't exceed  $\pm \Delta d$ . If  $\Delta d$  is small enough, the ODL acquired from this algorithm infinitely approaches its theoretical value. The flow chart of the full searching algorithm is shown in Fig. 2.



Fig. 2. Flow chart of full searching algorithm.

## 4. ODL-Based Cooperative Spectrum Sensing

#### 4.1 ODL-Based Cooperative Spectrum Sensing

Aiming to alleviate the negative impact brought by reporting errors and improve sensing performance, we propose an ODL-based cooperative spectrum sensing.

We defined Optimum Detection Region (ODR) as a disc with ODL as its center and r as its radius, where  $r \ll R$ , as shown in Fig. 1. Notate the member set of CPEs located in ODR as U, and the upper limit number of users for cooperative spectrum sensing as L.

The ODL-based cooperative spectrum sensing is conducted through the following steps:

- Step 1: BS calculates ODL and ascertains L according to information such as channel states information. Then, BS broadcasts ODR-related information to all CPEs;
- Step 2: After every  $CPE_i$  (*i*=1, 2, ..., *M*) receiving this message, it subsequently makes a judgment of whether itself is in ODR via positioning devices. If  $CPE_i \in U(i=1, 2, ..., M)$ , it will send back  $ACK_i$ , or  $NACK_i$  otherwise;
- Step 3: If  $|\mathbf{U}|=0$ , no spectrum sensing is conducted, BS randomly selects  $H_0$  or  $H_1$  as the final sensing result;
- Step 4: If  $0 < |\mathbf{U}| \le L$ , BS organizes all the CPEs in ODR to perform cooperative spectrum sensing using some kind of fusion rule;

Step 5: If  $|\mathbf{U}| > L$ , BS randomly selects *L* CPEs from U, to perform cooperative spectrum sensing using some kind of fusion rule.

By means of the above information exchanges, BS is able to pick out a specific number of CPEs situated in the vicinity of ODL. Evidently, these CPEs have better sensing performance compared with others in the network, so the veracity of our proposed sensing scheme can be greatly improved. The setting of upper limit number of users for cooperative spectrum sensing L should strike the balance of channel status, communication load caused to WRAN, and QoS requirement, etc.

## 4.2 Theoretical Performance Analysis of ODL-OR

Different decision rules can be applied to our proposed cooperative spectrum sensing scheme. Here, we adopt OR decision fusion rule to further evaluate the system performance of ODL-based cooperative spectrum sensing. We call this scheme ODL-OR.

In OR fusion rule, when at least 1 out of K secondary users detect the primary users, the final decision declares a primary user is present. Assume that the threshold  $\varepsilon$  in energy detection is identical for all CPEs, then the system false alarm probability  $P_F^{ODR-OR}$  and detection probability  $P_D^{ODR-OR}$  of the final decision are therefore, respectively

$$P_{F}^{ODR-OR} = 1 - \left(1 - P_{f}^{ODR-OR}\right)^{K},$$
(40)

$$P_D^{ODR-OR} = 1 - \left(1 - P_a^{ODR-OR}\right)^K \tag{41}$$

where  $P_{f,i}^{ODR-OR}$  is the probability of false alarm of CPE<sub>*i*</sub> (*i*=1, 2, ..., *K*) who is situated in ODR, and  $P_{d,i}^{ODR-OR}$  is the corresponding probability of detection. The second assumption mentioned above can lead to  $P_{f,j}^{ODR-OR} = P_{f,j}^{ODR-OR} = P_{d,j}^{ODR-OR} = P_{d,j}^{ODR-OR} = P_{d,j}^{ODR-OR} = P_{d,j}^{ODR-OR}$ (*i*, *j* = 1, 2, ..., *K*, *i* ≠ *j*).

Evidently, CPEs in ODR have the optimum sensing performance compared with other CPEs, that is

$$P_{a}^{ODR-OR} \ge P_{a,i}^{OR}, \quad i = 1, 2, ..., K$$

$$r_{a}^{ODR-OR} = P_{F}^{OR} = \overline{P}_{F}$$

$$(42)$$

where  $P_{d,i}^{OR}$  (*i* = 1, 2,..., *K*) stands for the probability of detection of arbitrary *K* CPEs randomly chosen from the network. Hereby, one can easily prove that

$$1 - \left(1 - P_{d}^{ODR - OR}\right)^{K} \ge 1 - \prod_{i=1}^{K} \left(1 - P_{d,i}^{OR}\right),$$
(43)

$$P_D^{ODR-OR} \ge P_D^{OR} . \tag{44}$$

Equation (44) implies that ODL-OR is capable to provide better protection to the primary user compared with other OR-rule sensing schemes.

Theorem 1: In WRAN scenario, assuming that M CPEs are uniformly distributed, the total number of CPEs

in ODR will vary with the time. On the assumption that the area of WRAN and ODR is *A* and *a* respectively, the total number of CPEs in ODR at a given time conforms to a Poisson process with parameter  $\lambda a$ , where  $\lambda = M/A$  stands for the average number of CPEs per unit area.

As we can see from Theorem 1, the number of cooperative users in ODR, denoted as *K*, conforms to a Poisson process with parameter  $\lambda a$ . So the system detection probability bears statistical significance, and it can be characterized as the average probability of detection. Therefore, with Poisson distribution expression, the average probability of detection for WRAN is formulated as

$$P_{d}^{Ave-OR} = 0.5 \cdot \Phi(k=0) + \sum_{K=1}^{L} \Phi(k=K) \cdot \left[ 1 - \left( 1 - P_{d}^{ODR-OR} \right)^{K} \right] \\ + \sum_{K=L}^{\infty} \Phi(k=K) \cdot \left[ 1 - \left( 1 - P_{d}^{ODR-OR} \right)^{L} \right] \\ = 0.5 \cdot e^{-\lambda a} + \sum_{K=1}^{L} \frac{e^{-\lambda a} (\lambda a)^{K}}{K!} \cdot \left[ 1 - \left( 1 - P_{d}^{ODR-OR} \right)^{K} \right] \\ + \sum_{K=L}^{\infty} \frac{e^{-\lambda a} (\lambda a)^{K}}{K!} \cdot \left[ 1 - \left( 1 - P_{d}^{ODR-OR} \right)^{L} \right]$$
(45)

Equation (45) reveals that the sensing performance of ODL-OR is influenced by two variables: L and  $\lambda a$ . When upper limit number of users for cooperative spectrum sensing L and the density of users in WRAN is getting larger, our proposed scheme will have better performance.

Under the CFAR, owing to ensure the utilization of idle channels, the average false alarm probability of ODL-OR is defined as the constant target value  $\bar{P}_{f}^{Ave}$ . It is assumed that the mapping relationship between  $P_{d}^{ODR-OR}$  and  $P_{f}^{ODR-OR}$  is defined as  $P_{d}^{ODR-OR} = \Psi(P_{f}^{ODR-OR})$ .

From (11)-(12), the mapping function  $\Psi$  is expressed as

$$P_{d}^{ODR-OR} = \Psi(P_{f}^{ODR-OR})$$

$$= Q \left[ Q^{-1} \left( \frac{P_{f}^{ODR-OR} - Q(\sqrt{2\gamma_{2}})}{1 - 2Q(\sqrt{2\gamma_{2}})} \right) - \sqrt{N} \cdot \gamma_{1} \right]$$

$$= Q \left[ Q^{-1} \left( \frac{1 - (1 - \overline{P}_{f}^{Ave})^{1/L} - Q(\sqrt{2\gamma_{2}})}{1 - 2Q(\sqrt{2\gamma_{2}})} \right) - \sqrt{N} \cdot \gamma_{1} \right]. \quad (46)$$

Under the constraint of  $\overline{P}_{f}^{Ave}$ , and according to equations (40)-(41), when the number of cooperative user is *K*, the detection probability of final decision is derived as

$$P_{D}^{ODR-OR} = 1 - \left(1 - \Psi(P_{f}^{ODR-OR})\right)^{K}$$

$$= 1 - \left[1 - \Psi\left(1 - (1 - \overline{P}_{f}^{Ave})^{1/K}\right)\right]^{K}$$

$$= 1 - \left[1 - \Psi\left(1 - (1 - \overline{P}_{f}^{Ave})^{1/K}\right)^{1/K}\right]^{K}$$
(47)

Theorem 2: Let  $\Lambda(K) = 1 - \left[1 - \Psi \left(1 - (1 - \overline{P}_f^{Ave})^{1/K}\right)\right]^n$ ,  $\Lambda(K)$  is a monotonously increasing function according to K, and the upper bound of  $\Lambda(K)$  is 1.

According to (45), when the average number of user in ODR is satisfied as  $\lambda a \rightarrow \infty$ , the average probability of decision is existed that

$$\lim_{\lambda a \to \infty} \sum_{K=0}^{L-1} \frac{e^{-\lambda a} (\lambda a)^K}{K!} = 0, \qquad (48)$$

then 
$$\lim_{\lambda a \to \infty} \sum_{K=L}^{\infty} \frac{e^{-\lambda a} (\lambda a)^K}{K!} = \lim_{\lambda a \to \infty} \sum_{K=0}^{\infty} \frac{e^{-\lambda a} (\lambda a)^K}{K!} = 1.$$
(49)

When the number of average user in ODR is satisfied as  $\lambda a \rightarrow \infty$ , the average probability of decision is derived as

$$\lim_{\lambda a \to \infty} P_d^{Ave} = 1 - \left(1 - P_d^{ODR - OR}\right)^L.$$
(50)

If  $\lambda a \rightarrow \infty$ , the number of cooperative user tends to the upper bound value *L*. Owing to the Theorem 2, we can obtain that the average probability of decision is a monotonous increasing function according to the number of average users in ODR and the number of cooperative users. So if  $\lambda a \rightarrow \infty$  and  $L \rightarrow \infty$ , the average probability of decision tends to the performance upper bound, and is expressed as

$$\lim_{L \to \infty} \lim_{\lambda_{d \to \infty}} P_d^{Ave} = \lim_{L \to \infty} 1 - \left(1 - P_d^{ODR - OR}\right)^L$$
$$= \lim_{L \to \infty} 1 - \left[1 - \Psi\left(1 - (1 - \overline{P}_f^{Ave})^{1/L}\right)\right]^L = 1$$
(51)

## 5. Simulation Results

We study some simulation results to demonstrate the performance of the ODL-based cooperative spectrum sensing. The radius of WRAN and ODR is set to 33 km and 0.33 km respectively, while BS is 120 km away from the primary user. During the sensing time, the number of received signal samples at each secondary user is set as 6000 samples. The path loss exponent factor  $\alpha$ , in (1) and (2), is set to be 3. The  $\beta$  and  $P_u$  are set at a value such that the primary user's signal to noise ratio at the secondary BS is -16 dB. For notation brevity, we refer to  $P_d$  as the average probability of detection for WRAN.



Fig. 3. Tendency of ODL under different CPE transmission power.

In Fig. 3, we show the tendency of  $P_d$  versus the distance between BS and a CPE  $d_2$  under different CPE transmission power. Under the CFAR requirement, the desired  $P_f$  is set as 0.1%. We observe that there does exist an optimum detection location in the line connecting BS and PU. When transmission power  $P_s$  increases, ODL moves forward to the edge of WRAN, and the sensing performance improves gradually. This phenomenon can demonstrate that it's feasible to ameliorate the sensing performance of WRAN system by enlarging transmission power of CPEs as long as it doesn't exceed the interference threshold tolerated by the primary user.

Fig. 4 shows the  $P_d$  of WRAN system with the total number of CPEs in the network increasing when using ODL-OR scheme under the constraint of  $P_f=0.1\%$ . It illustrates that the  $P_d$  increases as M and L grows, which tallies with our analysis in Section 4.



Fig. 4. The  $P_d$  of ODL-OR versus the total number of CPEs M in the network under different upper limit number of users for cooperative spectrum sensing L.

Fig. 5 compares the sensing performance of ODL-OR with conventional sensing scheme using OR-fusion rule. We suppose that the conventional sensing scheme randomly chooses 20 users for collaboration during one startup of cooperative spectrum sensing. For fairness, we set the upper limit number of users for cooperative spectrum sensing L in ODL-OR as 20. The  $P_f$  is also fixed on 0.1%, and the total number of CPEs M is assumed to be  $2 \times 10^5$ . In Fig. 5, the  $P_d$  of both schemes increases as  $P_s$ growing. However, conventional OR fusion scheme is inferior to ODL-OR scheme throughout the simulation range of  $P_s$ . When  $P_s = 0$  dB, the ODL achieves the  $P_d$  of 80%, while that of conventional OR fusion scheme is only slightly above 10%. In other words, the probability of detection can be enhanced by as much as 70% by our proposed scheme. Notably, when  $P_S$  is larger than 5 dB, ODL-OR scheme even outperforms conventional OR fusion scheme with no reporting errors. This phenomenon can be attributed to the exploitation of geographic advantages in user selection.



Fig. 5. The comparisons of  $P_d$  as the transmission power  $P_s$  increases under CFAR requirement.

Fig. 6 shows the complementary ROC curves, i.e. the probability of loss detection  $P_m$  versus the probability of false alarm  $P_f$  under CFAR requirement. The  $P_f$  is set to 0.1%, the transmission power is 10 dB, *M* is assumed to be  $2 \times 10^5$ . Fig. 6 demonstrates that the  $P_m$  of ODL-OR is much smaller than that of conventional OR fusion scheme, which again verifies the superiority of our proposed scheme over conventional sensing.



Fig. 6. The Complementary ROC curves (the probability of loss detection  $P_m$  versus the probability of false alarm  $P_{f}$ ) under CFAR requirement.

#### 6. Conclusion

In this paper, we proposed an optimum detection location-based cooperative spectrum sensing scheme to improve the sensing performance in the case of imperfect reporting channels. To exploit the geographic advantages to overcome the negative impact caused by reporting errors, we modeled the cooperative spectrum sensing process using energy detection and found out a specific location in WRAN where the CPE will have the optimum sensing performance using two algorithms. Numerical and simulation results both show that the sensing performance of our proposed scheme is improved considerably compared with conventional spectrum sensing.

#### Appendix

#### Proof of Theorem 1:

The distribution of *M* CPEs is uniform, so the probability  $P_n$  of every CPE<sub>*i*</sub> (*i* = 1, 2, ..., *M*) that falls into ODR is equal, and can be expressed as the following

$$P_n = \frac{a}{A} \tag{52}$$

where  $A = \pi R^2$  is the area of WRAN. Thus the probability of *K* CPES that fall into ODR is

$$\Phi(K) = C_M^K \cdot P_n^K \cdot (1 - P_n^K)^{M-K}.$$
(53)

According to Poisson Theorem, we obtain

$$\Phi(K) \approx e^{-\lambda a} (\lambda a)^K / K!.$$
(54)

conforms to a Poisson process with parameter  $\lambda a$ , where  $\lambda = M/A$  stands for the average number of CPEs per unit area.

#### Proof of Theorem 2:

Firstly, we proof that  $\Lambda(K)$  is a monotonously increasing function according to *K*, we can get the derivative of the function  $\Lambda(K)$ 

$$\frac{d\Lambda(K)}{dK} = \Lambda(K) \cdot \ln\left[1 - \Psi\left(1 - (1 - \overline{P}_f^{Ave})^{1/K}\right)\right] \\ \cdot \frac{d\Psi\left(1 - (1 - \overline{P}_f^{Ave})^{1/K}\right)}{dK}$$
(55)

where

$$\frac{d\Psi\left(1 - (1 - \overline{P_f}^{Ave})^{1/K}\right)}{dK} = -\frac{\exp\left[-\left(Q^{-1}\left(\frac{1 - (1 - \overline{P_f}^{Ave})^{1/K} - Q(\sqrt{2\gamma_2})}{1 - 2Q(\sqrt{2\gamma_2})}\right) - \sqrt{N} \cdot \gamma\right)^2/2\right]}{\sqrt{2\pi}} \cdot \frac{dQ^{-1}\left(\frac{1 - (1 - \overline{P_f}^{Ave})^{1/K} - Q(\sqrt{2\gamma_2})}{1 - 2Q(\sqrt{2\gamma_2})}\right)}{\sqrt{2\pi}}$$

and

$$\frac{dQ^{-1}(\frac{1-(1-\overline{P}_{f}^{Ave})^{1/K}-Q(\sqrt{2\gamma_{2}})}{1-2Q(\sqrt{2\gamma_{2}})})}{dx} = \frac{\sqrt{2\pi}(1-\overline{P}_{f}^{Ave})^{1/K-1}}{L(1-2Q(\sqrt{2\gamma_{2}}))}$$
$$/\exp\left[-\left(Q^{-1}(\frac{1-(1-\overline{P}_{f}^{Ave})^{1/K}-Q(\sqrt{2\gamma_{2}})}{1-2Q(\sqrt{2\gamma_{2}})})\right)^{2}/2\right] > 0$$

According to the range of Q function, we can obtain

:: 
$$0 < 1 - \Psi \left( 1 - (1 - \overline{P}_f^{Ave})^{1/K} \right) < 1$$
, (56)

$$\therefore \ln \left[ 1 - \Psi \left( 1 - (1 - \overline{P}_f^{Ave})^{1/K} \right) \right] < 0.$$
(57)

According to (55)-(57), we can proof that  $d\Lambda(K)/dK > 0$ ,  $\Lambda(K)$  is a monotonous increasing function according to *K*, in the following we will proof that the upper bound of  $\Lambda(K)$  is 1.

For  $\Lambda(K)$  is a monotonously increasing function according to *K*, the proof of the upper bound of  $\Lambda(K)$  is 1 equals to proof as following

$$\lim_{K \to \infty} \left[ 1 - \Psi \left( 1 - (1 - \overline{P}_f^{Ave})^{1/K} \right) \right]^K = 0 .$$
 (58)

Let 
$$\Phi(K) = K \cdot \ln \left[ 1 - \Psi \left( 1 - (1 - \overline{P}_f^{Ave})^{1/K} \right) \right], \text{ then}$$

$$\lim_{K \to \infty} \Phi(K) = \lim_{K \to \infty} \frac{\ln \left[ 1 - \Psi \left( 1 - (1 - \overline{P}_f^{Ave})^{1/K} \right) \right]}{\frac{1}{K}}.$$
 (59)

The limit of  $\Psi(1-(1-\overline{P}_f^{Ave})^{1/K})$  is expressed as

$$\lim_{K \to \infty} \Psi\left(1 - (1 - \overline{P}_{f}^{Ave})^{1/K}\right) = \\
\lim_{K \to \infty} Q\left[Q^{-1}\left(\frac{P_{f}^{ODR-OR} - Q(\sqrt{2\gamma_{2}})}{1 - 2Q(\sqrt{2\gamma_{2}})}\right) - \sqrt{N} \cdot \gamma_{1}\right] \\
= \lim_{K \to \infty} Q\left[Q^{-1}\left(\frac{1 - (1 - \overline{P}_{f}^{Ave})^{1/K} - Q(\sqrt{2\gamma_{2}})}{1 - 2Q(\sqrt{2\gamma_{2}})}\right) - \sqrt{N} \cdot \gamma_{1}\right]$$
(60)

where  $0 < \frac{1 - (1 - \overline{P}_f^{Ave})^{1/K} - Q(\sqrt{2\gamma_2})}{1 - 2Q(\sqrt{2\gamma_2})} < 1$ .

Since  $\lim_{K \to \infty} 1 - (1 - \overline{P}_f^{Ave})^{1/K} = 0$ , we can obtain  $Q(\sqrt{2\gamma_2}) \to 0$ .

Let 
$$x = \frac{1 - (1 - \bar{P}_f^{Ave})^{1/K} - Q(\sqrt{2\gamma_2})}{1 - 2Q(\sqrt{2\gamma_2})}$$
, then  $x \to 0$ .

Substituting into (60), we obtain

$$\lim_{K \to \infty} \Psi\left(1 - (1 - \overline{P}_f^{Ave})^{1/K}\right) = \lim_{x \to 0} \mathcal{Q}\left[\mathcal{Q}^{-1}(x) - \sqrt{N} \cdot \gamma_1\right] = 0.$$
(61)

Equation (59) is the type of 0/0, by Los Hospital Rule, we obtain

$$\lim_{K \to \infty} \frac{\ln \left[ 1 - \Psi \left( 1 - (1 - \overline{P}_{f}^{Ave})^{1/K} \right) \right]}{\frac{1}{K}} = \lim_{K \to \infty} \frac{K^{2}}{1 - \Psi \left( 1 - (1 - \overline{P}_{f}^{Ave})^{1/K} \right)} \cdot \frac{d\Psi \left( 1 - (1 - \overline{P}_{f}^{Ave})^{1/K} \right)}{dK}. \quad (62)$$

According to (55), we obtain

$$\lim_{K \to \infty} \frac{d\Psi \left( 1 - (1 - \overline{P}_{f}^{Ave})^{1/K} \right)}{dK} = \lim_{K \to \infty} -\frac{\sqrt{2\pi} \left( 1 - \overline{P}_{f}^{Ave} \right)^{1/K-1}}{K \left( 1 - 2Q(\sqrt{2\gamma_{2}}) \right)}$$
  
$$\cdot \exp \left[ \left( Q^{-1} \left( \frac{1 - (1 - \overline{P}_{f}^{Ave})^{1/K} - Q(\sqrt{2\gamma_{2}})}{1 - 2Q(\sqrt{2\gamma_{2}})} \right) - \sqrt{N} \cdot \gamma_{1} \right) \cdot \frac{\sqrt{N} \cdot \gamma_{1}}{2} \right]. \quad (63)$$

Substituting (63) into (62), we obtain

$$\lim_{K \to \infty} \Phi(K) = \lim_{K \to \infty} \frac{K^2}{1 - \Psi \left(1 - (1 - \overline{P}_f^{Ave})^{1/K}\right)} \cdot \frac{d\Psi \left(1 - (1 - \overline{P}_f^{Ave})^{1/K}\right)}{dK} = -\infty$$
(64)

Thus, (58) is valuable, and  $\Lambda(K)$  is a monotonous increasing function according to *K*, and the upper bound is 1, the proposition holds.

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