Broadband Approximations for Doubly Curved Reflector Antenna

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Abstract. *The broadband approximations for shaped-beam doubly curved reflector antennas with primary feed (rectangular horn) producing uniform amplitude and phase aperture distribution are derived and analyzed. They are very valuable for electromagnetic compatibility analyses both from electromagnetic interference and susceptibility point of view, because specialized more accurate methods such as physical optics are only used by antenna designers. To allow quick EMC analyses, typical values, beamwidth changes, sidelobe levels and aperture efficiencies are given for frequency changes approximately up to four times operating frequency. A comparison of approximated and measured patterns of doubly curved reflector antennas shows that the given approximation could be reliably used for analyses of pattern changes due to very broad frequency changes.*

Keywords

Broadband approximation, radiation patterns, shaped-beam reflector antennas, doubly curved reflector, electromagnetic compatibility.

1. Introduction

An antenna design requires a compromise between extensive calculations and the fabrication and measurement of prototypes. For example, when designing large and expensive antennas, the high fabrication cost justifies the time required for analysis. Silver [1] provides the foundation for an analysis based on aperture theory and physical optics (induced currents on the reflector). That remains the main techniques of design for reflectors [2], which are extremely large in wavelengths, as various numerical methods such as method of moments will converge to the correct solution but they could be very time consuming. The given methods could be reliable for very narrow frequency band but they are only used by antenna designers. If antenna patterns are considered for very broad frequency ranges, it could be very useful to use very simple approximations, which are not time consuming. That is very beneficial for electromagnetic compatibility (EMC) analyses both from electromagnetic interference (EMI) and susceptibility (EMS) point of view.

The characteristics of function $c(u,t)$, which approximates patterns of shaped-beam reflector antennas for broad band (out of operating frequencies) with primary feed (rectangular horn) producing uniform amplitude and phase aperture distribution are derived. A comparison of approximated and measured patterns of doubly curved reflector antennas is given.

2. Shaped-Beam Reflector Antennas

Various radar applications impose beam-shaping requirements upon the antenna. The shaped-beam doubly curved reflector antenna is a classical reflector type producing a narrow beam in one plane and a shaped beam in the other, which was described in 1940's [1]. Radar antenna design should be done using suitable software. The far-field radiation of antenna, shown in Fig. 1, can be calculated using aperture method or physical optics [1] – [8]. Considering radar antenna systems, it is necessary to analyze not only radiation pattern of an antenna but various related problems. In the early 1960's, the set of programs computing the shape and the radiation pattern of the doubly curved reflector has been developed [5]5], [6], [7]. Several Czech radar reflector antennas have been designed and hundreds of systems have been produced [9], [10].

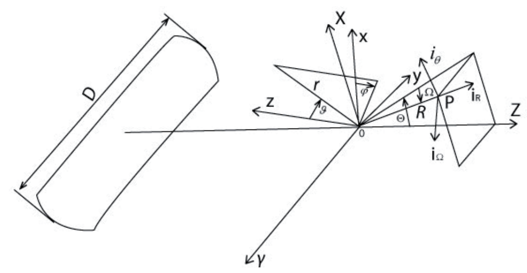

Fig. 1. Doubly curved reflector antenna with observation point *P*. The antenna system is defined by the *XYZ* *(RΘΩ)* system. The primary feed assembly is defined by the *xyz (rϑφ)* system with origin *O*.

The electric field of doubly curved reflector antennas (shown in Fig. 1) for constant Θ cuts, which are perpendicular to symmetry *ZX* plane, is given by [6]

$$E(\Omega) = K \int_{-D/2}^{D/2} f_1(Y) \exp\left(j \frac{2\pi}{\lambda} Y \sin \Omega \right) dY \tag{1}$$

where K is a constant, λ is the wavelength, and D is the reflector Y dimension. A function $f_1(Y)$ is given by both the reflector geometry and the primary feed pattern. This method is similar to [8] but it is more sophisticated. The antenna system is defined by the XYZ system, where the far-field observation point P is defined by distance R and Θ and Ω angles with unit vectors $\boldsymbol{i_R}$, $\boldsymbol{i_\Theta}$ and $\boldsymbol{i_\Omega}$. However, the assembly of primary feed, which is usually in the origin, O, of the xyz system, is defined by the xyz system. Primary feed radiation patterns for φ and ϑ angles could be calculated using various possibilities [6], [7], [11] (such as vertical or horizontal polarizations, pyramidal or conical horns, measured or calculated patterns and one, two or four horns). The phase of incident vector $\boldsymbol{E}_i$ could be changed, and therefore reflector systematic surface errors, primary feed phase errors and relatively small changes of primary feed positions (a few wavelengths), when amplitude changes could be neglected, may be computed. Whereas the integrals were evaluated by employing a 3000 point double Simpson's Rule for the maximum reflector dimension approximately 46 wavelengths [8] and the reflector should be subdivided into many patches because the phases of the currents change rapidly with position on the reflector, and the analysis must be repeated with finer and finer patches until the result converges [2], the used method of integral evaluations was much faster (time decreased by an order of magnitude – usually about 300 points were used for the reflector dimension approximately 170 wavelengths). That was thanks to fact that the special method [7], [12] were used for integration in the symmetry ZX plane and the Gaussian integration [13] for constant Θ cuts. Gaussian integration has been analyzed both analytically and experimentally (hundreds of several antenna types were measured) [6], [7], [9], [14] to [16] with the excellent agreement between the calculated and measured data.

Even if the above method is quite satisfactory and various kinds of general-purpose software are now available, they are only used by antenna designers. Therefore approximations, which are suitable for EMC specialists, are derived below. Since a function $f_1(Y)$ is given by both a reflector geometry and a primary feed pattern it is necessary to find out some simplifications. The effect of reflector geometry is relatively small for shallow reflectors (larger focal length-to-diameter ratio) and then the primary feed pattern is very important. If various details (such as vector character and phases of incident waves) are neglected, then primary feed patterns $F(\vartheta)$ could be used instead of $f_1(Y)$ function. When the E-plane of rectangular horn (waveguide) is perpendicular to symmetry ZX plane, then E-plane feed patterns $F(\vartheta)$ are given by

$$F(\vartheta) = \frac{1+\cos\vartheta}{2}\frac{\sin\alpha}{\alpha} \tag{2}$$

where $\alpha = (\pi a/\lambda)\sin\vartheta$, a is the width of the rectangular aperture, ϑ and φ are spherical coordinates of horn as is shown in Fig. 1.

Equation (1) could be approximately written as

$$E(\Omega) \approx K_1 c(u,t) = \frac{K_1}{2}\int_{-1}^{1}\frac{\sin tx}{tx}\cos uxdx \tag{3}$$

where K_1 is a constant, $t = (\pi a/\lambda)\sin\vartheta_z$, ϑ_z is the half subtended angle of the reflector, and $u = (\pi D/\lambda)\sin\Omega$.

Equation (3) contains two parameters, which are rather complicated functions of the wavelength. Therefore, the analyses of u and t parameter effect on $c(u,t)$ function are performed. Of course, the approximation (3) does not consider phases and neglect various details, and therefore it could not be expected that (1) and (3) have the same accuracy.

The following relation is valid for constant a and ϑ_z

$$t = (\pi a/\lambda)\sin\vartheta_z = K_2 f \tag{4a}$$

where K_2 is a constant and f is the frequency. Similarly, the following relation is applicable for constant D

$$u = (\pi D/\lambda)\sin\Omega = K_3 f\sin\Omega \tag{4b}$$

where K_3 is a constant.

Considering (3), (4a) and (4b) it can be seen that the function $c(u,t)$ could be proposed as pattern approximations for very broad frequency range. Increasing the feed beamwidth improves the illumination but increases the spillover. The efficiency peaks when the feed 10-dB beamwidth is approximately the subtended angle of the reflector. It is clear that the feed has a -10 dB reflector edge taper, when $t \approx 2.3$. Usually, $t \approx 2.3$ for the operating frequency. If pattern changes are analyzed for about four times operating frequency, it is necessary to consider t parameter changes approximately for range $0 \le t \le 10$ according to (4a).

3. Function *c*(*u*,*t*)

Equation (3) implies

$$c(u,t) = \frac{1}{2}\int_{-1}^{1}\frac{\sin tx}{tx}\cos uxdx = \int_{0}^{1}\frac{\sin tx}{tx}\cos uxdx \tag{5}$$

The $c(u,t)$ values could be calculated using a Gaussian integration [13].

The following equation is valid for $t = 0$, when $\sin(tx)/(tx) = 1$

$$c(u,0) = \frac{\sin u}{u}. \tag{6}$$

Of course, a uniform distribution corresponding to the above equation cannot be realized for any reflector.

The following equation is valid for $u = 0$

$$c(0,t) = \int_{0}^{1}\frac{\sin tx}{tx}dx = \frac{\mathrm{Si}(t)}{t} \tag{7}$$

where Si(t) is a sine integral [13].

The graphs of $N = 20 \log|c(u,t)|$ are shown in Fig. 2 for various constant t parameters. They could be used for determination of N sidelobe levels and the half-power beamwidths, u_3, which may be used for Ω_3 beamwidth calculations using (4b). The N sidelobe levels relative to the $c(0,t)$ values (solid line) and $c_{\max}(u,t)$ maximum values (dashed line) for given t are shown in Fig. 3. The u_3 beamwidths relative to the $c(0,t)$ values (solid line) and $c_{\max}(u,t)$ maximum values (dashed line) are shown in Fig. 4.

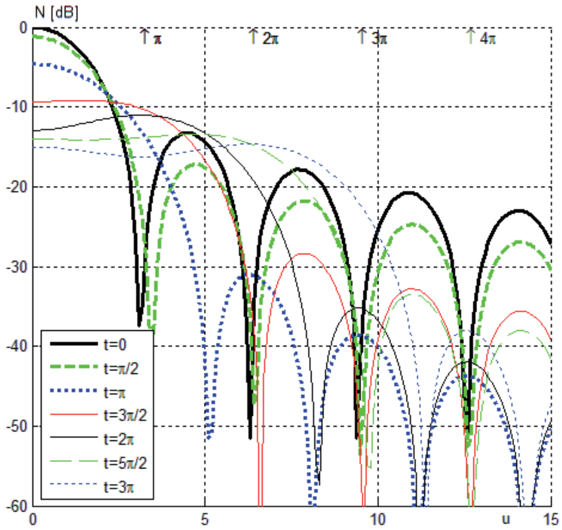

Fig. 2. Function $N = 20 \log|c(u,t)|$ for various constant t.

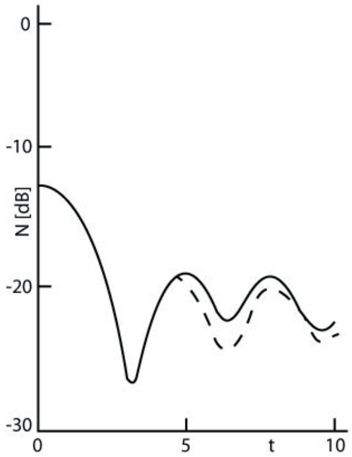

Fig. 3. Sidelobe levels relative to the $c(0,t)$ values (solid line) and $c_{\max}(u,t)$ maximum values (dashed line).

The half-power beamwidths and directivity (gain) of a relatively large planar array or aperture antennas are related by the well-known simple approximate equation. The half-power beamwidths in the symmetry *ZX* plane changes very slowly and therefore the u_3 beamwidths are substantial only [4]. Hence, the aperture efficiency q [1], [2], which is the ratio of the gain of the antenna and the gain of an ideal uniformly illuminated aperture, could be approximately given by

$$q = \left|\int_0^1 \frac{\sin tx}{tx}dx\right|^2 \Big/ \int_0^1 \frac{\sin^2 tx}{(tx)^2}dx \,. \qquad (8)$$

The aperture efficiency q is shown in Fig. 5.

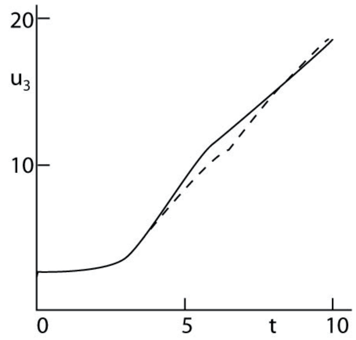

Fig. 4. The u_3 half-power beamwidths relative to the $c(0,t)$ values (solid line) and $c_{\max}(u,t)$ maximum values (dashed line).

4. Comparison of Approximations and Measurements

It is well known [1], [2], [3] that aperture theory does not accurately predict the pattern characteristics of electrically small horns (on the order of a wavelength or less in size). Diffraction effects from the flange or rim around the horn (also considered to be edge currents flowing on the outside surface of the horn) markedly influence the pattern. If the primary feed width is $a \approx \lambda$ then equation (2) gives *E*-plane pattern, which differs from measurements. Empirical data were collected and reduced to simple formulas for small rectangular horns based on aperture size only [1], [2]. Similarly, simple reflector feeds can be approximated with Gaussian beams [2], [7]. Another possibility is a concept of an equivalent a width, which is found by fitting radiation patterns to measured data [5], [6], [7]. Of course, the equivalent a dimensions are changing according to frequency changes and the equivalent a width could be equal to the physical a = 110 mm for higher frequencies.

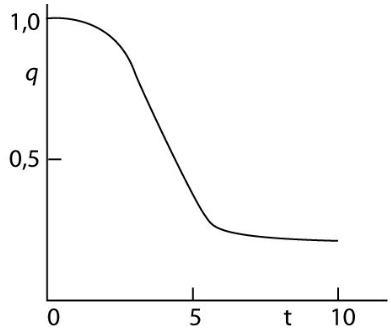

Fig. 5. Aperture efficiency q.

Several methods controlling antenna beamwidths for aperture widths approximately equal to one wavelength were proposed such as [18]. The mouth modifications can improve the radiation patterns and input impedance matching. However, the $c(0,t)$ values should be used with care for these cases. Probably an equivalent a width concept could be useful, especially for antenna designers.

The doubly curved reflector of RL-41 antenna with two beams [4], [9] was measured for comparison (measured frequency range of 2.7 to 10 GHz). Antenna parameters were $D = 5$ m, $a = 110$ mm and $\vartheta_z = 52^o$, i.e. $t = 2.54$ for $f = 2.8$ GHz. The parameter t is changed with frequency according to (4a). The u_3 beamwidths according to Fig. 4 could be easily used for calculation of Ω_3 beamwidths with the aid of (4b). The measured beamwidths of a lower beam for $\Theta = 0^o$ and $\Theta = 3^o$ cuts according to coordinate system shown in Fig. 1 are depicted in Fig. 6. The sidelobe levels of a lower beam for $\Theta = 0^o$ and $\Theta = 3^o$ cuts according to coordinate system shown in Fig. 1 are depicted in Fig. 7. It follows from Fig. 6 and 7 that the measurements, approximations and detailed calculations [6] for several frequencies were done. Therefore, some detailed results can be only shown here. The lower beam patterns for $\Theta = 3^o$ cut and 3 frequencies are only depicted in Fig. 8. That show measurements for f=2.8 GHz (solid line) and $f = 6$ GHz (dashed line) and approximations using $c(u,t)$ functions for $f = 2.8$ GHz (crosses) and $f = 6$ GHz (circles). The radiation patterns for radar operating frequency band (2.7 to 2.9 GHz) and near-by frequencies are very similar, and therefore the measurements and approximations are not presented. That is clearly demonstrated by calculations [6] for $f = 3$ GHz (dot-dashed curve).

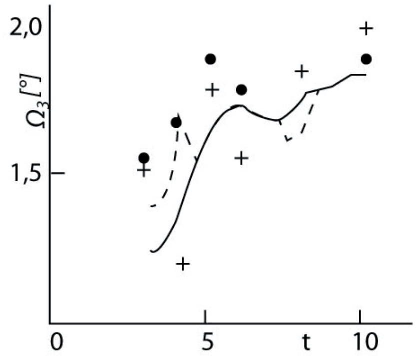

Fig. 6. The -3 dB beamwidths Ω_3 of the lower beam. Approximations using $c(u,t)$ for $a = 110$ mm (solid line) and approximations using $c(u,t)$ for equivalent (variable) a dimensions (dashed line). Measurements for $\Theta = 0^o$ (crosses) and $\Theta = 3^o$ (circles) cuts.

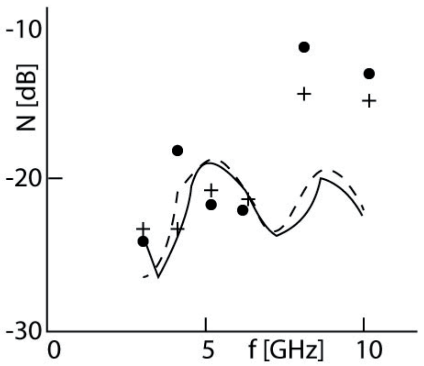

Fig. 7. The sidelobe levels relative to maximum values of given pattern cuts for the lower beam. Approximations using $c(u,t)$ for $a = 110$ mm (solid line) and approximations using $c(u,t)$ for equivalent (variable) a dimensions (dashed line). Measurements for $\Theta = 0^o$ (crosses) and $\Theta = 3^o$ (circles) cuts.

Of course, there are many valid reasons for the differences between approximations and measured values. They are created by approximation inaccuracies as phase characteristics of horn, amplitude horn pattern changes due to horn quadratic phase errors and horn higher mode effects, which are not considered. Manufacturing errors such as inaccuracies due to produced reflector and astigmatism (both the feed and the reflector could have unequal phase centers in different planes) create phase errors, which are directly proportional to the frequency.

It is well-known [1], [2] that a far-field measurement distance R is usually considered to be

$$R \geq \frac{2D^2}{\lambda} \tag{9}$$

where D is the reflector size – see Fig. 1. At that distance, the phase error across the aperture from a point source antenna is $\pi/8$. The distance is not sufficient for low-sidelobe antennas because quadratic phase error raises the measured sidelobes. The measurements has been taken in $R = 1\,240$ m, i.e. the condition (9) is only fulfilled for frequencies $f < 7.5$ GHz. Therefore, the higher sidelobes and broader beamwidths are obtained for measured patterns (especially for higher frequencies). As usual, for far-field measurements, the other measurement errors are due to multipath rays created by ground and nearby objects reflections. These errors change due to the frequency variations. They are not monotonic with frequency.

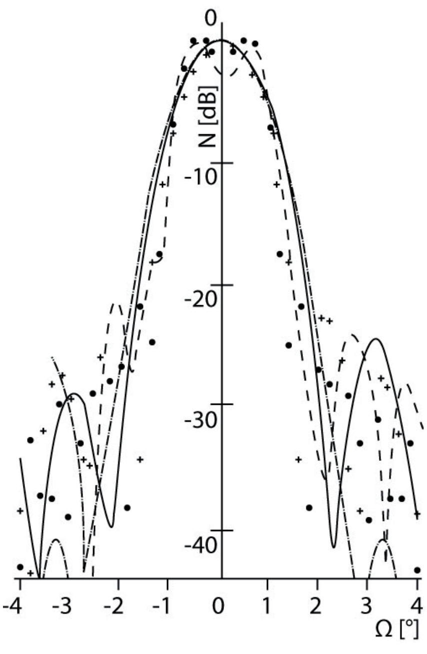

Fig. 8. Lower beam patterns. $\Theta = 3^o$ measurements for f=2.8 GHz (solid line) and $f = 6$ GHz (dashed line), approximations using $c(u,t)$ functions for $f = 2.8$ GHz (crosses) and $f = 6$ GHz (circles) and calculations [6] for $f = 3$ GHz (dot-dashed curve).

Considering the above errors the most important are manufacturing inaccuracies, which are discussed in detail [2], [3] and therefore they are not analyzed. However, it is necessary to notice that utilization of two beams (and therefore two horns, which cannot be placed at the same focus) causes that the antenna is more sensitive from the manufacturing tolerance point of view than an antenna with one horn at the focus. It could be deduced that the pattern approximations using $c(u,t)$ functions could be satisfactorily used for analyses of changes for broad frequency ranges.

The well-known simple formula of gain estimation considering beamwidths was analyzed both for various amplitude distributions and experimental data [4], [19], [20]. This could be very useful from EMC point of view as the $c(u,t)$ function gives beamwidth approximations. However, relatively small pattern changes for a perpendicular *ZX* plane could be rather complex as horns of doubly curved reflector antennas can be complicated - such as [9].

5. Conclusions

The properties of $c(u,t)$ functions, which approximate patterns of shaped-beam reflector antennas for very broad frequency range (out of operating frequencies), have been derived and analyzed. That could be very useful for EMC analyses both from EMI and EMS point of view as specialized more accurate methods are only used by antenna designers. Of course, thanks to dimensions of horn input waveguide, the frequencies below cutoff could not be analyzed. On the other hand, the $c(u,t)$ functions could improve analyses for higher frequencies. Considering EMI the parasitic oscillations of radar with doubly curved reflector antenna could be very dangerous, and therefore pattern information is demanded. The EMI filters (such as [21]) could be used for suppression of EMI, which penetrates through the power network. Similarly, due to external interferences, pattern information of doubly curved reflector antenna is demanded from radar EMS point of view. The $c(u,t)$ function gives beamwidth approximations that could be used for simple formula of gain estimation.

The typical properties of $c(u,t)$ functions are given. Graphs of $c(u,t)$ are shown in Fig. 2. Sidelobe level changes are depicted in Fig. 3. Fig. 4 and 5 are graphs of beamwidths and aperture efficiency q. The parameter t changes have been considered for range $0 \le t \le 10$. That usually corresponds four times operating frequency.

The following conclusions can be derived for $c(u,t)$ functions:

- Values of $c(0,t)$ functions decrease with increasing t as is shown in Fig. 2.
- The beam splitting is created for $t > 3\pi/2$ according to Fig. 2.
- The sidelobes are minimal for products of approximately π and maximal for odd products of approximately $\pi/2$ (Fig. 2 and 3). This phenomenon is connected with edge distribution [2]. Step transitions on the aperture edges produce high sidelobes, while tapering the edge reduces sidelobes. The sidelobe envelope of peaks is related to the derivative of the distributions at the edges.
- The beamwidth increases with increasing t as is shown in Fig. 4.
- The aperture efficiency q decreases with increasing t as is shown in Fig. 5.

The approximations and measured patterns of doubly curved reflector antenna with two beams are compared. The equivalent a width concept could be very useful, especially for antenna designers. However, that is usually unavailable for EMC experts. On the other hand, the changes are not substantial from the EMC point of view.

It can be concluded that pattern approximations using $c(u,t)$ functions characterize satisfactorily pattern changes due to frequency changes over a broad frequency range.

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