Behavioral Modeling of Memcapacitor

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Abstract. *Two behavioral models of memcapacitor are developed and implemented in SPICE-compatible simulators. Both models are related to the charge-controlled memcapacitor, the capacitance of which is controlled by the amount of electric charge conveyed through it. The first model starts from the state description of memcapacitor whereas the second one uses the memcapacitor constitutive relation as the only input data. Results of transient analyses clearly show the basic fingerprints of the memcapacitor.*

Keywords

Memcapacitor, memristor, SPICE, constitutive relation.

1. Introduction

In 1971, Leon Chua introduced the so-called memristor (= memory resistor) into the modern circuit theory [1]. 37 years later, the solid-state memristive device was fabricated in HP laboratories [2]. It led to a sharp rise of interest in memristive systems [3] in both the technological and the application domains. In December 2008, Chua proposed other hypothetical "mem-elements" from the nano-world, the memcapacitor and meminductor. Since all the above elements are not currently available as off-the-shelf devices, the role of modeling them increases, particularly with the aim of implementing such models in the current programs for circuit simulation.

One of the first SPICE models of memristor and memcapacitor were described in [4] and [5], respectively, starting from the general methodology given in [6]. These models were implemented in PSpice. Also note that the technique of mutators can be used for modeling the memcapacitor: It is shown in [7-9] that mutators can be designed that transform memristors into memcapacitors, and that memristors can be emulated via digital [10] or analog [11] means. For the purpose of modeling and simulating memcapacitor, one can use the fact that memristor models are well-known and that mutators can be modeled by simple linear dynamical systems [9].

In this paper, the memcapacitor is modeled directly, without being transformed from the memristor. Two different approaches to memcapacitor modeling are used. The first one, based on the state description of a memcapacitor, is an extension of the model from [5]. It is extended by virtue of new features, which Micro-Cap 10 can offer in contrast to the OrCAD PSpice program. The model implementation is in the form of a macro file, enabling a selection from among Joglekar, Biolek, and user-defined window functions for modeling the so-called boundary effects in the nano-components [4], [13], [14]. The model is related to the so-called charge- (or TIQ-) controlled capacitor, the capacitance of which is controlled by the amount of the time integral of electric charge (TIQ) conveyed through it. This amount affects the width of the dielectric.

It is well-known that the above windowing causes deviations from the theoretical behavior of the simulated mem-elements, and, as a consequence, the memcapacitor passes to the more general memcapacitive system, which need not show the memcapacitor fingerprints. That is why other models of memcapacitor are required which do not use windowing. The second model described in the paper starts directly from the constitutive relation of the charge/TIQ-controlled memcapacitor, which guarantees that the memcapacitance is unambiguously given by the TIQ passing through the memcapacitor. As a result, this model must show all the memcapacitor fingerprints irrespective of the memcapacitor interaction with external network. It is proved in Section 3 that the results of transient analysis clearly show three basic fingerprints of the memcapacitor: unambiguous constitutive relation [5], hysteretic effect in the Volt-Coulomb characteristic, and identical time instants when the voltage and charge waveforms cross zero levels.

2. Memcapacitor Model Based on State Space Description

The charge/TIQ-controlled memcapacitor is characterized by the following port equation (PE) and first-order state equation (SE) [5]:

$$\text{PE: } v = D_M(x,q,t)q\,, \quad \text{SE: } \dot{x} = \frac{d}{dt}x = f_q(x,q,t) \quad (1)$$

where v and q are the electric voltage and charge of the memcapacitor, $D_M = 1/C_M$ is an inverse memcapacitance, C_M is a memcapacitance, and x is an internal state variable of the memcapacitor. The charge/TIQ-controlled memcapacitor is a special type of time-varying capacitor, the (inverse) capacitance of which depends on the quantities indicated in (1).

The well-known relation between the voltage and current of the time-varying capacitor

$$v(t) = D_M(t)[C_M(0)v(0) + \int_0^t i(\xi)d\xi] = \\ = [C_M(0)v(0) + \int_0^t i(\xi)d\xi]/C_M(t) \quad (2)$$

where $v(0)$ and $C_M(0)$ are the initial voltage and capacitance of the memcapacitor at time $t = 0$, can be the starting point of the SPICE model, schematically described in Fig. 1.

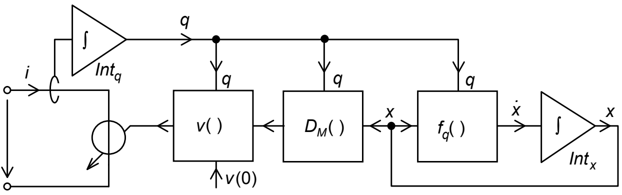


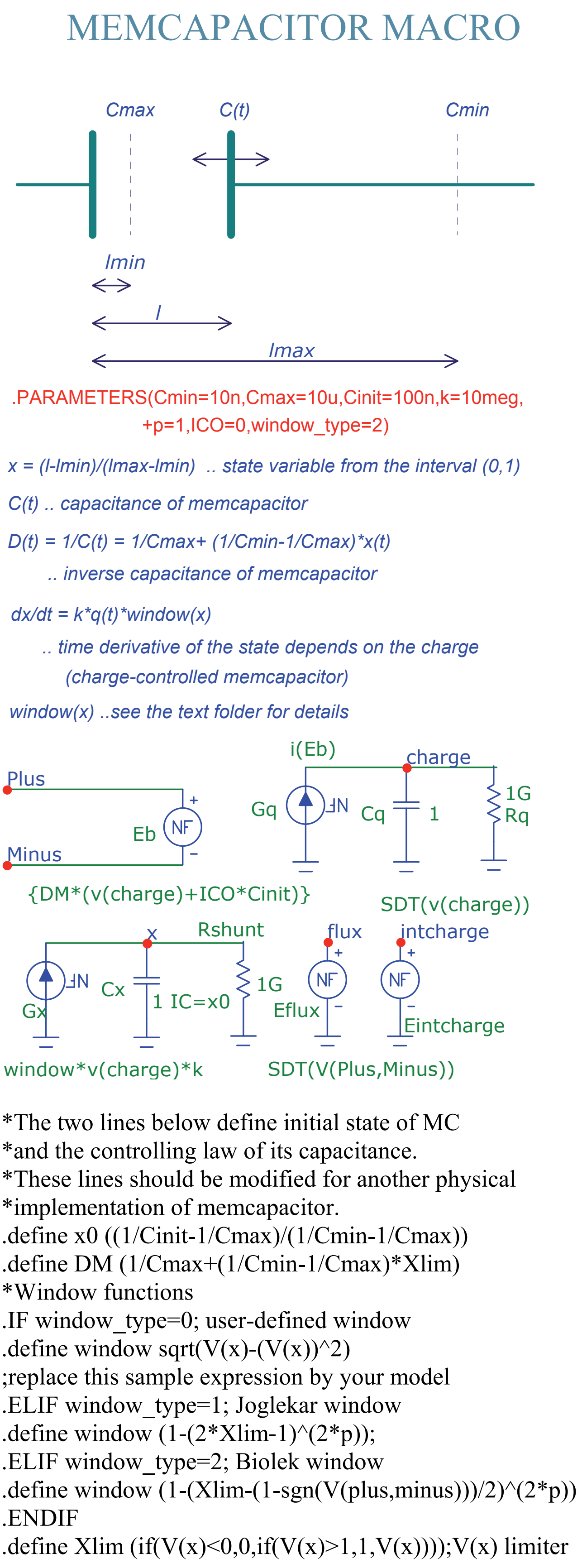
Fig. 1. Block diagram of the SPICE model of charge/TIQ-controlled memcapacitor.

Equation (2) is modeled via a voltage source which is controlled from the block labeled as "$v(\,)$". The input data of this block are the time-domain integral of current, i.e. charge q, the initial voltage $v(0)$, and the inverse memcapacitance as a function of charge and state x. The state variable is computed from the differential SE (1), which is modeled using the block of nonlinear function f_q and the integrator Int_x.

The above general model can be used for modeling charge/TIQ-controlled memcapacitors of arbitrary nature. The contents of the blocks $D_M(\,)$ and $f_q(\,)$ depend on the physical principle of a concrete memcapacitor.

A sample example of memcapacitor which is included in the Micro-Cap 10 release [12] is described below.

The contents of the Micro-Cap macro file of the memcapacitor is shown in Fig. 2. The schematic of a capacitor with moving right-side plate is accompanied by the basic notes how the capacitance depends on the position of this plate. The state variable x is derived from this position such that it can vary within the interval (0, 1). The state equation from [5] is considered. It contains the window function for modeling the boundary conditions [4-6], [8-9].



MEMCAPACITOR MACRO

.PARAMETERS(Cmin=10n,Cmax=10u,Cinit=100n,k=10meg,
+p=1,ICO=0,window_type=2)

x = (l-lmin)/(lmax-lmin) .. state variable from the interval (0,1)

C(t) .. capacitance of memcapacitor

D(t) = 1/C(t) = 1/Cmax+ (1/Cmin-1/Cmax)*x(t)
.. inverse capacitance of memcapacitor

dx/dt = k*q(t)*window(x)
.. time derivative of the state depends on the charge
(charge-controlled memcapacitor)

window(x) ..see the text folder for details

```
*The two lines below define initial state of MC
*and the controlling law of its capacitance.
*These lines should be modified for another physical
*implementation of memcapacitor.
.define x0 ((1/Cinit-1/Cmax)/(1/Cmin-1/Cmax))
.define DM (1/Cmax+(1/Cmin-1/Cmax)*Xlim)
*Window functions
.IF window_type=0; user-defined window
.define window sqrt(V(x)-(V(x))^2)
;replace this sample expression by your model
.ELIF window_type=1; Joglekar window
.define window (1-(2*Xlim-1)^(2*p));
.ELIF window_type=2; Biolek window
.define window (1-(Xlim-(1-sgn(V(plus,minus)))/2)^(2*p))
.ENDIF
.define Xlim (if(V(x)<0,0,if(V(x)>1,1,V(x))));V(x) limiter
```

Fig. 2. Complete memcapacitor macro.

The current source G_q together with the one-farad capacitor C_q and shunting resistor R_q for providing DC path to the ground models the integrator Int_q from Fig. 1 for transforming the memcapacitor current into charge. The charge value is available in the form of the voltage at node "charge". The current source G_x together with C_x and R_{shunt} models the integrator Int_x for evaluating the state equation (1). The auxiliary voltage sources Eflux and Eintcharge serve for computing the time-domain integrals of voltage (i.e. flux) and charge (i.e. TIQ, time-domain integral of charge). These quantities, which are important constitutive variables of the memcapacitor, can be then easily visualized in transient analysis results.

Results of the transient analysis of a memcapacitor driven by pulse-waveform voltage source with an internal resistance of 1 mΩ are presented in Fig. 3.

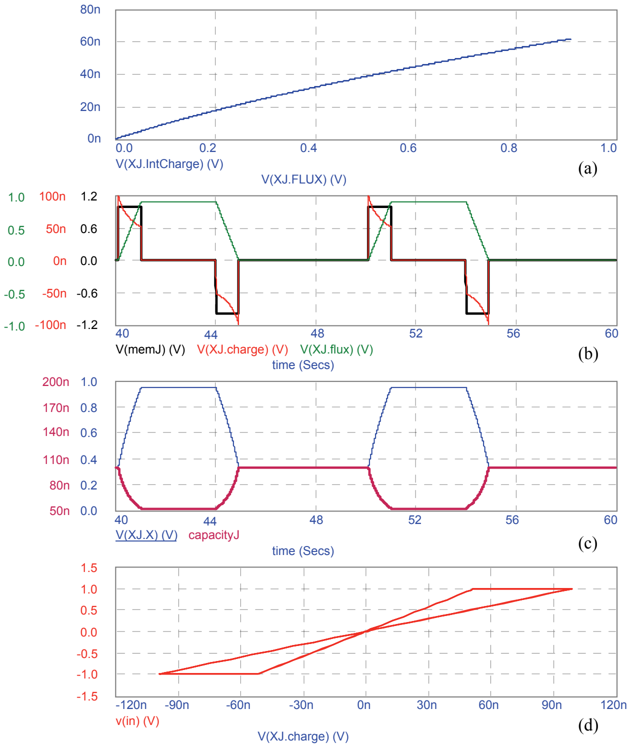


Fig. 3. Transient analysis of memcapacitor excited by pulse voltage source: (a) constitutive relation, (b) waveforms of input voltage V(memJ), charge V(XJ.charge), and flux V(XJ.flux), (c) time evolution of state variable V(XJ.X) and memcapacitance, (d) Volt-Coulomb hysteretic characteristic.

The memcapacitor has the following parameters: $C_{min} = 50$ nF, $C_{max} = 200$ nF, $C_{init} = 100$ nF, $ICO = 0$, $p = 10$, Joglekar window. The 900 ms width and 10 ms rise/fall time bipolar ±1V pulses in Fig. 3 (b) cause the pulse waveforms of the charge which modifies the position of the plate of the memcapacitor. The corresponding variation of the memcapacitance is shown in Fig. 3 (c). Fig. 3 (d) depicts the Volt-Coulomb characteristic with typical pinched hysteresis loop.

Detailed experiments with the above model prove that the constitutive relation can be modified from its form in Fig. 3 (a) under special constellations of excited signal, internal state of the element, and window function used. That is why models of a "true memcapacitor" are required. This problem is analyzed in the following section.

3. Memcapacitor Model Based on its Constitutive Relation

According to [8], the charge/TIQ-controlled memcapacitor is defined axiomatically by the nonlinear constitutive relation (CR)

$$\varphi = \hat{\varphi}(\sigma) \tag{3}$$

where φ and σ are the time-domain integrals of electric voltage v of the memcapacitor (TIV) and time integral of electric charge q, passing through the memcapacitor (TIQ). The inverse memcapacitance D_M (D_M=1/C_M, C_M is memcapacitance) at the operating point Q [15] can be derived from the constitutive relation as follows:

$$D_M(\sigma) = \left.\frac{d\hat{\varphi}(\sigma)}{d\sigma}\right|_Q . \tag{4}$$

Differentiating both sides of (3) yields the voltage-charge relations

$$\frac{d\varphi}{dt} = v(t) = \frac{d\hat{\varphi}(\sigma)}{d\sigma}\frac{d\sigma}{dt} = D_M(\sigma).q(t) . \tag{5}$$

Consider CR (3) in the form of the Taylor series

$$\varphi = \hat{\varphi}(\sigma) = \sum_{k=1}^{\infty} d_k \sigma^k \tag{6}$$

where d_k, $k = 1, 2, \ldots$ are real coefficients. Then equation (5) describes the conventional voltage-charge relation on the capacitor, with the inverse memcapacitance of the charge/TIQ-controlled memcapacitor being TIQ-dependent according to the formulae

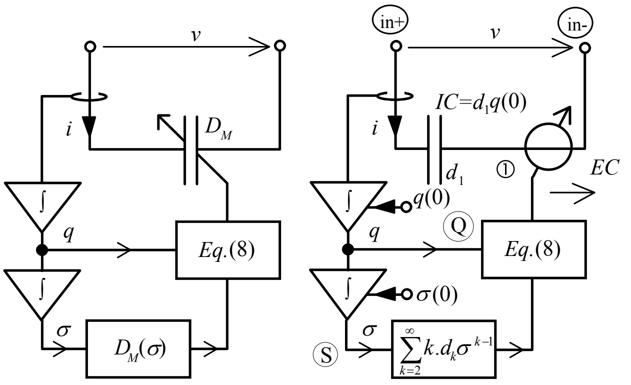
$$D_M(\sigma) = \sum_{k=1}^{\infty} k.d_k \sigma^{k-1} = d_1 + \sum_{k=2}^{\infty} k.d_k \sigma^{k-1} . \tag{7}$$

The above equation confirms that when the CR (3) is linear, i.e. when $d_k = 0$ for $k > 1$ in (6), the inverse memristance is then independent of the circuit variables, and the memcapacitor behaves as a linear capacitor. In other words, the memory effect is described by the remaining terms of the Taylor series just for $k > 1$.

Starting from the above results, the CR-based SPICE modeling of the memcapacitor could be as follows: The time-domain integration of the current should be performed in order to get the charge, and the time-domain of the charge yields the TIQ=σ. From the TIQ, the instantaneous value of the inverse memcapacitance can be computed via (7). If we know the inverse memcapacitance and charge, the memcapacitor can be modeled via a voltage source according to (2), which can be rewritten in the form

$$v = D_M q,\ q = q(0) + \int_0^t i(\xi)d\xi,\ q(0) = C_M(0)v(0)\,. \quad (8)$$

Figs. 4 (a) and (b) contain block-oriented models based on the above approach. The left-side general model is concretized on the right for (7), which takes into account the Taylor expansion of the CR. It is obvious from (7) that the memcapacitor can be modeled as a serial connection of a capacitor with fixed inverse capacitance d_1 and a capacitor with variable inverse capacitance which is given by the second term in (7). Since the SPICE does not enable a direct modeling of variable capacitor, the controlled voltage source EC is used in Fig. 4 (b), and its voltage is computed via (8) with D_M equal to the second term of (7).



(a) (b)

Fig. 4. Proposed models of the memcapacitor: (a) general, (b) concretized for SPICE implementation.

Note that Fig. 4 (b) contains the initial states of the integrators, i.e. the initial charge and TIQ which should be additional input data of the model. The initial charge determines the initial voltage of the fixed capacitor which should be defined in PSpice via IC attribute of the capacitor. This initial voltage can be derived, combining (7) and (8):

$$v = \left[d_1 + \sum_{k=2}^{\infty} k.d_k\sigma^{k-1}\right]\left[q(0) + \int_0^t i(\xi)d\xi\right] =$$
$$= d_1 q(0) + d_1\int_0^t i(\xi)d\xi + q\sum_{k=2}^{\infty} k.d_k\sigma^{k-1}\,. \quad (9)$$

The first two terms on the last row describe the capacitor with fixed capacitance $1/d_1$, with the initial voltage

$$v_1(0) = d_1 q(0)\,. \quad (10)$$

The third term describes the variable capacitor which is modeled via a controlled voltage source EC in Fig. 4 (b).

As an example, consider a memcapacitor with the CR

$$\varphi = d_1\sigma + d_3\sigma^3,\ d_1 = 10^5\ \text{F}^{-1},\ d_3 = 10^{15}\ \text{VA}^{-3}\text{s}^{-2}. \quad (11)$$

Differentiating (11) with respect to σ, one can prove that the inverse memcapacitance is under any circumstances, i.e. for an arbitrary TIQ, positive, which is a necessary condition for the memcapacitor passivity:

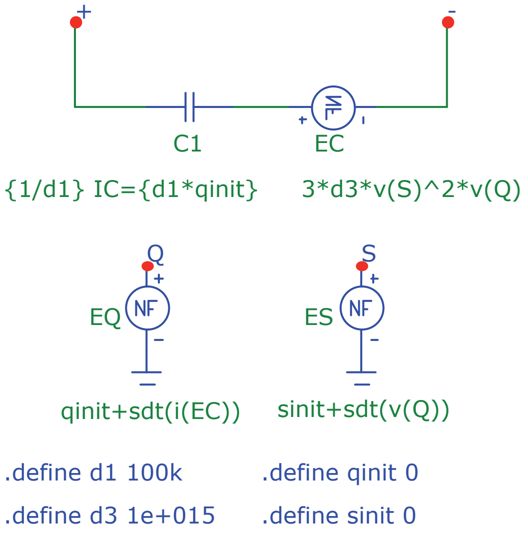
$$D_M = \frac{d\varphi}{d\sigma} = d_1 + 3d_3\sigma^2\,. \quad (12)$$

The PSPICE subcircuit of this memcapacitor, compiled on the basis of Fig. 4 (b), is given below:

```
 .subckt QC_memcapacitor in+ in- params: qinit=0
sinit=0
 .param d1 100k d3 1e15
 EQ Q 0 value={qinit+SDT(i(EC))}
 ES  S 0 value={sinit+SDT(v(Q))}
 C1 in+ 1 {1/d1} IC={d1*qinit}
 EC 1 in- value={3*d3*v(S)^2*v(Q)}
 .ends QC_memcapacitor
```

EQ- and ES-controlled sources provide time-domain integrations via the PSPICE internal function SDT. The parameters qinit and sinit represent the initial values of charge and TIQ.

The equivalent behavioral model in the Micro-Cap program is shown in Fig. 5.



.define d1 100k
.define d3 1e+015
.define qinit 0
.define sinit 0

Fig. 5. Memcapacitor modeling in Micro-Cap.

The above model was used for the simulation of a memcapacitor excited by the harmonic voltage source with an amplitude of 2 V, repeating frequency of 1 Hz, and internal resistance of 1 mΩ. The results of transient analysis in Fig. 6 show all the basic fingerprints of the memcapacitor: Unambiguous constitutive relation (11) (Fig.6 (a)), a pinched hysteresis loop in the Volt-Coulomb characteristic (Fig. 6 (b)), and identical time instants when the voltage and charge waveforms cross zero levels (Fig. 6 (c)). It can be also demonstrated that increasing the frequency causes an attenuation of the hysteretic effects in the Volt-Coulomb characteristic, and that the CR does not depend on the way the memcapacitor is excited.

4. Conclusions

Two different methods for memcapacitor modeling are proposed in the paper. The method in Fig. 1 starts from the physical model of a concrete implementation of the memcapacitor. In spite of the concrete example used here for the demonstration, this methodology is quite general because it can be used for modeling any memcapacitor the capacitance of which is controlled via various physical mechanisms. The second approach has nothing in common with the physical implementation of the memcapacitor. The element being modeled is considered as a hypothetical circuit component which is defined axiomatically from its constitutive relation. The only necessary condition for such modeling is to have a mathematical representation of the memcapacitance (or inverse memcapacitance) as a function of TIQ. Computer simulations clearly show all the basic fingerprints of the memcapacitor.

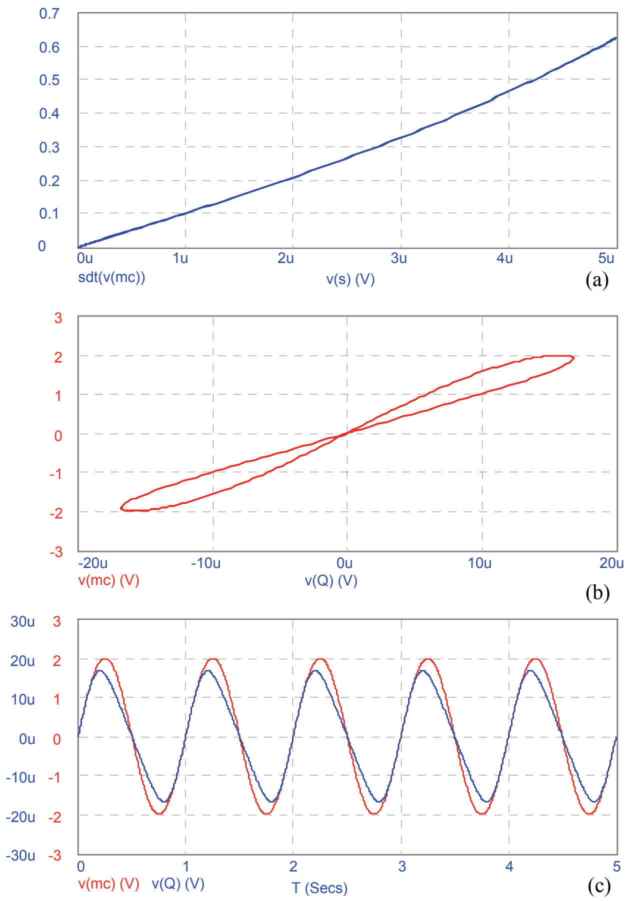


Fig. 6. Transient analysis of memcapacitor from Fig. 5 excited by sinusoidal voltage source: (a) constitutive relation, (b) Volt-Coulomb hysteretic characteristic, (c) waveforms of input voltage v(mc) and charge v(Q).

It is worth mentioning here that the above methodology can be easily used also for modeling the voltage/flux-controlled memcapacitor.

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