Switched Capacitor Realizations of Fractional-Order Differentiators and Integrators Based on an Operator with Improved Performance

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Abstract. *In this paper, switched capacitor realizations of discretized models of half differentiator and half integrator based on a new operator with improved performance have been proposed. This Al-Hsue operator is the weighted sum of the Al-Alaoui operator and the Hsue operator. The discretized models of the Al-Hsue operator have been expanded using Taylor Series Expansion and Continued Fraction Expansion, to be able to develop the Switched Capacitor realizations. These Switched Capacitor realizations are implemented using Spice and the results obtained are compared with the theoretical results of the continuous-time domain half differentiators and integrators. These Spice simulation results are also compared with the results of existing Al-Alaoui operator and Hsue operator based Switched Capacitor realizations of half differentiators and integrators of order 1/2. The results validate the effectiveness of the Switched Capacitor circuit implementation of the proposed approach.*

Keywords

Half differentiator, half integrator, Al-Alaoui operator, Hsue operator, Al-Hsue operator.

1. Introduction

Analog realizations of fractional-order circuits have been explored by many researchers [1-7], but work on digital realizations is still being considered [8-10]. In this paper, sampled-data realizations of fractional-order differentiators and integrators have been explored using SC circuits [11-16]. The technique for implementation of fractional-order circuits involves expansion of the z-domain transfer function into ladder form using continued fraction expansion. Each row of the Continued Fraction Expansion (CFE) is realized using Switched Capacitor (SC) amplifiers and integrators. Then, the expansions of the rows are implemented using parasitic insensitive SC realizations of integrators and amplifiers. The designed sampled data systems have been simulated using Spice.

Switched capacitor techniques [11], [12] have been developed for discrete data/signal processing to allow integration of both digital and analog functions on a single chip. In SC circuits, the function resistors are realized using MOS switches and capacitors. The advantage of using SC circuits is that the accuracy of the signal-processing function is proportional to the accuracy of capacitor ratios. The other advantages of SC circuits include – compatibility with CMOS technology, good accuracy of time constant, programmability, good voltage linearity, flexibility, good temperature characteristics, better accuracy and stability and ease of fabrication. For proper functioning of switched capacitor circuits, the sampling frequency should be at least ten times the maximum signal frequency [17]. This is in accordance with the Shannon-Kotelnik theorem which states that, "If in the process of sampling the information must not lose, a frequency of sampling ω_s and maximal frequency ω_m included in the signal spectrum have to comply with a condition $\omega_s \geq 2\omega_m$ [18]".

The organization of this paper is as follows: Section 2 defines fractional-order systems. Section 3 briefly introduces the Al-Hsue operator [19]. Section 4 discusses the CFE technique for realization of the Al-Hsue operator based models of half differentiator ($s^{1/2}$) and half integrator ($s^{-1/2}$). Spice simulation results of the half differentiators and integrators based on the Al-Hsue operator are presented in section 5. These simulation results are compared with the theoretical results of the half differentiators and integrators in continuous-time domain and the Spice simulation results of Al-Alaoui operator [20-22] and Hsue operator [23] based SC realizations of half differentiator and integrator in section 6. Section 7 concludes the paper.

2. Fractional-Order Calculus and Systems [1]

The word 'fractional calculus' is used for the theory of integrals and derivatives of arbitrary order. It is basically a generalization of differentiation and integration to a non-integer order fundamental operator ${}_aD_t^{\alpha}$, $\alpha \in R$, where α and t are the limits of operation [24].

The continuous integrodifferential operator [1] is defined as

$$ {}_aD_t^\alpha = \begin{cases} d^\alpha/dt^\alpha & \Re(\alpha)>0 \\ 1 & \Re(\alpha)=0 \\ \int_a^t (d\tau)^{-\alpha} & \Re(\alpha)<0 \end{cases} \quad (1)$$

where α is the fractional-order which can be a complex number and a is the constant related to the initial conditions. The two definitions used for the fractional differintegral ${}_aD_t^\alpha$ are the Grunwald Letnikov (GL) definition and the Riemann-Lioville (RL) definition [24].

A fractional-order system is represented by a fractional differential equation given by (2).

$$ a_n D_t^{\alpha_n} y(t) + \ldots.. + a_1 D_t^{\alpha_1} y(t) + a_0 D_t^{\alpha_0} y(t) = b_m D_t^{\beta_m} u(t) + \ldots.. + b_1 D_t^{\beta_1} u(t) + b_0 D_t^{\beta_0} u(t) \quad (2)$$

where β_k $(k = 0,1,2,\ldots,m)$, $\alpha_k(k = 0,1,2,\ldots,n)$ are real numbers and $a_k(k = 0,1,2,\ldots,n)$, $b_k(k = 0,1,2,\ldots,m)$ are arbitrary constants.

The discretized model of a fractional-order system is

$$ G(z) = \frac{b_m(\omega(z^{-1}))^{\beta_m} + \ldots\ldots + b_1(\omega(z^{-1}))^{\beta_1} + b_0(\omega(z^{-1}))^{\beta_0}}{a_n(\omega(z^{-1}))^{\alpha_n} + \ldots\ldots + a_1(\omega(z^{-1}))^{\alpha_1} + a_0(\omega(z^{-1}))^{\alpha_0}} \quad (3)$$

where $\omega(z^{-1})$ denotes the discrete operator, expressed as a function of the complex variable z or the shift operator z^{-1} [10].

3. The Al-Hsue Operator

The Al-Alaoui operator based integrator in z-domain is

$$ H_{al}(z) = \left[\left(\frac{7T}{8}\right)\left(\frac{1 - z^{-1}/7}{1 - z^{-1}}\right)\right] \quad (4)$$

and the integrator obtained by inverting the transformation of a wide-band differentiator [23] is

$$ H_{Hsue}(z) = \left[\left(\frac{T}{2}\right)\left(\frac{1 + 0.1658z^{-1}}{1 - z^{-1}}\right)\right] \quad (5)$$

where T is the sampling period.

To obtain a differentiator that fits better the ideal differentiator over the entire normalized frequency band, linear mixing of Al-Alaoui differentiator and the wide-band differentiator is performed as follows: (i) The transfer functions of the two integrators of (4), (5) are linearly mixed as in (6):

$$ \begin{aligned} &H_{new}(z) \\ &= \alpha H_{Hsue}(z) + (1-\alpha)H_{al}(z) \\ &= \alpha\left[0.28\frac{(1+0.1658z^{-1})}{(1-z^{-1})}\right] + (1-\alpha)\left[\frac{7T}{8}\frac{(1-z^{-1}/7)}{(1-z^{-1})}\right] \end{aligned} \quad (6)$$

where α, $(0 < \alpha < 1)$ determines the contribution of each operator in the new operator. (ii) The transfer function of (6) is inverted and the resulting transfer function of the new digital differentiator is

$$ G_{new}(z) = \frac{(z-1)}{z(0.875 - 0.375\alpha T) + (0.12495 - 0.04205\alpha T)} . \quad (7)$$

Using Jury's stability criterion, the differentiator $G_{new}(z)$ was found to be stable for the condition $\alpha T < 2.25$, $\forall(0 < \alpha < 1)$. Choosing $T = 0.05$ s, (sampling frequency = $2\pi(1/T) = 125.7$ rad/sec), the transfer function of the new differentiator [19], [22] is

$$ G_{new}(z) = \frac{(z-1)}{z(0.875 - 0.01875\alpha) + (0.12495 - 0.21025\alpha)} . \quad (8)$$

Now, α is varied from 0 to 1 in increments of 0.1. The magnitude response of the proposed differentiator is plotted for different values of α as shown in Fig. 1.

The percentage relative magnitude error of the new differentiator is compared with the magnitude response of the ideal differentiator and plotted in Fig. 2.

Observations show that best matching with ideal differentiator were for $\alpha = 0.9$. The error is within 2% upto 0.84 of the Nyquist frequency. Fig. 3 shows the phase of the new differentiator for different α. The response is almost linear with a maximum phase of 8.24° at 0.55 of the Nyquist frequency. The ideal linear phase corresponds to an ideal differentiator with half a sample of delay. These results are comparable with those of Al-Alaoui operator based differentiator as suggested in [20].

The transfer functions of the new differentiator for different values of T viz. 0.05 s, 0.00625 s, 0.001 s, and 0.000625 s with $\alpha = 0.9$ are:

$$ G_1(z)\big|_{T=0.05s} = \frac{23.1901(z-1)}{(z+0.1434)}, \quad (9)$$

$$ G_2(z)\big|_{T=0.001s} = \frac{1126.263(z-1)}{(z+0.1428)}, \quad (10)$$

$$ G_3(z)\big|_{T=0.00625s} = \frac{182.0131(z-1)}{(z+0.1428)}, \quad (11)$$

$$ G_4(z)\big|_{T=0.000625s} = \frac{1814.4141(z-1)}{(z+0.1428)}. \quad (12)$$

The magnitude response of (9-12) are plotted and compared with the magnitude of ideal differentiator (Fig. 4). The relative magnitude errors are plotted in dB in Fig. 5. The phase response of the new differentiator and the relative phase error is plotted in Fig. 6.

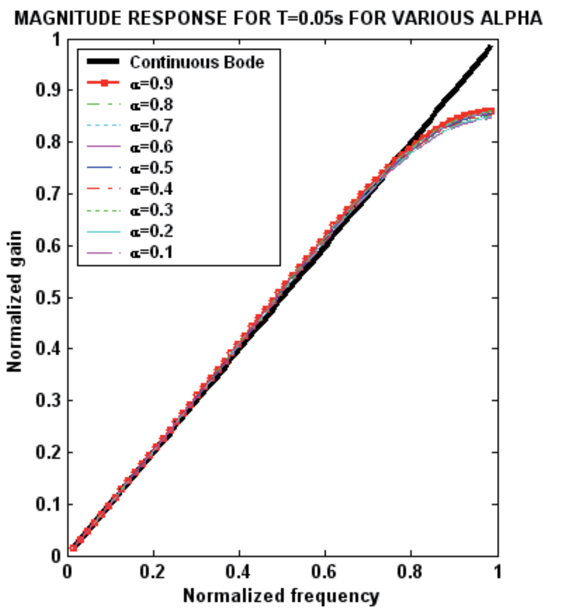

Fig. 1. Magnitude response of new operator for various α with $T = 0.05$ s.

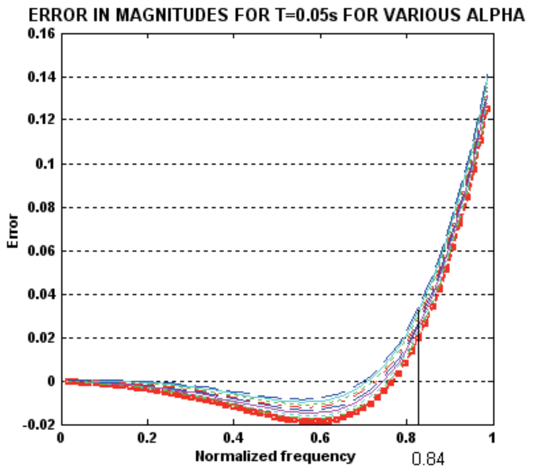

Fig. 2. Relative magnitude error as compared to the continuous-time differentiator for various α.

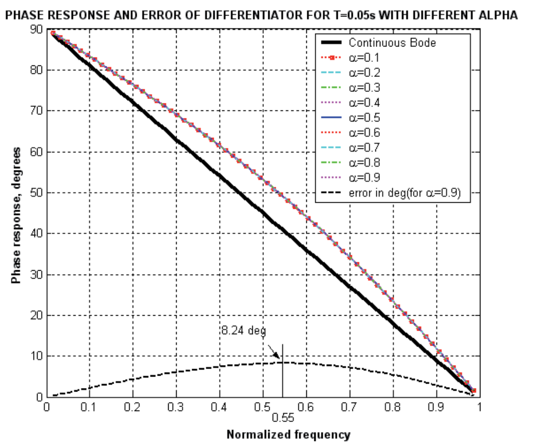

Fig. 3. Phase of new operator for various α and corresponding linear phase differentiator and phase error for $\alpha = 0.9$.

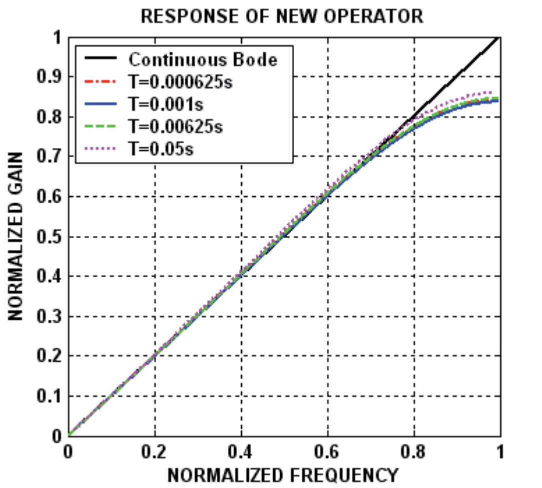

Fig. 4. Magnitude response of new operator for different T.

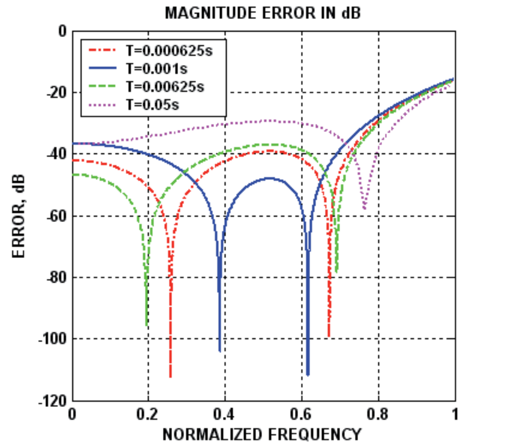

Fig. 5. Magnitude error (in dB) as compared to the continuous-time differentiator for different T.

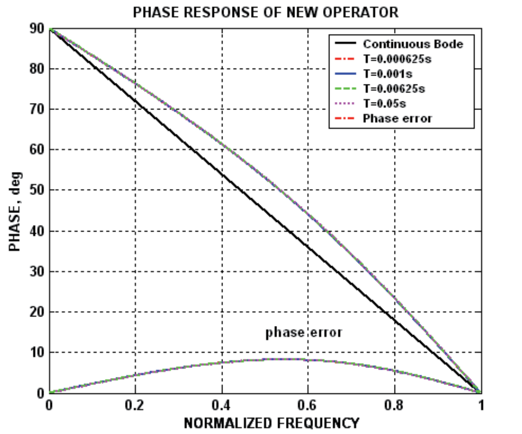

Fig. 6. Phase response for different T, corresponding linear phase differentiator and phase error.

Hence the Al-Hsue operator is a weighted combination of the Al-Alaoui operator and the Hsue operator.

The fractional-order models based on the Al-Hsue operator were obtained by expanding fractional powers of (10) using Taylor Series Expansion (TSE) and CFE. These fractional-order models showed improved performance in terms of both magnitude and phase, over the existing fractional-order models of differentiator and integrator. Hence, stable models of Al-Hsue operator based 3rd, 4th and 5th order half differentiator and half integrator were developed by the expansion of fractional powers of (10) (in this paper, the fractional powers used are $r = \pm 1/2$). To realize the above-mentioned models, the z-domain transfer functions were expanded using CFE as discussed in section 4.

4. Continued Fraction Expansion Technique and Switched Capacitor Realization of Al-Hsue operator Based Half Differentiators and Half Integrators

4.1 Continued Fraction Expansion Technique

A discrete transfer function of order n is expressed as

$$G_n(z) = \frac{a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0}{b_n z^n + b_{n-1} z^{n-1} + \cdots + b_0} \tag{13}$$

where a_i's and b_i's are the coefficients of numerator and denominator polynomials for $i = 0, 1, \ldots, n$. $G_n(z)$ can be expanded in different ways [12].

This transfer function $G(z)$ can be expanded in different ways. One of the expansions is:

$$G(z) = A_0 + \cfrac{1}{A_1 + B_1 z + \cfrac{1}{A_2 + B_2 z^{-1} + \cfrac{1}{\ddots + \cfrac{1}{A_n + B_n z^{-1}}}}} \tag{14}$$

where A_p and B_p are coefficients of CFE for $p = 1,2,\ldots,n$ and $A_p = a_n/b_n$.

There are several other expansion techniques involving different methods of division [25], but we have restricted our examples to only a selected few. Two of these transfer function expansion are of the following forms:

$$\text{a)}\quad G(z) = A_0 + G_{m,p}(z) \quad m = 1,2\,,$$

$$\text{b)}\quad G(z) = \frac{1}{A_0 + G_{m,p}(z)}\,. \tag{15}$$

Fig. 7 shows two schematic realizations of continued fraction expansion.

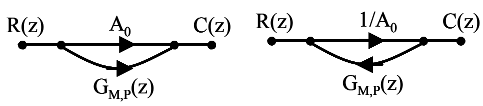

Fig. 7. Two schematic realizations of CFE.

All the expansions are based on mixed Cauer form involving terms of the form $(A_p + B_p z)$ and $(A_p + B_p z^{-1})$ [25].

Each row of the transfer function of (13) can be represented by any one of the following recursive relationships given in (16).

$$G_{m,p}(z) = \frac{1}{A_p + B_p z^{-1} + G_{m,p+1}(z)}\,,$$

or

$$G_{m,p}(z) = \frac{1}{A_p + B_p z + G_{m,p+1}(z)} \tag{16}$$

where $m = 1,2,\ldots,(n\text{-}1)$ and $p = m + 1$.

Equation (16) represents transfer functions of leaky inverting and non-inverting integrators, which are realizable using SC integrators. The constant term represents gain or attenuation, and is realized using SC amplifier. The SC circuit for the half differentiator and integrator are then obtained by connecting the different blocks of integrator and differentiator in ladder form.

4.2 Switched Capacitor Realization

To illustrate this technique, the procedure for SC realization of Al-Hsue operator based 3rd order half differentiator model obtained using CFE is discussed.

The 3rd order half-differentiator model ($T = 0.001$ s; where T is the sampling period) is

$$G_{32cfediff}(z) = \left(\frac{33.56z^3 - 52.74z^2 + 21.23z - 1.273}{z^3 - z^2 + 0.143z + 0.0204}\right). \tag{17}$$

Equation (17) is expanded as

$$G_{32cfediff}(z) = -A_0 G'_{32cfediff}(z)\,,$$

$$G'_{32cfediff}(z) = -1 + \cfrac{1}{g_1(z) - \cfrac{1}{g_2(z) - \cfrac{1}{g_3(z)}}}$$

$$A_0 = 33.56,\ g_1(z) = 1.749z - 0.2508, \tag{18}$$

$$g_2(z) = 6.9822z - 2.9934,\ g_3(z) = 1.7594z - 0.7529.$$

The term A_0 is a constant and the terms $1/g_i(z)\ \forall i = 1,2,3$ are transfer functions of integrator of the form

$$\frac{1}{g_i(z)} = \frac{1}{B_P z - A_P - G_{M,P+1}(z)} \tag{19}$$

where B_P, A_P are the coefficients of the expansion.

The z-domain transfer functions of the other third order models of half differentiator and half integrator (obtained using TSE and CFE) for $T = 0.001$ s are

$$G_{3newdifftay}(z) = \left(\frac{33.51z^3 - 16.75z^2 - 4.189z - 2.094}{z^3 + 0.07136z^2 - 0.002499z + 1.82e-4}\right), \tag{20}$$

$$H_{3new\,int\,cfe}(z) = \left(\frac{0.0298z^3 - 0.0298z^2 + 0.004259z + 0.0006068}{z^3 - 1.571z^2 + 0.6327z - 0.03793} \right), \quad (21)$$

$$H_{3new\,int\,tay}(z) = \left(\frac{0.0298z^3 + 0.002128z^2 - 7.45e\text{-}5z + 5.424e\text{-}6}{z^3 - 0.5z^2 - 0.125z - 0.0625} \right). \quad (22)$$

The basic building blocks of switched capacitor circuits are the SC amplifier/attenuator and the SC integrator [12]. Fig. 8 shows the stray-insensitive switched capacitor implementation of an amplifier with gain $A_0 = C_1/C_2$. Figs. 9, 10 show the signal flow graph and the SC realization of an integrator having a transfer function of the form of (19). The switches are controlled by two non-overlapping clock waveforms 'e' and 'o' shown in Fig. 11. The clocks have been realized using CMOS transmission gates (Fig. 12).

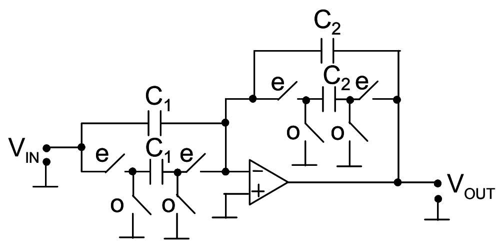

Fig. 8. Switched capacitor amplifier.

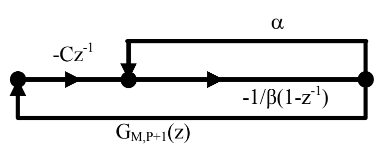

Fig. 9. Signal flow graph of integrator of (19).

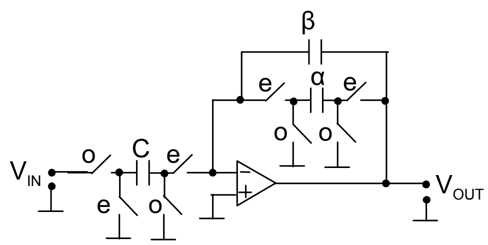

Fig. 10. Switched capacitor integrator.

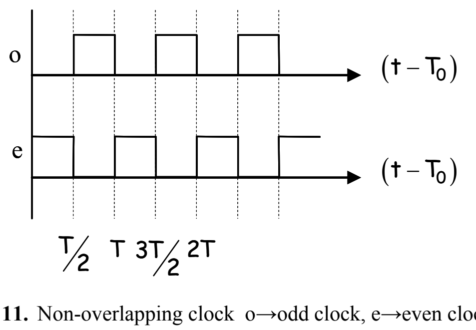

Fig. 11. Non-overlapping clock o→odd clock, e→even clock.

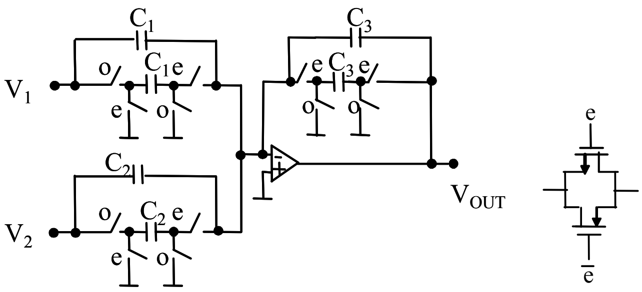

Fig. 12. SC summer and CMOS transmission gate.

The blocks of Figs. 8 and 10 are connected in ladder form to yield the SC realization of the third order half differentiator (obtained using CFE of the z-domain transfer function) shown in Fig. 13. In the figure, the three SC integrator blocks are of the form shown in Fig. 10. The summers used to connect various blocks of the ladder have been realized using switched capacitors (Fig. 12).

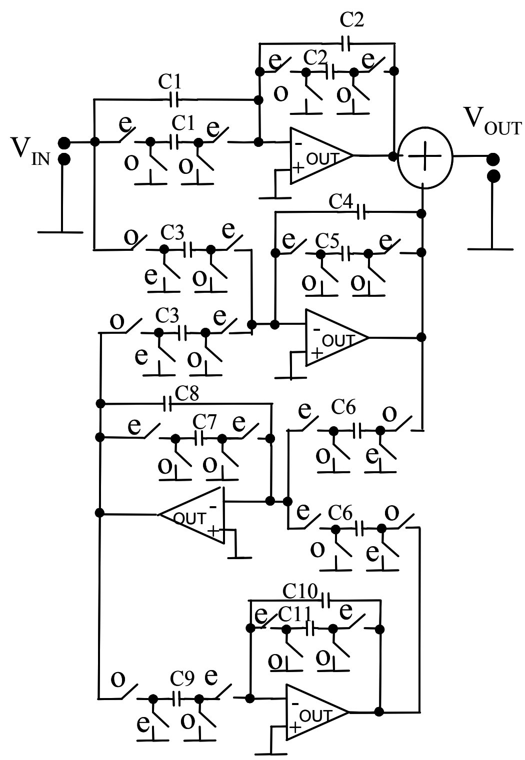

Fig. 13. Al-Hsue operator based third order SC half differentiator (using CFE).

The models given in (19-22) are expanded and realized in the same manner as discussed above, and their SC realizations are shown in Figs. 14-16.

Simulations of all the fractional-order differentiator discussed above are done on Spice using Level 3 MOSFET models with 2μ technology and CMOS transmission gates as switches.

The amplitude of the input and clock signal along with supply voltages applied are listed in Tab. 1.

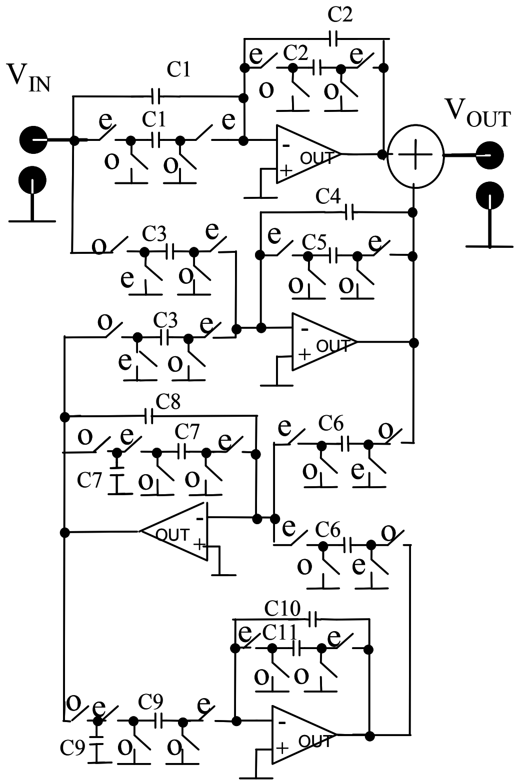

Fig. 14. Al-Hsue operator based third order SC half differentiator (using TSE).

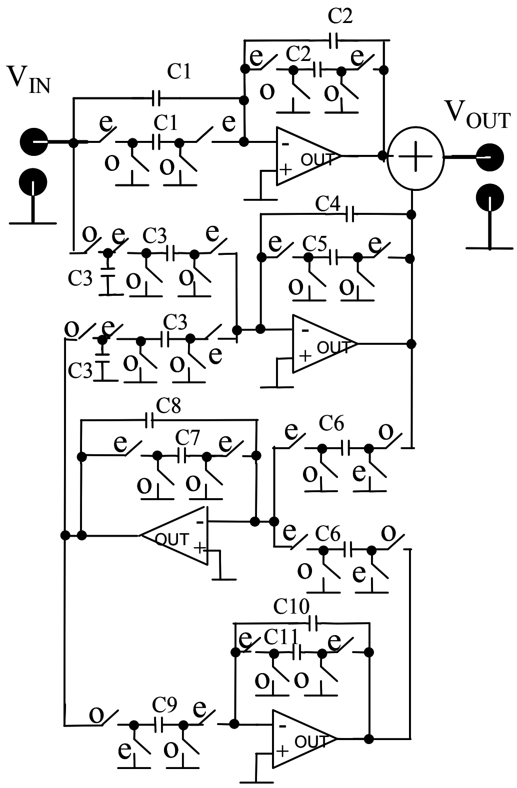

Fig. 15. Al-Hsue operator based third order SC half integrator (using CFE).

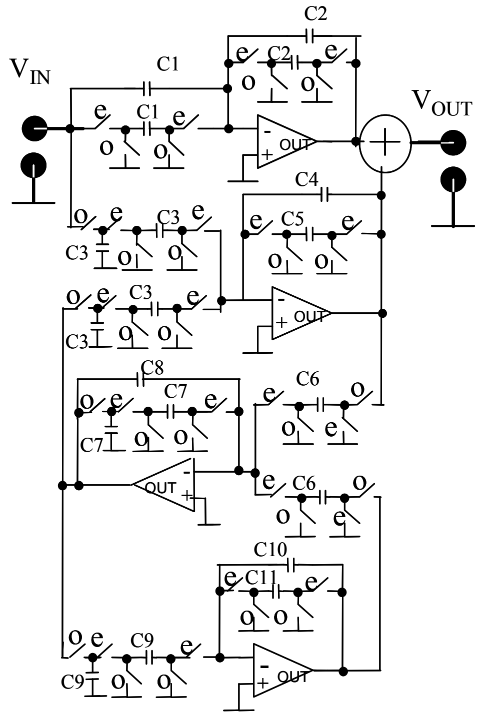

Fig. 16. Al-Hsue operator based third order SC half integrator (using TSE).

S. No.	Parameters	Values
1	Supply voltage	±7.5 V
2	Clock signal	-10 V to +10 V
3	Input	±2.5V

Tab. 1. Parameters of the clock and input.

5. Performance Results and Discussion

The simulation results are discussed as follows:

1) The frequency responses of the third order half differentiator based on the Al-Hsue operator obtained using TSE and CFE are shown in Fig. 17.

- The magnitude plots of both the forms are in close conformity to the theoretical results of continuous-time domain half differentiator and the MATLAB results presented in [19], but the third order half differentiator obtained using CFE shows better performance in terms of magnitude.
- The phase of Al-Hsue half differentiator approaches ~45° beyond 150 Hz. The phase shift of the third order half differentiator obtained using TSE is observed to be approximately linear.

2) The frequency responses of the third order half differentiator based on the Al-Alaoui operator and the Hsue operator are shown in Fig. 18.

- The magnitude responses of the Al-Hsue operator based half differentiator models match better with the theoretical results of half differentiator than the results of Al-Alaoui and Hsue operator based models.

- The phase response of Al-Alaoui operator based 3rd order half differentiator model approximates 45°better than the Al-Hsue operator (CFE) based model and the phase of Hsue operator and Al-Hsue operator (TSE) based 3rd order half differentiator vary linearly over the frequency range 0 to 600 Hz.

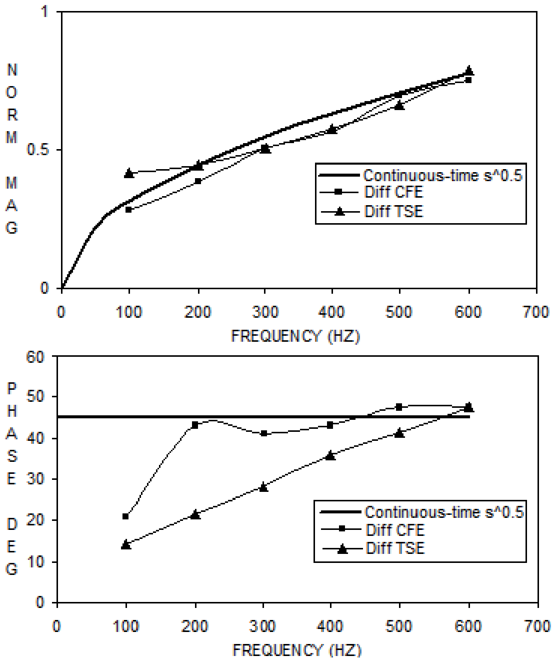

Fig. 17. SC results: Frequency response of Al-Hsue operator based half differentiator.

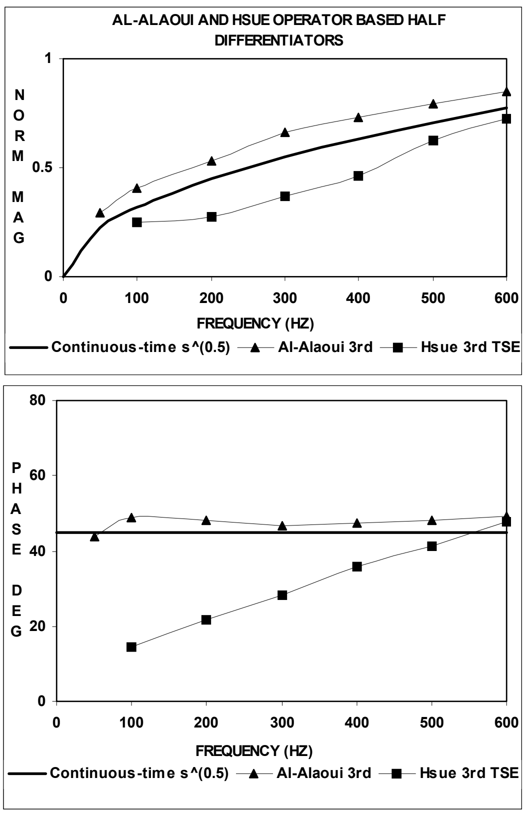

Fig. 18. SC results: Frequency response of Al-Alaoui and Hsue operator based half differentiators.

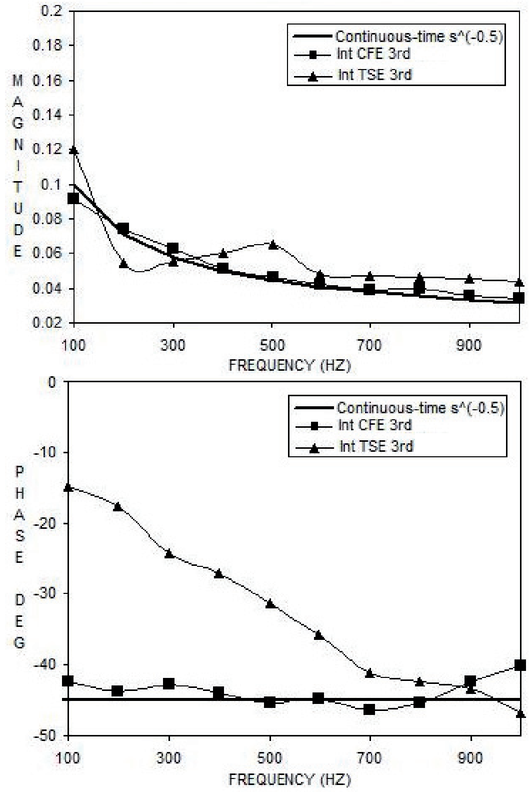

Fig. 19. SC results: Frequency responses of Al-Hsue operator based half integrator.

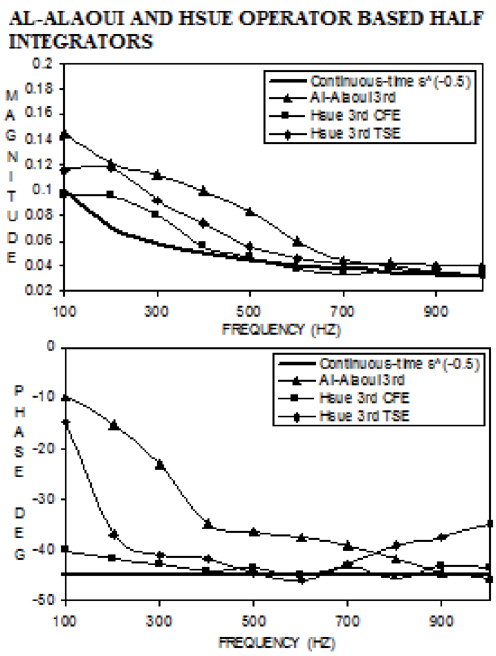

Fig. 20. SC results: Frequency response of Al-Alaoui and Hsue operator based half integrators

3) The frequency response of the third order half integrator based on the Al-Hsue operator obtained using TSE and CFE are shown in Fig. 19.

- The magnitude plot of the 3rd order half integrator obtained by CFE and TSE of the Al-Hsue operator follow with the theoretical results of continuous-time domain half integrator as well as the MATLAB simulation results presented in [19], but the CFE results match better.

- The phase of half integrator obtained using CFE approximates -45° over the frequency range 100 to 900 Hz with a maximum error of ±5°. The phase of half integrator obtained using TSE varies linearly from ~ -15° to ~ -50° over the frequency range 100 to 900 Hz.

4) The frequency responses of the third order half integrators based on the Al-Alaoui and the Hsue operator are shown in Fig. 20.

- The magnitude responses of both Al-Alaoui and Hsue operator based discretizations match with the MATLAB simulation results [19] and also with the theoretical result of half integrator in continuous-time domain, but best matching occurs for the Al-Hsue operator (CFE) based discretization of half integrator.
- The phase of the Al-Alaoui operator based third order integrator varies linearly in the frequency range 100 to 900 Hz, and that of the Hsue operator based third order half integrator realization using CFE is also ~-45°over the frequency range 100 to 900 Hz with a maximum error of ±5°.

Higher order models of the Al-Hsue operator based fractional-order differentiator can also be easily realized using the proposed method, but the complexity of the SC circuit increases.

6. Conclusions

In this paper, we have developed switched capacitor realizations of half differentiator and half integrator ($s^{\pm r}$; $r = ½$) based on the Al-Hsue operator which has improved performance over its parent operators. This method can be extended to develop fractional-order models and their switched capacitor realizations for different values of r; $\forall -1 < r < 1$.

The sampling period for all the approximations in this paper is chosen as 0.00 1s. However, approximate mathematical models of fractional-order differentiators and fractional-order integrators can be obtained for different values of T.

It is observed that the Spice results of the discretizations of Al-Hsue operator for half differentiators and half integrators show relatively better results as compared to its originators.

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