Analysis and Synthesis of the Digital Structures by the Matrix Method

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Abstract. *This paper presents a general matrix algorithm for analysis of digital filters. The method proposed in this paper allows not only the analysis of the digital filters, but also the construction of new structures of the canonic or non-canonic digital filter. Equivalent filters of different structures can be found according to various matrix expansions. The structures can be calculated even from transfer function or from state-space matrices and with the additional advantage of requiring minimum number of shifting elements. Traditional research methods are not able to construct the system with a minimum of the shifting operations.*

Keywords

Digital filter design, matrix method, analysis, filter synthesis.

1. Introduction

The digital system with multiple input and output is demonstrated in Fig. 1. It is very important for calculating state matrices **A**, **B**, **C** and **D** of the circuit to number the nodes in the order shown in the Fig. 1. The first numbers must be put to the inputs of the circuit. Following numbers must be placed to the output of the delay elements (z^{-1}) and the rest of numbers must be put to the output of the adders. In general the digital system with multiple input and output in Fig. 1 can be described by the following equations [1] [2]:

$$\begin{aligned} 0 &= -\mathbf{Y}(z)+\mathbf{F}_{YX}\mathbf{X}(z)+\mathbf{F}_{YU}\mathbf{U}(z)+\mathbf{F}_{YV}\mathbf{V}(z)\\ 0 &= -\mathbf{U}(z)+\mathbf{F}_{UX}\mathbf{X}(z)+\mathbf{F}_{UU}\mathbf{U}(z)+\mathbf{F}_{UV}\mathbf{V}(z)\\ 0 &= -\mathbf{V}(z)+\mathbf{F}_{VX}\mathbf{X}(z)+\mathbf{F}_{VU}\mathbf{U}(z)+\mathbf{F}_{VV}\mathbf{V}(z) \end{aligned} \quad (1)$$

or in matrix form (2), [2]

$$\mathbf{N}_S \cdot \begin{bmatrix} \mathbf{X}(z)\\ \mathbf{Y}(z)\\ \mathbf{U}(z)\\ \mathbf{V}(z) \end{bmatrix} = \mathbf{0} \quad (2)$$

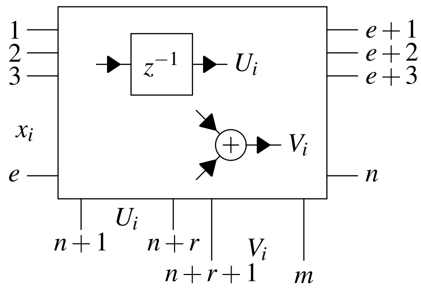

Fig. 1. Discrete structure with multiple input and output.

where $\mathbf{N}_S$ in (2) is the signal-flow matrix that represents the signal-flow graph of a digital system with multiple inputs and multiple outputs

$$\mathbf{N}_S = \left[\begin{array}{c|c|c|c} \mathbf{F}_{(YX)} & -\mathbf{I} & \mathbf{F}_{(YU)} & \mathbf{F}_{(YV)}\\ \hline \mathbf{F}_{(UX)} & \mathbf{0} & \mathbf{F}_{(UU)}-\mathbf{I} & \mathbf{F}_{(UV)}\\ \hline \mathbf{F}_{(VX)} & \mathbf{0} & \mathbf{F}_{(VU)} & \mathbf{F}_{(VV)}-\mathbf{I} \end{array}\right]. \quad (3)$$

X(z) is a vector of the input signals X_i, **Y**(z) is a vector of the output signals Y_i. $\mathbf{U}(z)$ is a vector of the delay elements output signals U_i and $\mathbf{V}(z)$ is a vector of the adders output signals V_i, see Fig 1. $\mathbf{F}_{YX}$ is the transfer matrix output/input, $\mathbf{F}_{YX} = \mathbf{Y}(z)/\mathbf{X}(z)$ if $\mathbf{U}(z) = \mathbf{V}(z) = 0$. If we reduce the signals in the outputs of the adders V_i in (2), we obtain

$$\mathbf{N}_E \cdot \begin{bmatrix} \mathbf{X}(z)\\ \mathbf{Y}(z)\\ \mathbf{U}(z) \end{bmatrix} = \mathbf{0} \quad (4)$$

where $\mathbf{N}_E$ in (4) is a flow-state matrix and the matrices **A**, **B**, **C** and **D** are state matrices of the digital system presented in equation (5) as [3], [4], [5]

$$\mathbf{N}_E = \left[\begin{array}{c|c|c} \mathbf{D} & -\mathbf{I} & \mathbf{C}\\ \hline z^{-1}\mathbf{B} & \mathbf{0} & z^{-1}\mathbf{A}-\mathbf{I} \end{array}\right]. \quad (5)$$

In the flow-state matrix, the matrices **I** and **0** are the identity and zero matrices respectively. If we reduce the matrix equation (4), not taking into account the vector of the signals U_i, we get the expression

$$\mathbf{N}_T^{(2)} \cdot \begin{bmatrix} \mathbf{X}(z)\\ \mathbf{Y}(z) \end{bmatrix} = \mathbf{0} \quad (6)$$

where the transfer matrix $\mathbf{N}_T^{(2)}$ can be defined by (7) as [5], [6]

$$\mathbf{N}_T^{(2)} = \left[\ \mathbf{D}+\mathbf{C}\cdot(z\mathbf{I}-\mathbf{A})^{-1}\mathbf{B};\quad -\mathbf{I}\ \right]. \tag{7}$$

Sign $\mathbf{N_T}^{(2)} = \left[n_{21}^{(2)}; n_{22}^{(2)}\right]$,

then the element $n_{21}^{(2)}$ of the transfer matrix $\mathbf{N}_T^{(2)}$ is the transfer function H(z) and $n_{22}^{(2)} = -1$

$$n_{21}^{(2)} = H(z) = \mathbf{D}+\mathbf{C}\cdot(z\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}. \tag{8}$$

2. Analysis of Parallel Adapter

The block of the parallel adaptor [7] [8] and its signal-flow diagram is shown in Fig 2.

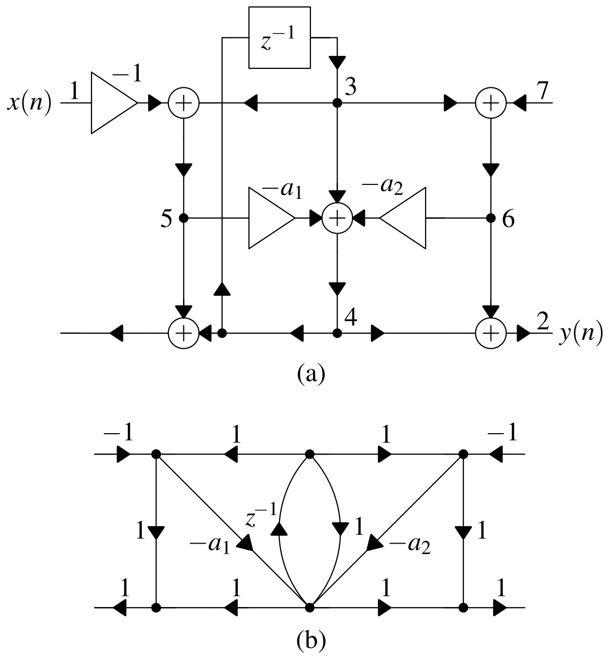

Fig. 2. Parallel adapter (a) and its signal flow graph (b).

To obtain signal-flow matrix of the parallel adapter in Fig. 2 (a) it is necessary to mark the nodes first. We label the input with the number one and output with the number two. To the outputs of the delay elements we shall put the number 3 and 4. Outputs of the adders are marked with numbers 5, 6 and 7. Signal-flow matrix $\mathbf{N}_S^{(7)}$ of the parallel adapter in Fig. 2(a) is

$$\mathbf{N}_S^{(6)} = \begin{array}{c} \\ 2\\3\\4\\5\\6 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cccccc} 0 & -1 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & z^{-1} & 0 & 0 \\ 0 & 0 & 1 & -1 & -a_1 & -a_2 \\ -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right] \end{array}. \tag{9}$$

The element $n_{45}^{(6)} = -a_1$ because the multiplier $-a_1$ is connected in the signal-flow graph in Fig. 2 (a) between the nodes 5 and 4. The element $n_{34}^{(6)} = z^{-1}$ because between the nodes 4 and 3 a delay element z^{-1} is connected. The element $n_{23}^{(6)} = 0$ because no direct path from the node 3 to node 2 occurs. Furthermore in the signal-flow matrix $n_{ii} = -1$ and $n_{i2} = 0$ for $i \geq 3$. In order to obtain the transfer function we must reduce the nodes 6, 5, 4 and 3. Reducing the 6th column and row we get $\mathbf{N}^{(5)}$

$$\mathbf{N}^{(5)} = \begin{array}{c} \\ 2\\3\\4\\5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \left[\begin{array}{ccccc} 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & z^{-1} & 0 \\ 0 & 0 & 1-a_2 & -1 & -a_1 \\ -1 & 0 & 1 & 0 & -1 \end{array}\right] \end{array} \tag{10}$$

and reducing 5th column and row we obtain $\mathbf{N}^{(4)}$

$$\mathbf{N}^{(4)} = \begin{array}{c} \\ 2\\3\\4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0 & -1 & 1 & 1 \\ 0 & 0 & -1 & z^{-1} \\ a_1 & 0 & 1-a_2-a1 & -1 \end{array}\right] \end{array}. \tag{11}$$

To obtain state-flow matrix we must reduce the vector of the signal V_3, it means the 4th column and row in the matrix $\mathbf{N}^{(4)}$

$$\mathbf{N}_E^{(3)} = \begin{array}{c} \\ 2\\3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \left[\begin{array}{ccc} a_1 & -1 & 2-a_1-a_2 \\ z^{-1}a_1 & 0 & -1+z^{-1}(1-a_1-a_2) \end{array}\right] \end{array}. \tag{12}$$

If we compare equations (12) and (5) we can construct the state-space scalars A, B, C and D as

$$D = a_1, \qquad C = 2-a_1-a_2,$$
$$B = a_1, \qquad A = 1-a_1-a_2.$$

Substituting state-space matrices to the equation (8) we obtain the transfer function $H(z)$

$$H(z) = a_1 + (2-a_1-a_2)\cdot(z-1+a_1+a_2)^{-1}\cdot a_1$$

$$H(z) = \frac{a_1+a_1z^{-1}}{1-z^{-1}(1-a_1-a_2)}$$

The same result is obtained by reduction of the last column and row in (12) as

$$\mathbf{N}_T^{(2)} = \begin{array}{c} \\ 2 \end{array}\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \left[\begin{array}{cc} \frac{-a_1-a_1z^{-1}}{-1+z^{-1}(1-a_1-a_2)} & -1 \end{array}\right] \end{array}. \tag{13}$$

The transfer function of the parallel adapter in Fig. 2 (a) is

$$H(z) = \frac{a_1+a_1z^{-1}}{1-z^{-1}(1-a_1-a_2)}. \tag{14}$$

The transfer function of the parallel adapter is derived also by means of a signal-flow graph reduction or by Mason rules. In Fig. 3 there are loops-gains and transfer path input/output of the parallel adapter Fig. 2 (b). Using Mason rules (15) we get the transfer function (14)

$$H(z) = \frac{P_1\cdot d_1 + P_2\cdot d_2}{1-(S_1+S_2+S_3)}. \tag{15}$$

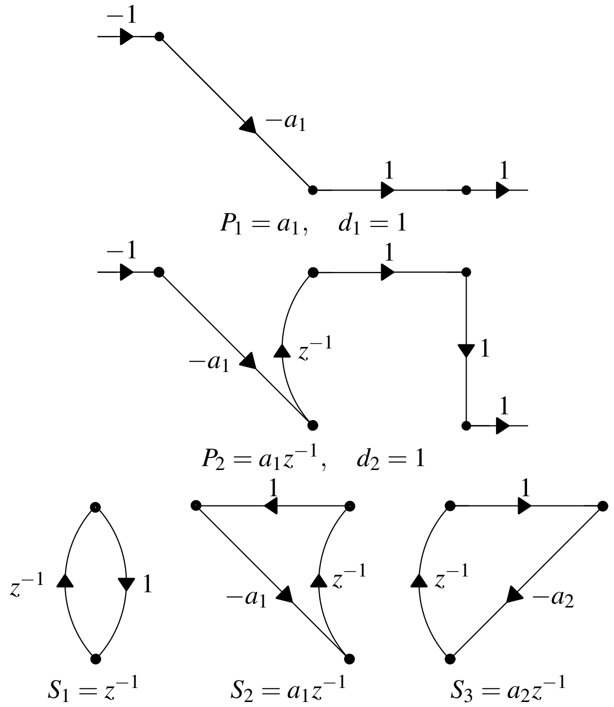

Fig. 3. Signal flow graph and its loop-gains.

2.1 Example of Digital Filter Analysis

In this section we shall calculate the transfer function of the circuit in Fig. 4. As the first step we shall calculate signal-flow matrix $\mathbf{N}_S^{(7)}$ of the circuit [9].

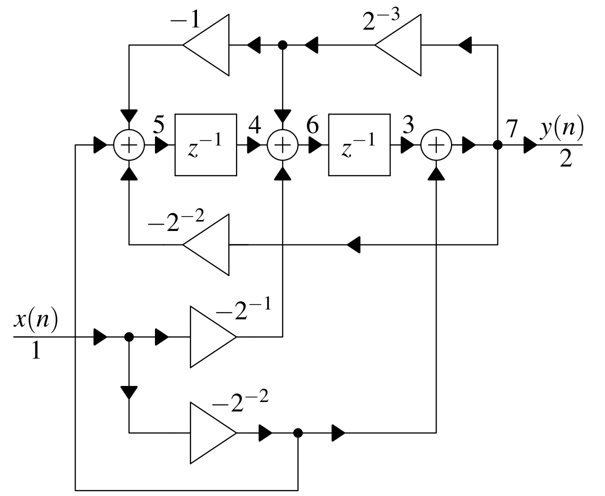

Fig. 4. Filter with four shifting elements and two delay elements.

$$\mathbf{N}_S^{(7)} = \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \left[\begin{array}{ccccccc} 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & z^{-1} & 0 \\ 0 & 0 & 0 & -1 & z^{-1} & 0 & 0 \\ 2^{-2} & 0 & 0 & 0 & -1 & 0 & -2^{-2}-2^{-3} \\ 2^{-1} & 0 & 0 & 1 & 0 & -1 & 2^{-3} \\ 2^{-2} & 0 & 1 & 0 & 0 & 0 & -1 \end{array} \right] \end{array} \quad (16)$$

To obtain $\mathbf{N}^{(6)}$ we must reduce the node 7, it means the last column and the last rows. Reduction of the matrices can be done by the relation

$$n_{ij}^{n-1} = \frac{n_{ij}^n n_{nn}^n - n_{in}^n n_{nj}^n}{n_{nn}^n} \quad (17)$$

where $i = 2,3,\dots n$ and $j = 1,2,3,\dots n$.

$$\mathbf{N}^{(6)} = \begin{bmatrix} 0.25 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & z^{-1} \\ 0 & 0 & 0 & -1 & z^{-1} & 0 \\ 0.15625 & 0 & -0.375 & 0 & -1 & 0 \\ 0.53125 & 0 & 0.125 & 1 & 0 & -1 \end{bmatrix} \quad (18)$$

Similarly we obtain $\mathbf{N}^{(5)}$ and $\mathbf{N}^{(4)}$.

$$\mathbf{N}^{(5)} = \begin{bmatrix} 0.25 & -1 & 1 & 0 & 0 \\ 0.53125z^{-1} & 0 & -1+0.125z^{-1} & z^{-1} & 0 \\ 0 & 0 & 0 & -1 & z^{-1} \\ 0.15625 & 0 & -0.375 & 0 & -1 \end{bmatrix} \quad (19)$$

The matrix $\mathbf{N}_E^{(4)}$ is flow-state matrix. From the flow-state matrix we can calculate state matrices **A**, **B**, **C** and scalar D, (21).

$$\mathbf{N}_E^{(4)} = \begin{bmatrix} 0.25 & -1 & 1 & 0 \\ 0.53125z^{-1} & 0 & -1+0.125z^{-1} & z^{-1} \\ 0.15625z^{-1} & 0 & -0.375z^{-1} & -1 \end{bmatrix} \quad (20)$$

$$D = 0.25 \qquad \mathbf{C} = [1 \quad 0]$$
$$\mathbf{B} = \begin{bmatrix} 0.53125 \\ 0.15625 \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 0.125 & 1 \\ -0.375 & 0 \end{bmatrix} \quad (21)$$

Substituting state-space matrices into (8) we get (22) and the transfer function (23)

$$H(z) = 0.25+$$
$$+[1 \quad 0]\left(z\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.125 & 1 \\ -0.375 & 0 \end{bmatrix}\right)^{-1} \cdot \begin{bmatrix} 0.53125 \\ 0.15625 \end{bmatrix} \quad (22)$$

Calculating equation (22) the transfer function of the circuit in Fig. 4 is

$$H(z) = \frac{0.25+0.5z^{-1}+0.25z^{-2}}{1-0.125z^{-1}+0.375z^{-2}} = $$
$$= \frac{2^{-2}+2^{-1}z^{-1}+2^{-2}z^{-2}}{1-2^{-3}z^{-1}+(2^{-2}+2^{-3})z^{-2}}. \quad (23)$$

3. Realization of the Circuit

This algorithm can be used also for realization of the circuit from state-space matrices (21). Provided that the state-space matrices (21) are known we can write the state-flow matrix $\mathbf{N}_E^{(4)}$ (20). This state-flow matrix has four columns and three rows. In this matrix we can expand one row and column to obtain matrix $\mathbf{N}^{(5)}$ with five columns and four rows (19). To obtain the matrix $\mathbf{N}^{(5)}$ with five columns and four rows (19) from the matrix $\mathbf{N}_E^{(4)}$ which contains four columns and three rows, we must select elements in the last column and row of the matrix $\mathbf{N}^{(5)}$. For example if we choose the element $n_{45}^{(5)}$ of the matrix $\mathbf{N}^{(5)}$ z^{-1} and $n_{51}^{(5)} = 2^{-2} - 2^{-2}(2^{-2} + 2^{-3}) = 0.15625$ then $n_{(41)}^{(5)}$=0. If we choose the element $n_{25}^{(5)}$=0, then the first row of the matrix $\mathbf{N}^{(5)}$ and $\mathbf{N}_E^{(4)}$ remains unchanged. In case of choosing the element $n_{54}^{(5)}$=0 of the matrix $\mathbf{N}^{(5)}$ then the forth columns of $\mathbf{N}^{(5)}$ and $\mathbf{N}_E^{(4)}$ remain unchanged. Similarly we can obtain the matrix $\mathbf{N}^{(6)}$ (18) and $\mathbf{N}_S^{(7)}$ (16). Matrix $\mathbf{N}_S^{(7)}$ is the signal flow matrix because all of the elements $n_{ij}^{(7)}$ of the matrix $\mathbf{N}_S^{(7)}$ are simple expressions. From the signal flow matrix the circuit can be sketched. Element $n_{36}^{(7)} = z^{-1}$ must be connected in the circuit, Fig. 4, between the nodes 6 and 3. Element $n_{67}^{(7)} = 2^{-3} = 0.125$ must be connected in the circuit, Fig. 4, between the nodes 7 and 6. Digital structure that corresponds to the signal-flow matrix $\mathbf{N}_S^{(7)}$ is presented in Fig. 4. Equation (24) can be used for the expansion of matrixes

$$n_{ij}^{n} = n_{ij}^{n-1} - n_{in}^{n} n_{nj}^{n} = 0 \tag{24}$$

where the elements of n_{in}^{n} and n_{nj}^{n} can be chosen. In the next paragraph we shall demonstrate how the circuit is obtained if the state space matrices are known.

4. Design of the Third Order Direct Form State-Space Structure

In this example the procedure of circuit realization described in Section 3 is explained. The following example demonstrates the proposal of the third order State-Space structure. State matrices of the third order State-Space filter can be written in the form [10], [11]

$$\mathbf{D} = d, \qquad \mathbf{C} = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix},$$
$$\mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}. \tag{25}$$

State-flow matrix (27) can be obtained by substitution (25) in general State-flow matrix (26)

$$\mathbf{N}_E^{(5)} = \left[\begin{array}{c|c|c} \mathbf{D} & -\mathbf{E} & \mathbf{C} \\ \hline z^{-1}\mathbf{B} & \mathbf{0} & -\mathbf{E} + \mathbf{A}z^{-1} \end{array}\right], \tag{26}$$

$$\mathbf{N}_E^{(5)} = \\ = \begin{bmatrix} d & -1 & c_1 & c_2 & c_3 \\ z^{-1}b_1 & 0 & -1 + a_{11}z^{-1} & a_{12}z^{-1} & a_{13}z^{-1} \\ z^{-1}b_2 & 0 & a_{21}z^{-1} & -1 + a_{22}z^{-1} & a_{23}z^{-1} \\ z^{-1}b_3 & 0 & a_{31}z^{-1} & a_{32}z^{-1} & -1 + a_{33}z^{-1} \end{bmatrix}. \tag{27}$$

To expand the State-flow matrix (27) which contains five columns and four rows in the matrix with six columns and five rows (29), we use the equation (28) only in the case if we choose $n_{nn} = -1$,

$$n_{ij}^{n} = n_{ij}^{n-1} - n_{in}^{n} n_{nj}^{n}. \tag{28}$$

If we choose the elements of the new matrix $n_{26}^{(6)} = n_{46}^{(6)} = n_{56}^{(6)} = 0$, then the first, third and fourth rows in the new matrix $\mathbf{N}^{(6)}$ remain unchanged (29).

$$\mathbf{N}^{(6)} = \\ = \begin{bmatrix} d & -1 & c_1 & c_2 & c_3 & 0 \\ n_{31}^{(6)} & n_{32}^{(6)} & n_{33}^{(6)} & n_{34}^{(6)} & n_{35}^{(6)} & n_{36}^{(6)} \\ z^{-1}b_2 & 0 & a_{21}z^{-1} & -1 + a_{22}z^{-1} & a_{23}z^{-1} & 0 \\ z^{-1}b_3 & 0 & a_{31}z^{-1} & a_{32}z^{-1} & 1 - a_{33}z^{-1} & 0 \\ n_{61}^{(6)} & n_{62}^{(6)} & n_{63}^{(6)} & n_{64}^{(6)} & n_{65}^{(6)} & n_{66}^{(6)} \end{bmatrix}. \tag{29}$$

The elements of the matrix (29), $n_{61}^{(6)}$, $n_{62}^{(6)}$, $n_{63}^{(6)}$, $n_{64}^{(6)}$, $n_{65}^{(6)}$, $n_{66}^{(6)}$ and $n_{36}^{(6)}$ can be selected and the remaining elements $n_{31}^{(6)}$, $n_{32}^{(6)}$, $n_{33}^{(6)}$, $n_{34}^{(6)}$ and $n_{35}^{(6)}$ can be obtained by means of equation (28). If we choose

$$\begin{aligned} &n_{26}^{(6)} = 0, \quad n_{46}^{(6)} = 0, \quad n_{56}^{(6)} = 0, \quad n_{62}^{(6)} = 0, \\ &n_{66}^{(6)} = -1, \quad n_{65}^{(6)} = a_{13}, \quad n_{64}^{(6)} = a_{12}, \quad n_{63}^{(6)} = a_{11}, \\ &n_{36}^{(6)} = z^{-1}, \quad n_{61}^{(6)} = b_1, \end{aligned}$$

then the elements of the new matrix take the form

$$\begin{aligned} n_{31}^{(6)} &= n_{31}^{(5)} - n_{36}^{(6)} n_{61}^{(6)} = z^{-1}b_1 - z^{-1}b_1 &&= 0, \\ n_{32}^{(6)} &= n_{32}^{(5)} - n_{36}^{(6)} n_{62}^{(6)} = 0 - z^{-1}0 &&= 0, \\ n_{33}^{(6)} &= n_{33}^{(5)} - n_{36}^{(6)} n_{63}^{(6)} = -1 + a_{11}z^{-1} - a_{11}z^{-1} &&= -1, \\ n_{34}^{(6)} &= n_{34}^{(5)} - n_{36}^{(6)} n_{64}^{(6)} = z^{-1}a_{12} - z^{-1}a_{12} &&= 0, \\ n_{35}^{(6)} &= n_{35}^{(5)} - n_{36}^{(6)} n_{65}^{(6)} = z^{-1}a_{13} - z^{-1}a_{13} &&= 0 \end{aligned}$$

and we obtain the matrix $\mathbf{N}^{(6)}$

$$\mathbf{N}^{(6)} = \\ = \begin{bmatrix} d & -1 & c_1 & c_2 & c_3 & 0 \\ 0 & 0 & -1 & 0 & 0 & z^{-1} \\ z^{-1}b_2 & 0 & a_{21}z^{-1} & -1 + a_{22}z^{-1} & a_{23}z^{-1} & 0 \\ z^{-1}b_3 & 0 & a_{31}z^{-1} & a_{32}z^{-1} & -1 + a_{33}z^{-1} & 0 \\ b_1 & 0 & a_{11} & a_{12} & a_{13} & -1 \end{bmatrix}. \tag{30}$$

Similarly, we can obtain the matrices $\mathbf{N}^{(7)}$ and $\mathbf{N}^{(8)}$. After a very simple calculation, we can get the matrix (31) and signal-flow matrix (32). For example it is advantageous to choose the element $n_{47} = z^{-1}$, in the matrix $\mathbf{N}^{(7)}$, because each element in row 3 of the matrix $\mathbf{N}^{(6)}$ contains z^{-1}. Provided that we choose the matrix element n_{71} equal to b_2, the element n_{41} is equal to zero, marked by (0). The same procedure can be applied to matrix $\mathbf{N}^{(7)}$, equation (31), in order to get equation (32):

$$\mathbf{N}^{(7)} = \begin{bmatrix} d & -1 & c_1 & c_2 & c_3 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & z^{-1} & 0 \\ (0) & 0 & 0 & -1 & 0 & 0 & (z^{-1}) \\ z^{-1}b_3 & 0 & a_{31}z^{-1} & a_{32}z^{-1} & -1+a_{33}z^{-1} & 0 & 0 \\ b_1 & 0 & a_{11} & a_{12} & a_{13} & -1 & 0 \\ (b_2) & 0 & a_{21} & a_{22} & a_{23} & 0 & (-1) \end{bmatrix}, \tag{31}$$

$$\mathbf{N}^{(8)} = \begin{bmatrix} d & -1 & c_1 & c_2 & c_3 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & z^{-1} & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & z^{-1} & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & z^{-1} \\ b_1 & 0 & a_{11} & a_{12} & a_{13} & -1 & 0 & 0 \\ b_2 & 0 & a_{21} & a_{22} & a_{23} & 0 & -1 & 0 \\ b_3 & 0 & a_{31} & a_{32} & a_{33} & 0 & 0 & -1 \end{bmatrix}. \tag{32}$$

The digital structure that corresponds to the signal flow matrix $\mathbf{N}^{(8)}$ is presented in Fig. 5. From the State-Space filter of the third order in Fig. 5 the general structure for the State-Space filter of the arbitrary order can be derived.

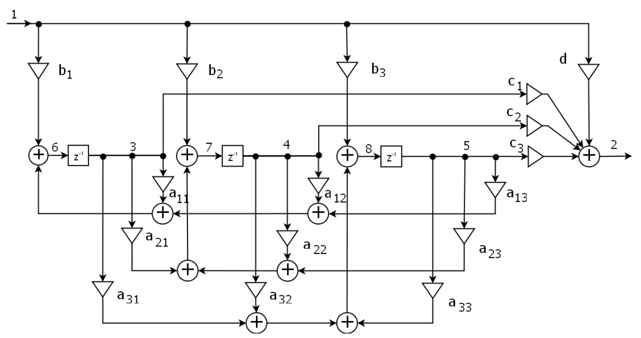

Fig. 5. State-Space filter of the third order.

The equivalent digital structure in Fig. 6 can be obtained from the State-Space filter in Fig. 5 by changing the summator to node, the node to summator, the input port to output port and changing directions of the multipliers.

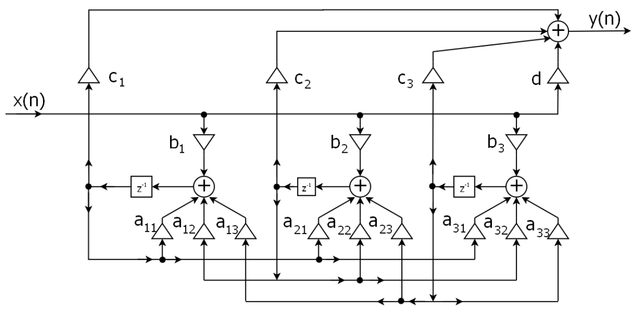

Fig. 6. State-Space filter of the third order in second form.

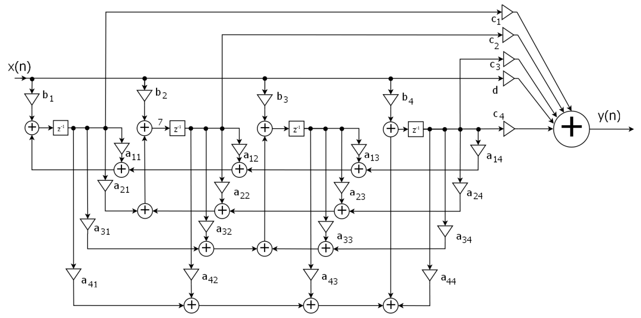

Fig. 7. Direct realization of the State-Space Filter.

Analogously we can write from $\mathbf{N}^{(8)}$ the matrix $\mathbf{N}^{(10)}$ for the State-Space digital filter of the fourth order (33). The digital structure that corresponds to the signal flow matrix $\mathbf{N}^{(10)}$ is presented in Fig. 7,

$$\mathbf{N}^{(10)} = \begin{bmatrix} d & -1 & c_1 & c_2 & c_3 & c_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & z^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & z^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & z^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & z^{-1} \\ b_1 & 0 & a_{11} & a_{12} & a_{13} & a_{14} & -1 & 0 & 0 & 0 \\ b_2 & 0 & a_{21} & a_{22} & a_{23} & a_{24} & 0 & -1 & 0 & 0 \\ b_3 & 0 & a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 & -1 & 0 \\ b_4 & 0 & a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 & 0 & -1 \end{bmatrix}. \tag{33}$$

5. Conclusion

The method proposed in this paper allows not only analysis of digital networks but also construction of new digital filters. Equivalent filters of differing structures can be found according to various matrix expansions. However, some of these structures can be sensitive to the error of quantization. This matrix synthesis method of the digital structures seems to be laborious, but in fact it is very simple and the effects are satisfactory when seen from the analysis of the structures.

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