Theoretical Model of the Bistable Semiconductor Laser Diode Based on the Rate Equations

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Abstract. The paper describes theoretical and experimental results that enabled the authors to proof optical bistability behavior of a specially modified bistable semiconductor laser diode (BLD) created on a structure with a saturable absorption section. A mathematical model of the light-current characteristic, condition for bistability and the basic parameters of the hysteresis loop were derived by solving a system of three rate equations. That system was used for simulation of the light-current characteristic and conditions of bistability of the realized BLDs. For selected operating points of the simulated light-current characteristic the parameters of hysteresis loop and element values of the BLD electrical equivalent circuit for small signal variations were calculated. The bistability was experimentally measured by the new time method devised for impulse bistability verification (IBV). The basic measured and calculated parameters of the hysteresis loop of the BLD light-current characteristic were compared.

Keywords
Optical bistability, bistable semiconductor laser diode, rate equation.

1. Introduction

Recently, in the realm of integrated optoelectronics appear devices exploiting new principles. One of them is the bistable semiconductor laser diode (BLD), utilizing the effect of absorption-based optical bistability. Optical bistability is one of the effects that were first forecast and only later observed. Lasher attracted attention to optical bistability in 1964 [1] and it was confirmed experimentally by Basov in 1968 [2].

The absorption-based optical bistability of the multisection laser diode (LD) belongs to the nonlinear dynamic optical phenomena in LD radiation that are based on Q switching [3]. This physical mechanism is a result of nonlinear dependence of the absorption and gain coefficients of the absorbing and gain region of the divided active layer of the semiconductor LD and manifests itself in the LD radiation as self-pulsations or optical bistability. A few years ago several papers were published describing the origin of bistability in LDs comprising a double heterostructure that was purposefully modified by dividing the stripe contact for achieving the bistable mode. The papers differed in semiconductor types, ways of strip contact splitting and also according to physical mechanism of bistability [4], [5] and [7].

An important dependence expressing the bistable properties of the BLD is its transmission characteristic (light-current characteristic). Its analytic expression can be derived by solving three rate equations. The BLD transmission characteristic was theoretically and experimentally determined in [7], [8] and [9]. If we rewrite the sentence with proper ordering, it is possible to derive the principal condition for bistability appearance as well as the conditions necessary for the creation of pulsations in the LD radiation [10], [11].

This article deals with theoretical results and experiments that enabled the authors to mathematically and experimentally describe bistability of a specially modified double heterostructure Ga1-xAlxAs/GaAs LD with a saturable absorption section (see Section 2). The mathematical model of the light-current characteristic was derived by static solution of the rate equation and it was used to simulate the characteristics of the realized BLDs. The characteristics obtained by the numerical modeling were optimized according to the measurement results. Numerical values parameters of the BLD linear equivalent circuit for small signal variations were calculated for selected operating points of the simulated light-current characteristics.

To prove the bistability of the BLD a IBV method is proposed. Using this method, the principal parameters of the hysteresis loop in the light-current characteristic for the realized BLD samples are determined.

2. Realization and Measurement of the BLD

The double heterostructure (DH) laser diode with a split contact on the top p-side was realized by a standard
LPE (liquid phase epitaxy) process in our laboratory. Typical doping levels are given in Fig. 1. The device was realized in a classical ternary DH Ga$_{1-x}$Al$_x$As/Ga$_{1-y}$Al$_y$As/GaAs where $x=0.4$, $y=0.1$ and with multiple-divided stripe contact.

![Diagram of device](image)

Fig. 1. Ga$_{1-x}$Al$_x$As/ Ga$_{1-y}$Al$_y$As/GaAs double heterostructure with multiple-divided stripe contact.

The strip contact masking was arranged according to [5]. The distance between contact islets was increased from 10 to 20 $\mu$m. The axial length of the contact islet was 30 $\mu$m and width 10 $\mu$m (see Fig. 1).

To prove bistability, measurements were carried out during the first stage of work selecting only samples showing bistability. All the selected BLD samples operated in the impulse regime only. Because a BLD is a sequential device whose internal state depends on the amplitude and timing of driving pulses, the bistable behavior was verified by the method of gradual setting and resetting the device by a set of current pulses and observing the radiation response timing of the BLD by the IBV method. The block diagram of the measuring system is shown in Fig. 2.

![Block diagram of pulse method](image)

Fig. 2. Block diagram of pulse method for bistability verification IBV.

The knowledge of the waveforms enabled us to determine the BLD impulse light-current characteristic and the basic parameters of the hysteresis loop (see Fig. 6, Tab. 3).

Also the temperature dependence of the pulsed light-current characteristic was found. For this purpose a system for integral light-current characteristic measurement was used, recording the monitored optical output power on drive pulse current and temperature.

![Waveforms](image)

Fig. 3. The waveforms a) represent the BLD driving current pulses, the waveforms b) represent the optical output ($T_1$ and $T_2$ are setting and resetting times) [6].

The bistable characteristic depends on device temperature in the range of thirty degrees. The BLD threshold current values were $I_{th} = 140$ to 220 mA and hysteresis width $\Delta I_{th} = 5$ to 10 % $I_{th}$. The sizes of discontinuous jumps of the optical output were $\Delta P \approx 0.5$ to 3.5 mW, see Tab. 3. The transition times between the stable states were $t_s = 0.5$ to 1 ns, $t_f = 3$ ns. The amplitudes of the current pulses and the magnitude of the radiation transition from the lower to the higher level were used to determine the static and dynamic parameters of the BLD hysteresis loop, used as initial values for the light-current characteristic simulation, shown in Fig. 6.
3. Mathematical Model Using the Rate Equations

In this section a mathematical model of the light-current characteristics is derived and was justified by comparison to the experimentally measured data obtained by BIV method. The model was based on the set of the rate equations [4]. We analyzed the stationary solution of the three rate equations system from [4]. The first equation was formulated for the density of minority carriers in the gain section (1), the second one for those in the absorbing section (3) and the third one for the density of photons in the lasing mode (5). Measurements show that, when this laser is based for stable operation, the light is emitted into one longitudinal mode. This simple optical behavior can be modeled by one rate equation for the density of photons in the lasing mode. The carrier and photons densities have been averaged over the length of the device. The three equations are given (1), (3) and (5),

\[
\frac{dN_g}{dt} = \frac{I_g}{eV_g} - g_g \left( N_g - N_{og} \right) N_{ph} - \frac{N_g}{\tau_g},
\]

where

\[
\frac{1}{\tau_g} = B_g \left( N_g + N_a \right),
\]

\[
\frac{dN_a}{dt} = \frac{I_a}{eV_a} - g_a \left( N_a - N_{oa} \right) N_{ph} - \frac{N_a}{\tau_a},
\]

where

\[
\frac{1}{\tau_a} = B_a \left( N_a + N_k \right),
\]

\[
\frac{dN_{ph}}{dt} = g_g \alpha_1 \left( N_g - N_{og} \right) N_{ph} + g_a \alpha_2 \left( N_a - N_{oa} \right) N_{ph} + \\
+ \beta \left[ \alpha_1 B_g \left( N_g + N_a \right) N_{ph} + \alpha_2 B_a \left( N_a + N_k \right) N_{ph} \right] - \frac{N_{ph}}{\tau_{ph}}.
\]

We analyzed the stationary solution of equation (1) – (5) in [6]. The light-current characteristics given by (7), (8) and (9) were derived from the static solution of the rate equations under condition (6):

\[
\frac{dN_g}{dt} = \frac{dN_a}{dt} = \frac{dN_{ph}}{dt} = 0.
\]

The static solutions of the rate equations are parametric quadratic system

\[
I_g = f_i(N_{ph}, N_g),
\]

\[
N_g = f_2(N_{ph}, N_a)
\]

and

\[
N_a = f_3(N_{ph}),
\]

where we assume \( I_g = 0 \). A parameter of the system (7), (8), (9) is \( \alpha_2 / \alpha_1 \).

\[
I_g = eV_g B_g \left[ N_g^{\frac{1}{2}} + \left( \frac{g_a N_{ph} + g_g N_{ph}}{B_g} \right) N_g \right],
\]

\[
N_g = \left( \frac{g_a N_{ph} + N_g}{2} \right) \pm \sqrt{\left( \frac{g_a}{2B_g} N_{ph} \right)^2 + N_g^2} \]

\[
N_a = \left( \frac{g_g}{2B_g} N_{ph} \right) \pm \sqrt{\left( \frac{g_g}{2B_g} N_{ph} \right)^2 + N_{ph}^2}.
\]

For a graphical presentation of the results we use the following normalization (10), (11):

\[
I_{ph} = eV_g B_g \left[ \frac{g_a \alpha_1}{g_g \alpha_2} N_{oa} + \frac{1}{\alpha_g \tau_{ph}} \right],
\]

\[
N_{ph} = \left( \frac{g_a \tau_{ph}}{g_a \tau_{ph}} \right)^{1/2}.
\]

A calculated light-current characteristic is shown in Fig. 4.

Fig. 4. Normalized static light-current characteristics with parameter \( \alpha_1 / \alpha_2 \).

The physical constants were determined by dynamic method measuring of the cut-off frequency of the modulation characteristic of a LD, excited below the threshold and by measuring frequency of the photon-electron resonance of samples excited above the threshold. A LD was prepared by identical technology, but without absorption regions. The values of the physical constants were optimized throughout the simulation process, see Tab. 1.
4. The Small Signal Equivalent Circuit of the BLD

An equivalent electrical small signal circuit for the BLD in Fig. 5 was derived for selected operating points A, B and D on the simulated light-current characteristics shown in Fig. 6. The element values of the circuit are specified in Tab. 2.

This circuit is described by a set of circuit equations (12) through (14), which are obtained from small signal analysis of equations (1) to (3) according to [4]. Based on the electrical equivalent circuit knowledge the radiation stability of a semiconductor LD can be analyzed or derived the signal drivers aimed at suppressing the unwanted dynamic effects in the radiation response of a LD or BLD, respectively.

\[
\frac{du_g}{dt} = \frac{1}{C_g} \left( I_g - \frac{u_g}{R_g} \right) - i_L, \tag{12}
\]

\[
\frac{du_a}{dt} = \frac{1}{C_a} \left( I_a - \frac{u_a}{R_a} + K_1 i_L \right), \tag{13}
\]

\[
\frac{di_L}{dt} = \frac{1}{L} \left( u_g + N_1 u_a - R_a i_L \right) \tag{14}
\]

where

\[
R_g = \left[ C_g \left( \frac{1}{\tau_g} + g_a \bar{N}_{ph} \right) \right]^{-1}, \tag{15}
\]

\[
R_a = \left[ C_a \left( \frac{1}{\tau_a} + g_a \bar{N}_{ph} \right) \right]^{-1}, \tag{16}
\]

\[
C_g = \frac{e^2 V_g N_{\bar{g}}}{m_k kT}, \tag{17}
\]

\[
C_a = \frac{e^2 V_a N_a}{m_k kT}, \tag{18}
\]

\[
L = \left[ C_g \alpha_1 g \left( \bar{N}_g - N_{\bar{ag}} \left( \frac{\beta}{\tau_g} + g_a \bar{N}_{ph} \right) \right) \right]^{-1}, \tag{19}
\]

\[
R_s = L \left[ \alpha_1 g \frac{I_{gh} \tau_g}{e V_g} - \alpha_1 g \bar{N}_g - \alpha_2 g_a \bar{N}_a \right]. \tag{20}
\]
\[ B = \alpha_1 g N_{og} \tau_{ph}, \]  
\[ C = \alpha_2 g N_{oa} \tau_{ph}, \]  
\[ x = \frac{N_{ph}}{N_{phs}}, \]  
\[ K = \frac{\alpha_1 \tau_{ph} I_a}{N_{phs} e V_a}. \]

We have analyzed function (21) under the condition \( \beta = K = 0 \). Equation (27) was differentiated and its roots were found revealing two real local extrema (28) – a minimum and a maximum.

\[ x^2 + 2x + \left[ 1 + C(1 - A) \right] = 0, \]  
\[ x_{1,2} = \pm \left[ C(A-1) \right]^{1/2} - 1. \]

By applying formulas (22), (24) and (25) we express (29) as

\[ I_{g_{\text{max}}} = I_{gh} = \frac{eVN_{phs}}{\tau_{ph}} A(1+B+C), \]  
\[ I_{g_{\text{min}}} = \frac{eVN_{phs}}{\tau_{ph}} \left[ A(1+B) + C + 2[C(A-1)]^{1/2} - 1 \right]. \]

The parameters of the hysteresis loop \( \Delta I_{gh} \) and \( \Delta N_{ph} \) is connected with points B, C and D (see Fig. 6) and are simply related as follows

\[ \Delta I_{gh} = I_{g_{\text{max}}} - I_{g_{\text{min}}}. \]

The measured and simulated parameters of the light-current characteristic hysteresis loop are shown in Tab. 3, the values of the physical constants are specified in Tab. 4.

<table>
<thead>
<tr>
<th></th>
<th>Measured</th>
<th>Simulated</th>
</tr>
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<tbody>
<tr>
<td>( \Delta P ) [mW]</td>
<td>5.6</td>
<td>5.2</td>
</tr>
<tr>
<td>( \Delta I_a ) [mA]</td>
<td>14</td>
<td>16</td>
</tr>
</tbody>
</table>

\[ \text{Tab. 3. The measured and simulated parameters of the light-current hysteresis loop.} \]

From extreme analysis of function (21) the condition for origination of a BLD bistability mode after (27), (28) has the following form

\[ \frac{[C(A-1)]}{1} > 1. \]

This can be expressed as

\[ \frac{R_a}{R_{ac}} > 1 \]

where \( R_a \) is the rate of photon absorption in the saturable absorption centers in the absorption regions expressed by (36), and \( R_{ac} \) is the critical photon absorption rate at the non-saturable centers in the absorption regions of a multiple-divided strip contact determined by relation (37). This condition can be expressed by stating that non-linear behavior of the two areas must be distinct enough from each other and that the origination of a LD bistable mode must be conditioned by the fact that the photon absorption rate at the saturable absorption centers must be greater than the absorption rate caused nonsaturable optical losses.
6. Conclusion

A mathematical model for bistable laser diode - BLD was created using the system of three rate equations and derived the relationship for the light-current characteristic. The model uses physical constants the values of which were determined by methods employing modulation and optimized by simulation. A BLD was designed and confirmed that the presence ofinhomogeneous excitation caused by nonuniform contacts can produce bistability in the output radiation of BLD. A simple model is analyzed which displays the main features ofsemiconductor laser with inhomogeneous current injection – BLD, namely a hysteresis in the light current characteristic and negative resistance and capacitance, which isoptoelectronic in origin across the absorber sections. A small signal analysis of this model leads to an electrical equivalent circuit which clarifies the frequency limitations of the device up to several hundred megahertz.

The physical constants values of the model were optimized in relation to the measured values of the static light-current characteristic $I_{ph}, A_{gh}$ and $\Delta P$. The comparison of measured and optimized light current function is presented in Fig. 6. The optimized values of physical constants are shown in Tab. 1. For selected points $P_1, P_2, P_{max}$ the elements of the small signal equivalent electrical circuit depicted in Fig. 5 are given in Tab. 2. The circuit elements $R_g$ and $C_g$ characterizing the influence of absorption areas have negative values. This means that the BLD produces unstable radiation in an area of excitation evident in its light-current characteristics. The small signal equivalent electrical circuit contains only negative $R_g$ for the self-pulsation type instability of the LD. The bistable mode can only exist when a sufficiently large negative $C_g$ capacitance is present in addition to the negative $R_g$. If this capacitance is not large enough, the device pulsates with a frequency represented by the $L$ and $C_g$ values.

BLD is a perspective element; it can be used as an electro-optical limiter, a simple logic device and an optical memory element. However, its direct use in practice is hindered by some problems. The higher threshold current and strong temperature dependence of the bistable mode prevented the introduction of continuous bistable operating mode in the prepared samples.

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