Acceleration of Marching on in Time Method for TD-EFIE by Equivalent Dipole Moment Method and its Analysis

Jaroslav LACIK

Dept. of Radio Electronics, Brno University of Technology, Purkynova 118, 612 00 Brno, Czech Republic

lacik@feec.vutbr.cz

Abstract. *The paper is focused on the application of the equivalent dipole moment (EDM) method to accelerate the marching on in time (MOT) method for the time domain electric field integral equation (TD-EFIE). The implicit MOT scheme with the EDM method for the TD-EFIE is derived and analyzed. It is shown that the derived scheme is faster than the conventional one, even if it is not used for modeling electrically large structures. Since the conventional implicit MOT scheme for the TD-EFIE is sensitive to small changes of its coefficients, the full-value using of the MOT scheme with the EDM approximation requires an appropriate technique (e.g. a preconditioning) to obtain a well-conditioned scheme.*

Keywords

Marching on in time method, time domain electric field integral equation, equivalent dipole moment method.

1. Introduction

For transient analysis of antennas or scatterers, the time domain electric field integral equation (TD-EFIE) can be solved. For its numerical solution, the marching-on-in-time (MOT) [1] method can be applied, so the TD-EFIE is solved in space by the method of moments (MoM).

The MoM employing triangular surface patches is a versatile technique for variety EM problems. However, one of its major disadvantages is a high computational complexity which is proportional to the square of a number of surface unknowns since the electromagnetic coupling between all discretized elements has to be considered.

In recent years, several techniques for decreasing the computational complexity of the MOT method have been proposed. The plane wave time domain (PWTD) algorithm [2] and the time domain adaptive integral equation method (TD-AIM) [3] belong between the most popular ones. Although the computation complexity of these methods is lower than the conventional MOT approach, they are suitable mainly for modeling electrically large structures.

This paper deals with the acceleration of the conventional MOT method for the TD-EFIE by the equivalent dipole moment (EDM) method. The EDM method has been originally applied for the acceleration of the MoM in the frequency domain [4] - [6]. In this paper, the EDM approach is applied on the MOT method, and the resultant scheme is analyzed. The basic idea of the EDM method consists in computing the interaction between the source and testing function locations directly (the approximation of the radiated field by an infinitely small dipole with the equivalent moment) for a separation distance larger than the nominal value, without evaluating double integrals.

The paper is organized as follows: Section 2 presents the MOT scheme with the EDM method derived for the TD-EFIE. Section 3 investigates the limitation of the derived method. Section 4 presents numerical examples, and section 5 concludes the paper.

Note that the whole paper, and the investigation carried out in this paper is limited to open perfectly electric conducting structures to avoid troubles with internal resonances.

2. MOT Scheme with EDM Method for TD-EFIE

Let us analyze the scattering of an open perfectly conducting structure illuminated by a transient electromagnetic wave. The TD-EFIE is solved by the method of moments. The surface of the analyzed structure is approximated by planar triangular patches, and RWG [7] functions are used to expand the spatial variation of the electric current density. In time, the TD-EFIE is approximated by central finite differences. The goal is to find the surface current due to the incident field. The resultant implicit MOT scheme can be written in the following matrix form [1]

$$[\lambda_{mn}][I_m(t_k)] = [\beta_m(t_k)] = \Delta t[V_m(t_{k-1/2})] - \Delta t\left[\sum_{n=1}^{N_S} \chi_{mn}(t_k)\right] \quad (1)$$

where $[\lambda_{mn}]$ denotes a matrix of time invariant coefficients, $[I_m(t_k)]$ is a column vector of the unknown current coefficients at time t_i, $[\beta_m(t_k)]$ is a column vector related to the incident field $V_m(t_{k-1/2})$ located at the m-th testing function, and the coefficient $\chi_{mn}(t_k)$ depending on the location of the m-th testing and the n-th source function and the known current coefficients from time t_0 to t_{k-1}, Δt is the length of the time step. Detailed derivation is given in [1].

The most time consuming part of the implicit scheme (1) is the evaluation of the coefficient $\chi_{mn}(t_i)$ at each time step which consists in computing surface integrals over testing and expansion functions involving Green's functions. These integrals for MOT schemes are usually computed by numerical or analytical-numerical techniques [8], [9]. However, in this paper to speed up the computing of the coefficient $\chi_{mn}(t_i)$, we exploit the equivalent dipole moment (EDM) method [4] originally proposed in the frequency domain.

The EDM method stands on the idea that if the size of triangles for approximating a body of an analyzed structure is sufficiently small, the fields radiated due to the current on a triangle pair may be approximated by radiation of an infinitely small dipole with the equivalent moment. The approximation is valid beyond the nominal value R_0 [4].

The equivalent moment for RWG function (Fig. 1) can be defined [7]

$$\mathbf{m}_n = \frac{l_n}{2}\left(\boldsymbol{\rho}_n^{c+} + \boldsymbol{\rho}_n^{c-}\right) = l_n\left(\mathbf{r}_n^{c-} - \mathbf{r}_n^{c+}\right) \qquad (2)$$

where $\boldsymbol{\rho}_n^{c\pm}$ is the vector between the free vertex and the centroid of the triangle pair $T_n^{\pm}$, with $\boldsymbol{\rho}_n^{c-}$ directed toward and $\boldsymbol{\rho}_n^{c+}$ directed away from the vertex, $\mathbf{r}_n^{c\pm}$ is the position vector of the centroid of $T_n^{\pm}$ with respect to the global origin, and the l_n is the length of the n-th common edge of the triangle pair $T_n^{\pm}$.

The radiation of the infinitely small dipole [10] after the transformation to the time domain can be described

$$\mathbf{E}(\mathbf{r},t) = \frac{\eta}{4\pi}\left[\frac{(\hat{\mathbf{r}}\cdot\mathbf{m}_n)\hat{\mathbf{r}} - \mathbf{m}_n}{cR}\frac{\partial I_0\left(t-\frac{R}{c}\right)}{\partial t} + \frac{3(\hat{\mathbf{r}}\cdot\mathbf{m}_n)\hat{\mathbf{r}} - \mathbf{m}_n}{R^2} I_0\left(t-\frac{R}{c}\right) + \frac{3(\hat{\mathbf{r}}\cdot\mathbf{m}_n)\hat{\mathbf{r}} - \mathbf{m}_n}{R^3}\int_0^{t-\frac{R}{c}} I_0(\tau)d\tau\right] \qquad (3)$$

where I_0 is the current on the infinitely small dipole, η is the intrinsic impedance of the medium, and c is the velocity of the wave propagation in that space, R=|$\mathbf{r}$-$\mathbf{r}$'| is the distance between the observation point $\mathbf{r}$ and the position of the infinitely small dipole $\mathbf{r}$', $\hat{\mathbf{r}} = (\mathbf{r}-\mathbf{r}')/R$ is the unit vector.

In order to speed up the scheme (1), let us approximate the contribution of the current at the n-th source function to the m-testing function by an infinitely small di

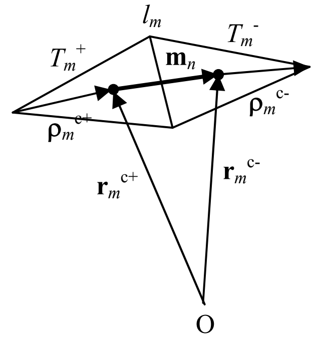


Fig. 1. Definition of the equivalent moment for RWG function.

pole (3) with the equivalent moment (2). After substituting R=R_{mn}=|$\mathbf{r}_m$-$\mathbf{r}_n$|, where the $\mathbf{r}_m$ and $\mathbf{r}_n$ are the position vectors of the center of the m-the and n-th edge (Fig. 2), respectively, and $\hat{\mathbf{r}} = \hat{\mathbf{r}}_{mn} = (\mathbf{r}_m - \mathbf{r}_n)/R_{mn}$ to (3), after the discretization of (3) in time in the same way as the scheme (1), and, finally, after the testing procedure (in the same way as in [4]), the coefficient $\chi_{mn}(t_i)$ can be expressed

$$\chi_{mn}(t_k) = \frac{\eta}{4\pi}\mathbf{m}_m \cdot \left[\frac{(\hat{\mathbf{r}}_{mn}\cdot\mathbf{m}_n)\hat{\mathbf{r}}_{mn} - \mathbf{m}_n}{cR_{mn}\Delta t}\left(I_n\left(t_k-\frac{R_{mn}}{c}\right) - I_n\left(t_{k-1}-\frac{R_{mn}}{c}\right)\right) + \frac{3(\hat{\mathbf{r}}_{mn}\cdot\mathbf{m}_n)\hat{\mathbf{r}}_{mn} - \mathbf{m}_n}{2R_{mn}^2}\left(I_n\left(t_k-\frac{R_{mn}}{c}\right) + I_n\left(t_{k-1}-\frac{R_{mn}}{c}\right)\right) + \frac{3(\hat{\mathbf{r}}_{mn}\cdot\mathbf{m}_n)\hat{\mathbf{r}}_{mn} - \mathbf{m}_n}{2R_{mn}^3}\left(Q_n\left(t_k-\frac{R_{mn}}{c}\right) + Q_n\left(t_{k-1}-\frac{R_{mn}}{c}\right)\right)\right] \qquad (4)$$

where

$$Q_n\left(t_k-\frac{R_{mn}}{c}\right) = \int_0^{t_k-\frac{R_{mn}}{c}} I_n(\tau)d\tau . \qquad (5)$$

Although expression (3) for the radiation of the infinitely small dipole is valid at arbitrary distance from the dipole, the coefficient $\chi_{mn}(t_k)$ can be computed for the implicit MOT scheme (1) according to (4) only if the distance between the center of the m-th and n-th edge is larger, than the nominal value R_0. If this condition is not met, the approximation (4) cannot be used, and the coefficient $\chi_{mn}(t_k)$ have to be computed in the conventional way.

Note that the integral in (5) is computed numerically.

In order to meet the basic requirement of the EDM method in the time domain that the size of triangles for approximating the analyzed structure is sufficiently small, we have to compare the size of all triangles and the wavelength $\lambda(f_{max})$ at the maximum frequency f_{max} of the important part of the spectrum of the excitation pulse. The requirement can be met if the lengths of all triangle edges are much smaller than the wavelength $\lambda(f_{max})$. In accordance with the application of the EDM method in the frequency domain [4]-[6], we choose the average edge length of triangles comparable to $0.1\lambda(f_{max})$.

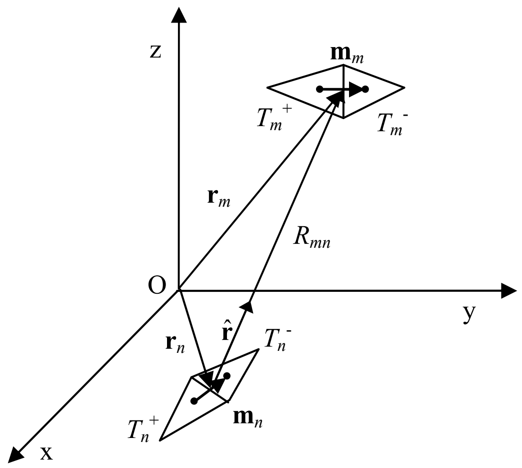


Fig. 2. Configuration of m-th and n-th triangular pair and associated vectors.

3. Analysis of MOT Scheme with EDM Method for TD-EFIE

Before the MOT scheme with EDM method is used, it has to be known how the conventional MOT scheme is sensitive to small change of its coefficients, and how accurate the approximation of the coefficient $\chi_{mn}(t_k)$ in the MOT scheme by the EDM method is. Subsequently, if it is possible, a rule for the determination of the nominal value R_0 should be defined.

3.1 Sensitivity Analysis of MOT Scheme

It is known that the frequency domain electric field integral equation (FD-EFIE) suffers from the low frequency breakdown [11], [12] which means that a reliable solution is difficult to find if the discretization becomes very fine in terms of subdivisions per wavelength. Finer discretization usually causes larger condition numbers of a system matrix, and consequently, an exploited numerical scheme for solving a given task is more sensitive on the evaluation of its coefficients. Thus, the low frequency breakdown can arise. Since TD-EFIE is the counterpart of the FD-EFIE, similar behavior can occur for MOT scheme for TD-EFIE [13]. To remedy this phenomenon, an appropriate technique (e.g. a preconditioning, or the modification of the MOT scheme to its hierarchical version [13]), can be used.

To avoid the above trouble without applying additional technique in this paper, let's carry out the condition number analysis of the MOT scheme (1), and define requirements on an excitation signal since the breakdown is a matter of low frequencies.

Note that the MOT scheme defined by (1) represents linear invariant discrete system, so the condition number of its system matrix can be computed after transforming the scheme to the Z-domain.

We carried out extensive numerical experiments, and computed the condition number for different structures which were located in free space. The analyzed structures were discretized with respect to the frequency f= 300 MHz (λ= 1 m) with the average edge length of triangles comparable to $0.1\lambda(f_{\max})$ as it was discussed at the end of section 2.

The results for three structures, the strip (2 m × 0.08 m), the square plates (1 m × 1 m), and the rectangular plate (2 m × 1 m), are depicted in Fig. 3. Obviously, the MOT scheme has large condition numbers at low frequencies as we expected. Similar results were observed for other numerical experiments. Considering mentioned facts and results of our investigation, we can conclude:

1. If the spectrum of the excitation signal has important components at the frequencies where the condition number of the system matrix is high (a low frequency region), all coefficients of the scheme (1) has to be computed with high accuracy. This fact imposes high demand on the accuracy of evaluation of the surface integrals of the MOT scheme (1) and on the scheme itself, otherwise, the scheme is unstable.
2. The MOT scheme (1) is less sensitive to small change of its coefficients at higher frequencies (low condition numbers). Thus, the troubles can be avoided by an excitation signal whose ratio of the maximum and minimum frequency of the important part of its spectrum is not high. According to the results of our numerical experiments, the ratio should not exceed 4 for the given discretization criterion (this ratio should not be exceeded for the MOT scheme with EDM method). It has to be stressed that although the extensive numerical experiments were carried out, their number was limited, so the given ratio is not general, and it is used only for the investigation in the rest of this paper.

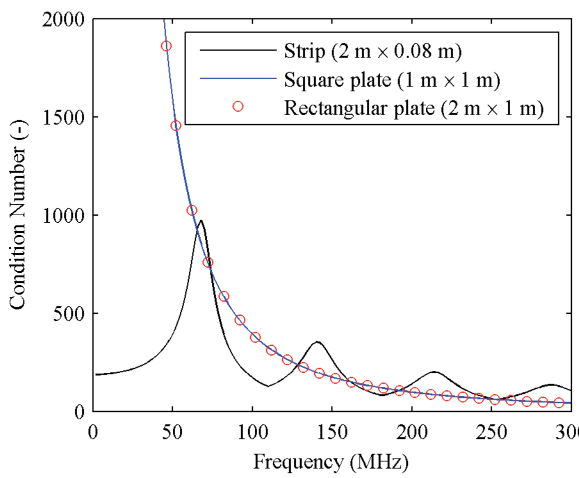


Fig. 3. Dependence of condition number on frequency for different structures.

3.2 Accuracy of Approximation of χ_{mn} by EDM Method and Determination of Nominal Value

The accuracy investigation is carried out on the analysis of the strip with the dimensions 2 m × 0.08 m. The body of the strip is modeled with respect to the frequency f = 300 MHz by 44 patches to meet the discretization criterion. Let's transform the coefficient $\chi_{mn}(t_i)$ to the Z-domain, firstly as it is defined for the scheme (1) in [1], and secondly as it is defined by the EDM approximation (4), and compute a relative error of the EDM approximation depending on the distance between the center of the m-th and n-th edge for the different lengths of the time step. The relative error is computed at the frequency f = 300 MHz where we expect the highest error because the electrical size of triangles is the largest.

From Fig. 4 we can see that for small distances between the centers of the edges, the relative error has downward tendency, but it is high. However, from a certain distance, the relative error is small, but not negligible. Evidently, this error depends on the length of the time step ($R_{\min}$ is the minimum distance between any two centers of triangular patches). Since the relative error is not negligible even for long distances, and depends on the length of the time step, the nominal value R_0 cannot be determined from such kind of investigation. Thus, it has to be proceeded in a different way, directly in the time domain.

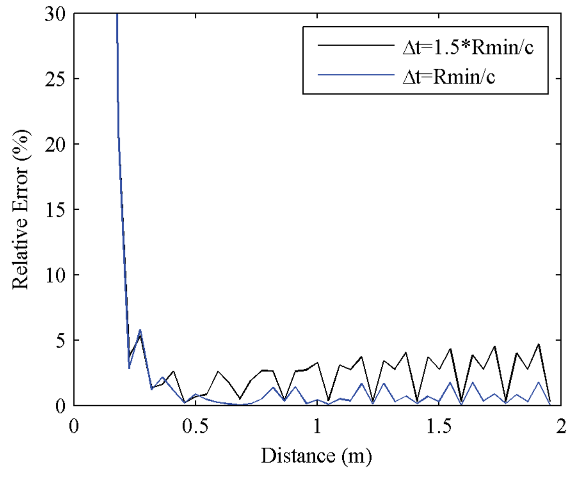


Fig. 4. Relative error of approximation (4) depending on the distance between the center of m-th and n-th edge.

Based on the results of our numerical experiments in the previous subsection that the ratio of the maximum and minimum frequency of the important part of the spectrum of the excitation pulse should not exceed 4 for the given discretization criterion and the modeled structures, we carried out extensive numerical experiments, and computed transient responses of different structures (the same ones as in the previous subsection) and compared the responses obtained by the MOT scheme (1), and by the MOT scheme with the EDM approximation (4) to find a relative error of those responses lower than 3 %. It was observed that the nominal value R_0 depends on the length of the time step, as it was expected, but even on the ratio of the maximum distance between any two centers of triangular patches $R_{\max}$ and the wavelength $\lambda(f_{\max})$.

The normalized nominal values R_0 are depicted in Fig. 5. Apparently, the MOT scheme with the EDM method saves more computational time in comparison to the conventional scheme (1), as the electrical size of an analyzed structure is larger, and the length of the time step is smaller since more interactions between the m-th and n- triangular pair is computed by the EDM method.

Note that the ratio $R_{\max}/\lambda(f_{\max})$ is very close to the electrical size of the analyzed structure at the maximum frequency of the important part of the excitation signal.

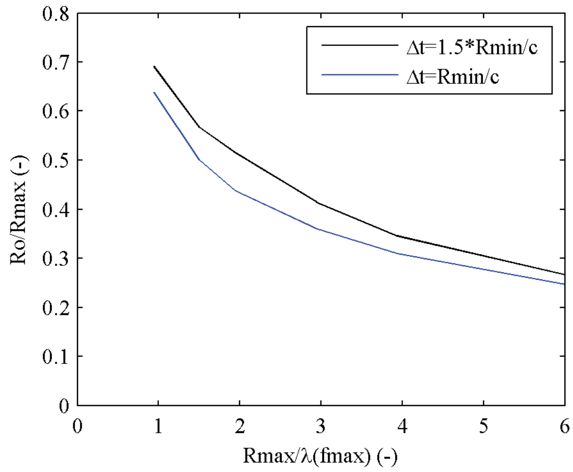


Fig. 5. Normalized nominal value.

4. Numerical Examples

Let's demonstrate the efficiency of the MOT scheme with the EDM method, and illuminate two strips of different lengths, a square plate, and a rectangular plate by a harmonic plane wave modulated by the Gaussian pulse

$$\mathbf{E}(\mathbf{r}^i,t) = \mathbf{E}_0 \frac{4}{\sqrt{\pi}Tc} e^{-\left[\frac{4}{T}\left(t-t_0-\frac{\mathbf{r}.\hat{\mathbf{k}}}{c}\right)\right]^2} \cos\left[2\pi f_0\left(t-t_0-\frac{\mathbf{r}.\hat{\mathbf{k}}}{c}\right)\right] \quad (6)$$

where $\hat{\mathbf{k}}$ is the unit vector in the direction of the propagation of the incident wave, $\mathbf{E}_0.\hat{\mathbf{k}} = 0$, $\mathbf{r}$ is a position vector relative to the origin, T is the width of the Gaussian pulse, c is the velocity of the electromagnetic wave in vacuum, f_0 is the frequency of the harmonic signal, and t_0 is the time delay of the pulse. The parameters of the incident plane wave are set to: $\mathbf{E}_0 = 120\pi\hat{\mathbf{x}}$, T = 24 ns, t_0 = 25 ns, f_0 = 187.5 MHz, $\hat{\mathbf{k}} = -\mathbf{z}$. The bandwidth of this wave is 225 MHz. The time and frequency characteristics are depicted in Fig. 6.

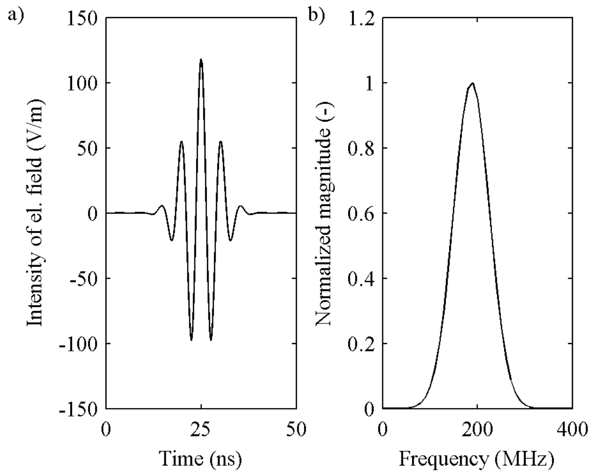


Fig. 6. Harmonic plane wave modulated by Gaussian pulse a), the spectrum b).

The analyzed structures are located in the xy plane, and discretized with respect to the frequency $f = 300$ MHz to meet the discretization criterion. Their dimensions, the number of triangular patches N_{tr} necessary for the approximation of their body, and the distances R_{max} are summarized in Tab. 1. The nominal values were computed with the help of Fig. 5 for different lengths of the time step. The structures are firstly analyzed by the conventional MOT scheme and than by the MOT scheme with the EDM method.

Structure	Dimensions [m × m]	N_{tr} [-]	R_{max} [m]	R_0^1 [m]	R_0^2 [m]
Strip 1	2.00 × 0.08	44	1.94	1.00	0.85
Strip 2	5.00 × 0.08	111	4.94	1.52	1.38
Square plate	1.00 × 1.00	264	1.33	0.80	0.72
Rectangular plate	2.00 × 1.00	484	2.15	1.05	0.90

Tab. 1. Geometrical properties of the analyzed structures. Nominal values R_0^1 and R_0^2 are computed for different lengths of the time step, $\Delta t = 1.5R_{min}/c$, and $\Delta t = R_{min}/c$, respectively.

The results of the analysis recorded in the percentage saved time for different lengths of the time step are summarized in Tab. 2. Evidently, the MOT scheme with the EDM method saves more computational time in comparison to the conventional MOT scheme, as the electrical size of the analyzed structure is larger, and the length of the time step is smaller. Thus, we confirmed our assumption mentioned at the end of subsection 3.2.

Structure	$R_{max}/\lambda(f_{max})$	Saved time for $\Delta t = 1.5R_{min}/c$ [%]	Saved time for $\Delta t = R_{min}/c$ [%]
Strip 1	1.94	17	24
Strip 2	4.94	38	42
Square plate	1.33	10	15
Rectangular plate	2.15	20	28

Tab. 2. Saved time by the MOT scheme with the EDM method in comparison to the conventional MOT scheme for different lengths of the time step.

The transient responses of the current at the center of the strip 2, and the rectangular plate obtained by both approaches are depicted in Fig.7, and Fig. 8, respectively. Apparently, the agreement is very good.

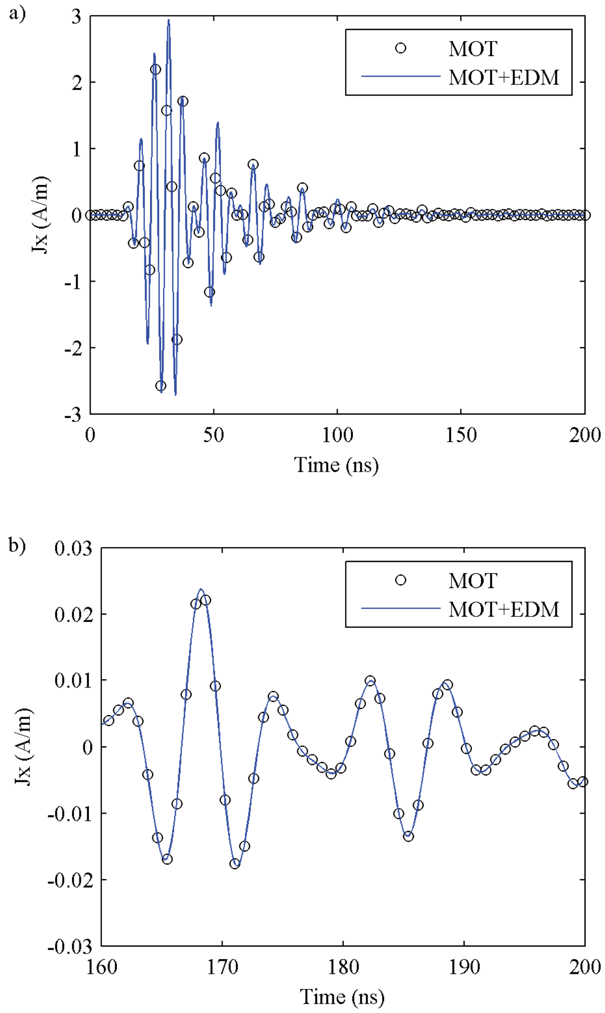
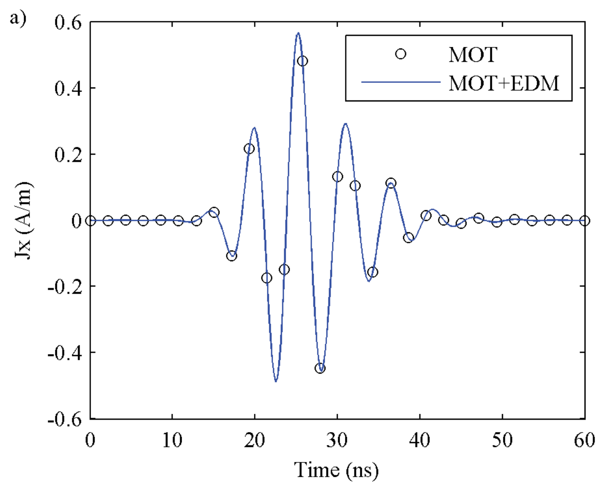


Fig. 7. Current response at the center of the strip 2 a), and its enlarged detail b). The length of the time step is $\Delta t = 1.5R_{min}/c$.



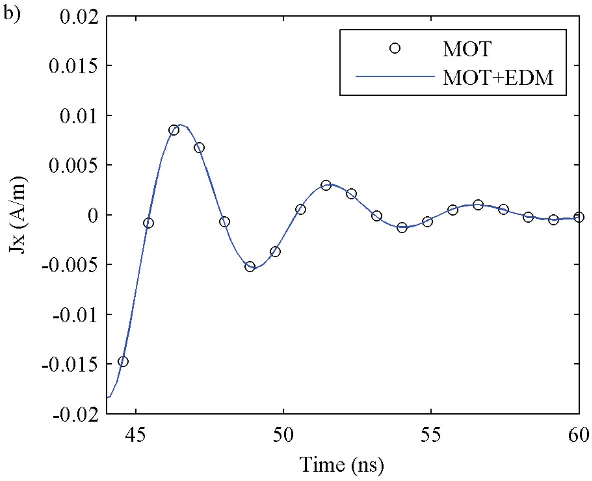
b)


Fig. 8. Current response at the center of the rectangular plate a), and its enlarged detail b). The length of the time step is $\Delta t = 1.5R_{min}/c$.

Note that the accuracy of the conventional MOT scheme was verified by the results obtained by the inverse discrete Fourier transform of the frequency domain solution in [1]. Thus, here, we take it as a reference for the MOT scheme with the EDM method.

5. Conclusion

Although the exploitation of the EDM method for the acceleration of the MOT scheme for the TD-EFIE seems perspective, its full-value using requires an appropriate technique for the decreasing sensitivity of the MOT scheme to obtain a well-conditioned scheme. Actually, such a technique should be used even if the conventional MOT scheme is used to increase the stability of obtained results.

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About Authors ...

Jaroslav LÁČÍK was born in Zlín, Czech Republic, in 1978. He received the Ing. (M.Sc.) and Ph.D. degrees from the Brno University of Technology (BUT). Since 2007, he has been an assistant at the Dept. of Radio Electronics, BUT. He is interested in modeling antennas and scatterers in the time- and frequency-domain.