A Low Complexity Partial Transmit Sequence for Peak to Average Power Ratio Reduction in OFDM Systems

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Abstract. *Partial transmit sequence (PTS) is one of the most important techniques for reducing the peak to average power ratio (PAPR) in OFDM systems. This paper presents a low complexity PTS scheme by applying a new phase sequence. Unlike the conventional PTS which needs several inverse fast Fourier transform (IFFT) operations, the proposed scheme requires half IFFT operations only at the expense of slight PAPR degradation. Simulation and results are examined with QPSK modulation and OFDM signal and power amplifier with memory effects.*

Keywords

CCDF, digital predistortion, Orthogonal frequency division multiplexing, partial transmit sequence, peak to average power ratio.

1. Introduction

An orthogonal frequency division multiplexing (OFDM) system has been proposed as a standard for the mobile communication systems. Despite the advantages of OFDM signals like high spectral efficiency and robustness against ISI, the OFDM signals have some disadvantages among which the main one is the high PAPR [1], [2]. This high PAPR signal when transmitted through a nonlinear power amplifier creates spectral broadening and also an increase in the dynamic range of the digital to analog converter (DAC). The result will be an increase in the cost of the system and reduction in efficiency. To overcome this impact, several techniques for reducing the PAPR have been proposed. Some of the most important techniques are selected mapping (SLM) [3] which is in frequency domain and PTS [4], [5] [7] which is in time domain. Authors have proposed a combinational method to reduce the complexity of the PTS method [10]. In [4], [5] authors proposed phase weighting method, subblock phase weighing but they didn't achieve complexity reduction. Here with applying the new phase sequence by first generating the matrix of phase sequence and then partitioning it based on the requirement for PAPR reduction and also complexity. With this new phase sequence the complexity of PTS reduces significantly as it reduces the number of IFFT but it only degrades the PAPR performance slightly. In simulation the proposed PTS scheme is examined with considering the power amplifier (PA) model with memory effects [8].

This paper is organized as follows, section 2 is the basic introduction about OFDM systems and definitions, in section 3 the new PTS method is proposed. Section 4 and 5 discuss the simulation results and conclusions respectively.

2. PAPR Definition

In OFDM systems, a fixed number of successive input data samples are modulated first (e.g. PSK or QAM), and then jointly correlated together using IFFT at the transmitter side. IFFT is used to produce orthogonal data subcarriers. Mathematically, IFFT combines all the input signals (superposition process) to produce each element (signal) of the output OFDM symbol $\{x_n\}_{n=0}^{N-1}$. The time domain complex baseband OFDM signal can be represented as [1]:

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{n}{N} k}, \quad n = 0, 1, 2, \ldots, N-1 \tag{1}$$

where x_n is the n-th signal component in OFDM output symbol, X_k is the k-th data modulated symbol in OFDM frequency domain, and N is the number of subcarriers.

The PAPR (in dB) of the transmitted OFDM signal can be defined as [2]:

$$PAPR = \frac{\max\limits_{0 \le t \le T} \left[|x(t)|^2 \right]}{E\left[|x(t)|^2 \right]} \tag{2}$$

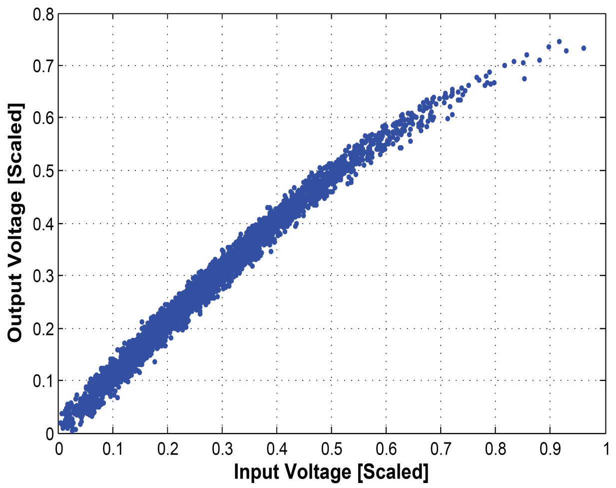
where $E[\cdot]$ is the expected value operator. The theoretical maximum of the PAPR for N number of subcarriers is as follows:

$$PAPR_{\max} = 10 \log(N) \quad \text{dB}. \tag{3}$$

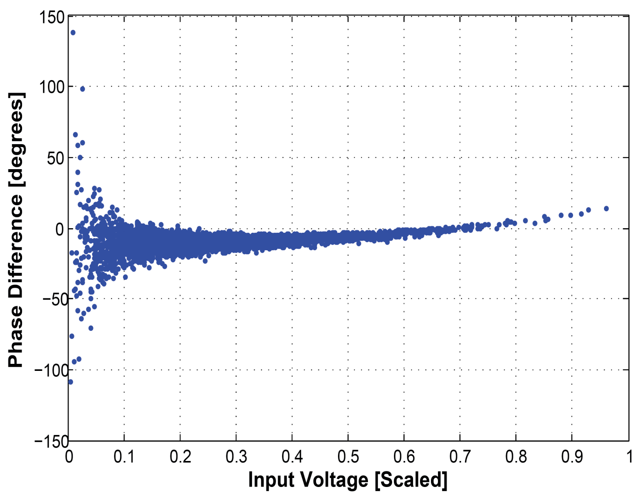
PAPR is a random variable, because it is a function of the input data and the input data are random variable. Therefore PAPR can be calculated by using level crossing rate theorem that calculates the average number of times that the envelope of a signal crosses a given level. Knowing the amplitude distribution of the OFDM output signals, it is easy to compute the probability that the instantaneous amplitude will be above a given threshold and the same goes for power. This is performed by calculating the complementary cumulative distribution function (CCDF) for different PAPR values as follows:

$$CCDF = \Pr(PAPR > PAPR_0)\,. \tag{4}$$

In this paper, the power amplifier with memory effects [9], [10] and [11] is applied for demonstrating the effec-tiveness of the proposed PTS method in reducing the spectral broadening.



(a)



(b)

Fig. 1. AM-AM and AM-PM characteristics of the PA with memory effects: (a) AM/AM (b) AM/PM.

The power amplifier with memory effects is based on the memory polynomial method [8] as follows:

$$y(n) = \sum_{k=1}^{K}\sum_{q=0}^{Q-1} a_{kq}\, x(n-q)\left|x(n-q)\right|^{k-1} \tag{5}$$

where a_{kq} are the coefficients of the power amplifier, K is the order of nonlinearity and Q is the memory length. The extracted coefficients when $Q = 2$ and $K = 3$ are given by:

$$a_{10} = 1.4513 + 0.132\mathrm{i}\ a_{11} = \text{-}0.123 - 0.023\mathrm{i}\ a_{12} = 0.012 - 0.0043\mathrm{i},$$
$$a_{30} = \text{-}0.132 - 0.430\mathrm{i}\ \ a_{31} = 0.322 + 0.243\mathrm{i}\ a_{32} = \text{-}0.0123 - 0.12\mathrm{i},$$
$$a_{50} = \text{-}0.755 - 0.654\mathrm{i}\ a_{51} = \text{-}0.213 - 0.411\mathrm{i}\ a_{52} = 0.233 + 0.233\mathrm{i}.$$

The AM/AM and AM/PM characteristics of this PA are shown in Fig. 1. Fig. 1a and 1b show the AM/AM and AM/PM characteristics of the power amplifier with memory effects respectively. As it can be observed, the impact of memory causes these curves to spread over their linear behavior. The type of memory effects that cause these effects is electrical memory effects or also called short term effects and the power amplifier for modeling it, is based on the specific case of Volterra series [9]. This memory effect causes spectrum broadening.

3. Proposed Phase Sequence

In this section the proposed phase sequence of PTS method is presented. But fist the conventional PTS (C-PTS) is discussed in details.

3.1 Conventional PTS (C-PTS)

Let X denote a random input signal in frequency domain with length N. X is partitioned into M disjoint subblocks $\mathbf{x_v}=[X_{v,0},X_{v,1},\ldots,X_{v,N-1}]^T$, $v=1, 2, \ldots, M$ such that

$$\sum_{v=1}^{M} X_v = X$$

and then these subblocks are combined to minimize the PAPR in time domain. By applying the phase rotation factor $b_v = e^{j\phi_v}$, $v = 1, 2, \ldots, M$ to the IFFT of the v^{th} subblock X_v, the time domain signal after combining is given by:

$$\mathbf{x}'(b) = \sum_{v=1}^{M} b_v x_v \tag{6}$$

where $\mathbf{x}'(b) = [x_0'(b),\ x_1'(b),\ \ldots\ x_{NL-1}'(b)]^T$ and L is the oversampling factor. The objective is to find the optimum signal $\mathbf{x}'(b)$ with the lowest PAPR. Both b and x can be shown in matrix form as follows:

$$\mathbf{B} = \begin{bmatrix} b_1, & b_1 & ,\ldots, & b_1 \\ \vdots & \vdots & & \vdots \\ b_M, & b_M & ,\ldots, & b_M \end{bmatrix}_{M\times N}, \tag{7}$$

$$\mathbf{X} = \begin{bmatrix} x_{1,0}, & x_{1,1}, & \ldots, & x_{1,NL-1} \\ \vdots & \vdots & & \vdots \\ x_{M,0}, & x_{M,1}, & \ldots, & x_{M,NL-1} \end{bmatrix}_{M\times NL}. \tag{8}$$

It should be noted that all the elements of each row of matrix **B** are of the same values and this is in accordance with the C-PTS method. Now, the process is performed by choosing the optimization parameter $\tilde{b}$ with the following condition:

$$\hat{b} = \arg\min(\max_{0 \le k \le NL-1} \left| \sum_{v=1}^{M} b_v x_v \right| . \qquad (9)$$

After finding the optimum $\hat{b}$, the optimum signal is transmitted to the next block. To obtain the optimum $\tilde{b}$, we should perform exhaustive search for (M-1) phase factors since one phase factor can remain fixed, $b_1 = 1$. Hence to find the optimum phase factor, W^{M-1} iterations should be performed, where W is the number of allowed phase factors.

3.2 Proposed Phase Sequence

In order to decrease the complexity of C-PTS, we generate a new phase sequence. This new phase sequence is based on the generation of N random values of {1 -1}, If we consider the number of allowed phase factors is $W = 2$. Hence the new phase subsequence has a formation as follows:

$$\hat{\mathbf{B}} = \begin{bmatrix} b_{1,1}, & b_{1,2} & ,...., & b_{1,N} \\ \vdots & \vdots & \vdots & \vdots \\ b_{M/2,1}, & b_{M/2,2}, & ..., & b_{M/2,N} \end{bmatrix}_{M/2 \times N} . \qquad (10)$$

According to (10), N random phase sequences are generated and periodically $M/2$ times will be generated where M is the number of subblock partitioning. Despite C-PTS where each row of the matrix $\hat{\mathbf{B}}$ has same values and followed by multiplying with $\mathbf{x}$ according to (6), in the proposed phase sequence, each row of the matrix in (10) has different phase factors which are random values of {1 -1} and this will cause reduction in PAPR due to having more possibility of low PAPR. This will be shown later in simulations.

The random phase sequence also can be in the form of interleaved and adjacent as follows:

$$\hat{\mathbf{b}} = \left[\overbrace{b_{1,1}, b_{1,2}, ..., b_{1,N/D}, ..., b_{1,1}, b_{1,2}, ..., b_{1,N/D}}^{D} \right]_{1 \times N} , \qquad (11)$$

$$\hat{\mathbf{b}} = \left[\overbrace{b_{1,1}, ..., b_{1,1}}^{D}, \overbrace{b_{1,2}, ..., b_{1,2}}^{D}, ..., \overbrace{b_{1,N/D}, ..., b_{1,N/D}}^{D} \right]_{1 \times N} \qquad (12)$$

where (11) and (12) are the interleaved the adjacent phase sequence respectively and D is the number of partition in the phase sequence and is defined as:

$$D = VW^{M/2-1} \quad , \; V = 1, 2, ..., \frac{N}{W^{M/2} - 1} . \qquad (13)$$

After generating the matrix of phase sequences, the matrix should be extended for D rows as follows:

$$\hat{\mathbf{B}} = \begin{bmatrix} b_{1,1} & ,..., & b_{1,N} \\ \vdots & \vdots & \vdots \\ b_{M/2,1} & ,..., & b_{2,N} \\ b_{M/2+1,1} & ,..., & b_{M/2+1,N} \\ \vdots & \vdots & \vdots \\ b_{D,1} & ,..., & b_{D,N} \end{bmatrix}_{[D \times N]} \qquad (14)$$

where D is from (13) and W=2.

In this paper we use random phase sequence matrix in (14). For computing the actual PAPR, the oversampling needs to be considered. Hence the matrix in (14) can be expressed as:

$$\hat{\mathbf{B}} = \begin{bmatrix} b_{1,1} & ,..., & b_{1,NL} \\ \vdots & \vdots & \vdots \\ b_{M/2,1} & ,..., & b_{2,NL} \\ b_{M/2+1,1} & ,..., & b_{M/2+1,NL} \\ \vdots & \vdots & \vdots \\ b_{D,1} & ,..., & b_{D,NL} \end{bmatrix}_{[D \times NL]} \qquad (15)$$

where L is the oversampling factor. It should be noted that in order to have exact PAPR calculation, at least 4 times oversampling is necessary [5]. As oversampling will add zeros to the vector, then after multiplying phase sequence $\hat{\mathbf{B}}$ with $\mathbf{X}$, the only section that counts in the multiplication will be N elements, hence the new phase sequence matrix in (15) still has N rows and the oversampling factor does not have any effect on that.

Fig. 2 shows the block diagram of the proposed PTS scheme with PA. By applying the new phase sequence, (6) can be expressed as follows for the specific case of $M = 2$:

$$\mathbf{x}'(b) = x'_{1,k} b_{l,k} + b_{l+1,k} x_{2,k} \qquad (16)$$

where $l = 1, 2, ..., D$-1 and $k = 1, 2, ..., N$.

There is a trade off for choosing D, whereas lower number of partitions results in less PAPR reduction but less complexity and higher number of division has higher PAPR reduction with higher complexity. The side information is the same as the C-PTS because the number of iterations is the same, however as the size of the phase sequence is increased larger memory space is required to store the phase sequence matrix. As an example assume $N = 256$, and the number of allowed phase factor and subblock partitioning are $W = 2$ and $M = 4$ respectively, with C-PTS there are $W^{M-1} = 8$ possible iterations, whereas for the proposed method, in the case of $V = 1$, $D = 8$, the phase sequence is a matrix of [8x256] elements according to (16). In this case, we have the same number of iterations for finding the optimum phase sequence compared to C-PTS,

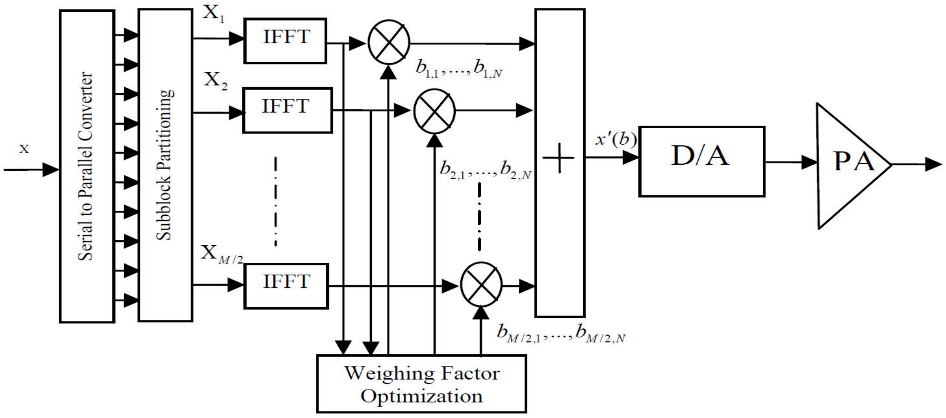


Fig. 2. Block diagram of the proposed PTS scheme with power amplifier.

Number of Subblocks	Complex Computation	C-PTS	Proposed Method			
			$V = 1$	CCRR	$V = 2$	CCRR
$M = 4$	Multiplication	14336	7168	50 %	12288	14.2 %
	Addition	14336	7168	50 %	10240	28.5 %
$M = 8$	Multiplication	303104	151552	50 %	299008	1.3 %
	Addition	245760	122880	50 %	237568	3.3 %

Tab. 1. Computational complexity comparison for $N = 256$, $W = 2$.

and rows of matrix of (14) multiply point-wise with the time domain input signal x with length [2x256].

The reduction of subblocks to 2 is done because it gives almost the same PAPR reduction as C-PTS with $M = 4$. It should be noted that if $V = 2$, $D = 16$ then the complexity increase and PAPR reduces more.

3.3 Computational Complexity

When the number of subcarriers is $N = 2^n$ and oversampling factor is $L = 2^l$, the total complexity including IFFT and phase factor combination and PAPR calculation can be expressed as [5]:

$$n_{mul} = 2^{n+l-1}(n+l)M + W^{M-1}N(M+1), \tag{17}$$

$$n_{add} = 2^{n+l}(n+l)M + W^{M-1}N(M-1) \tag{18}$$

where M is the number of subblocks and W is the number of allowed phase factors.

These values for proposed method are:

$$n_{mul} = 2^{n+l-1}(n+l)\frac{M}{2} + D\left(\frac{M}{2}+1\right)N, \tag{19}$$

$$n_{add} = 2^{n+l}(n+l)\frac{M}{2} + D\left(\frac{M}{2}-1\right)N. \tag{20}$$

The number of IFFT in the proposed method reduces to half and the number of iterations is D according to (13). It should be noted that the main reason that the number of subblocks in the proposed method as mentioned in the latter section is the same PAPR performance can be achieved compared to C-PTS. This is also shown in the simulation results.

Tab. 1 shows the computational complexity of C-PTS and the proposed method. Here we calculate the complexity for $N = 256$, $M = 4$, $W = 2$ and $l = 0$.

The computational complexity reduction ratio (CCRR) of proposed technique over the C-PTS is defined as [2]:

$$\text{CCRR} = \left(1 - \frac{\textit{Complexity of the new PTS}}{\textit{Complexity of the } C-PTS}\right)\times 100\% \tag{21}$$

It is clear that the CCRR is improved for both values of V. the amount of improvement is more when $V = 1$ whereas the PAPR reduction is less compared to C-PTS.

4. Simulation and Results

In order to evaluate and compare the performance of the proposed method with C-PTS, Matlab simulation is performed. We employed QPSK modulation with IFFT length of $N = 256$. To obtain the complementary cumulative distribution function (CCDF), 100 000 random OFDM symbols are generated and the oversampling factor is 4.

Fig. 3 shows the CCDF of three different types of phase sequences, interleaved, adjacent and random. From this figure, PAPR reduction with random phase sequence outperforms the other types and hence this type of phase sequence is applied in the following simulations.

Fig. 4 shows the comparison of CCDF of the proposed method and C-PTS. It can be observed that the PAPR reduction for our proposed PTS scheme degrades

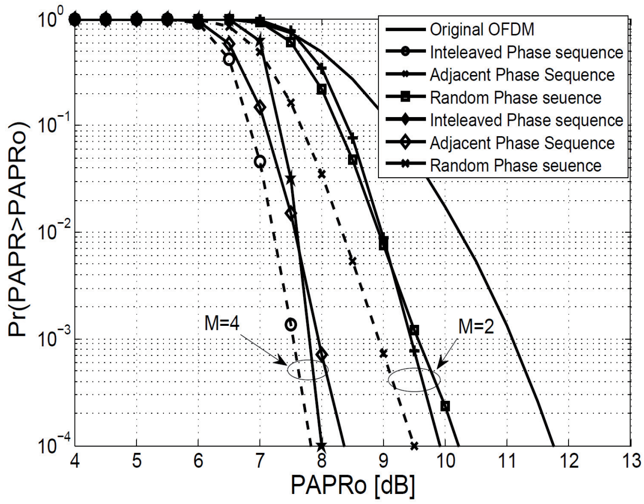


Fig. 3. CCDF of PAPR of the different phase sequence forms for $M = 2$ and 4.

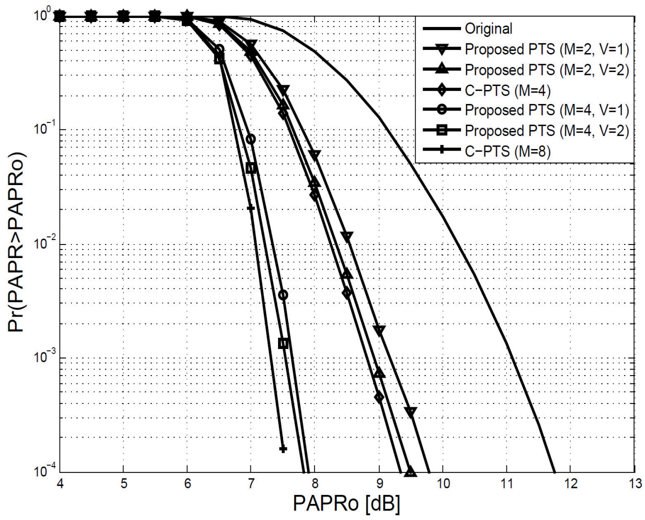


Fig. 4. CCDF of PAPR of proposed method compared to C-PTS for $M = 4$ and 8.

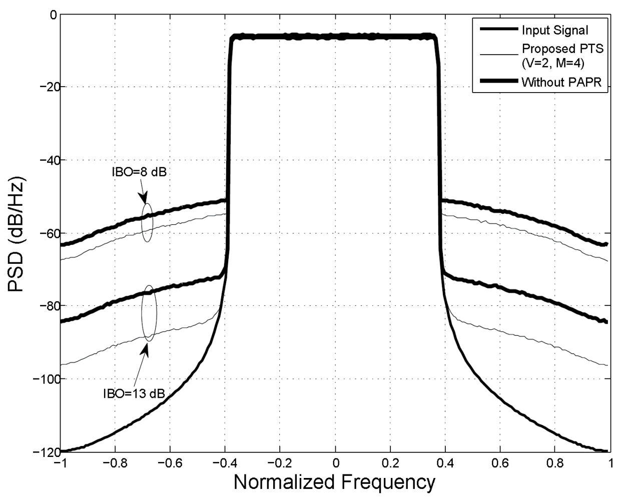


Fig. 5. Power spectral density of the PA with memory effects with the proposed PTS.

compared to C-PTS while complexity is enhanced. The simulation is examined for $V = 1$ and 2 and $M = 4$ and 8 respectively. It is clear that the PAPR reduction of C-PTS when $M = 4$ is almost the same as PAPR reduction of the proposed method when $M = 2$. If $V = 2$ the PAPR reduction is higher than with $V = 1$ because more partitions are applied in the phase sequence matrix.

Fig. 5 shows the power spectral density (PSD) for the power amplifier with memory effects. This figure shows the effectiveness of PTS method in reducing the out of band distortion. The bold line indicates the power amplifier without PAPR and the light color line indicates the PSD when PTS with new phase sequence is applied which has the highest PAPR reduction among others with $M = 4$ and $V = 2$. The results are shown when input back off (IBO) is 8 dB and 13 dB. It can be seen that for higher IBO the effect of applying PAPR is higher and this is because the PA works in more linear region. As the PAPR is almost the same for the proposed method and C-PTS, there is no difference in PSD and hence it is not shown for the C-PTS

5. Conclusion

In this paper a new phase sequence of PTS scheme has been proposed. In this approach, matrix of possible random phase factors is first generated and then multiplies point-wise with the input signal. By applying this technique the number of IFFT is reduced to one half which results in lower complexity compared to C-PTS at the expense of slight PAPR degradation. By adding the PA with memory effects the performance of the PAPR is also examined and it proves the effectiveness of the proposed method in reducing the out of band distortions.

Acknowledgements

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