Current-Mode Universal Biquad Using Current Followers: A Minimal Realization

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Abstract. A new universal biquad filter is presented which employs a minimum number of active elements (only three current followers (CF)) along with a minimum number of passive components (i.e. only two resistors and two capacitors). The new circuit provides all the five standard filter responses (namely lowpass, bandpass, highpass, notch and allpass) from the same structure without requiring any component matching conditions and with explicit current outputs available from high output impedance terminals. The workability of the proposed universal biquad, realized with unity-gain current followers implemented in 0.35 µm CMOS and operated from 1.65 volts DC power supplies, is established by SPICE simulations.

Keywords

Universal filters, current followers, current-mode circuits.

1. Introduction

Apart from their usual applications as standard building blocks for realizing second-order and higher order active filter designs, multifunction active biquad filters (e.g. those realizing lowpass (LP), bandpass (BP) and highpass (HP) from three different output terminals, simultaneously) have other applications in phase locked loop FM stereo demodulators, touch tone telephone systems and cross over networks used in threeway high fidelity loud speakers [1]. A number of currentmode biquads have been reported in the literature which have the potential of being utilized in one or more of such applications (for example, see [1] and the references cited therein).

Because of the advantages of wider bandwidth, low power consumption and simpler architecture, unity-gain current followers (CF) have attracted considerable attention in the literature for the realization of biquad filters [2-12]. The configurations reported in [2] employ one/two CFs but realize only one/two functions at a time and provide at the most one explicit current-mode (CM) output. On the other hand, a close look at the structures presented in [3-6] reveals that while one of the circuits of [3] can realize all the five standard filter re-

sponses in current-mode, it does not provide explicit current output (i.e. from high output impedance node) and requires the nature of various circuit elements to be changed (resistive or capacitive) to realize the various filtering functions (i.e. it does not have a fixed topology). The circuits of [4-7] do provide explicit current outputs and are also able to provide all the five standard filter responses (namely, LP, BP, HP, notch and allpass (AP)) from the same structure, however, none of them is able to do so without requiring any component-matching conditions or equality constraints. Such conditions are invariably required in realizing notch and AP functions. Moreover, the circuits of [4] require three CFs and three voltage followers (VF), those of [5] require three CFs and four VFs and those of [6] require three CFs and two VFs. The circuit of [7] can realize all the five standard filter functions in voltage-mode as well as in current-mode but requires four voltage followers and four current followers as active elements. Explicit voltagemode/current-mode outputs are available from low output impedance terminals and high output impedance terminals, respectively. Thus, apart from the requirement of the component matching conditions, all the circuits from [4-7] use number of active elements, which are in excess of the minimum number i.e. three. The structure of [13] employs multiple output current follower (MO-CF) to realize a universal filter with four inputs and as many as seven outputs but two of the output currents therein are flowing in the two grounded capacitors and thus these output currents are not available explicitly. Although, the circuit is minimal from the passive component point of view. The SIMO type universal filter of [14] with adjustable parameters employs three MO-CFs along with a differential adjustable current amplifier (DACA) for providing digital adjustment of Q factor thereby requiring a total number of four active elements. In [17], use of three MO-CF and three current amplifiers (CA) with adjustable gain leads to a universal digitally adjustable filter in single ended mode and in fully differential mode where although all the filter parameters can be adjusted electronically but number of active devices required is as many as six. Lastly, the circuit in [18] employs two current controlled transconductance amplifier (CCTA) to realize a current-mode filter providing LP, HP and inverting BP responses but two CFs are needed therein to provide explicit HP and BP outputs thereby making active component count as four. Thus, it is seen that in spite of attaining

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different properties the circuits from [13], [14], [1]7, [18] also employ more than three active elements.

Recently, the authors presented new configuration using only unity gain current followers in [8]. The circuit in [8] (reproduced here in Fig. 1) can realize all the five standard filter functions employing only unity-gain current followers as active elements and provides explicit current-mode outputs from high output impedance terminals. The universal biquad of Fig. 1 provides the following advantageous features simultaneously: (i) availability of all the five standard filtering functions from the same configuration; (ii) no constraints/ additional cancellation conditions are required for notch/ AP responses; (iii) explicit availability of current-mode outputs from high output impedance terminals; (iv) sequential/ independent controllability of various parameters; (v) use of grounded/virtually grounded capacitors.

During the course of these investigations, a question was posed as to what could be that universal biquad which employs a bare minimum of only four passive elements (i.e. two capacitors and two resistors) while employing a minimum^{*} number of (only three) CFs and still be capable of providing all the five standard current-mode filter outputs explicitly without requiring any component matching constraints/ conditions. Although such a configuration may not possess feature (iv) mentioned above but would be useful in applications where reduced power consumption is of interest.



Multiple Output CF



Fig. 1. Universal current-mode Biquad of [8].

The aim of this paper is, therefore, to present such a *fixed-topology-type* universal current-mode biquad which, by contrast to the circuits of [4-8] (employing five to seven

active elements), employs only a minimum number of (only three) CFs along with a minimum number of passive elements (only two resistors and two capacitors) and provides all the five standard filter responses *explicitly* from the same structure without requiring any component-matching conditions or equality constraints. As far as is known, no such CF-based *minimal* universal biquad circuit has yet been published in the literature earlier.

2. Proposed Fixed-Topology-Type Minimal Universal Biquad

The proposed biquad is shown in Fig. 2 where CF2 is a CF2; CF1 and CF3 are multiple-output CFs. These CFs are characterized by the equations $v_x = 0$, $i_z^+ = -i_z^- = i_x$, with v_z being arbitrary.



Fig. 2. The proposed universal current-mode biquad.

A straightforward analysis of the circuit of Fig. 2 reveals the following three transfer functions:

$$\frac{I_{HP}}{I_{in}} = s^2 / D(s) = H_0 s^2 / D(s),$$
(1)

$$\frac{I_{BP}}{I_{in}} = -\left(\frac{1}{C_1 R_1}\right) s / D(s) = -H_0\left(\frac{\omega_0}{Q_0}\right) s / D(s), \quad (2)$$

$$\frac{I_{LP}}{I_{in}} = \left(\frac{1}{C_1 C_2 R_1 R_2}\right) / D(s) = H_0 \omega_0^2 / D(s)$$
(3)

where

$$D(s) = s^{2} + \left(\frac{1}{C_{1}R_{1}}\right)s + \frac{1}{C_{1}C_{2}R_{1}R_{2}} = s^{2} + \left(\frac{\omega_{0}}{Q_{0}}\right)s + \omega_{0}^{2}$$
⁽⁴⁾

and the parameters ϖ_0 , H_0 and Q_0 have their usual meanings. Please note that $H_0 = 1$ in all the cases.

From (4), it is seen that after adjusting the $(\overline{\omega}_0/Q_0)$ by R_1 , $\overline{\omega}_0$ can be adjusted by R_2 . It may be observed that if the cur rent-mode outputs $I_{\rm HP}$ and $I_{\rm LP}$ are tied together, current-mode notch response will be available directly without any condition and similarly, by adding $I_{\rm HP}$, $I_{\rm LP}$ and $I_{\rm BP}$ together one can obtain current-mode AP response also without any condition. These current-mode TFs are given by:

^{*} Although, one can possibly manage a circuit with two resistors, two capacitors and two CFs with multiple outputs such that they realize explicitly LP and HP responses thereby, permitting to obtain a notch filter by adding the two. However, such a circuit can not realize all five responses by any means. Therefore, a minimal structure for realizing three basic current-mode functions explicitly namely LP, BP and HP (thereby enabling the remaining two responses, namely notch and all pass as shown here) will have to employ at least 3 CFs.

$$\frac{I_{Notch}}{I_{in}} = \frac{\left(s^2 + \frac{1}{C_1 C_2 R_1 R_2}\right)}{D(s)} = \frac{H_0 \left(s^2 + \omega_0^2\right)}{D(s)}, \quad (5)$$

$$\frac{I_{AP}}{I_{in}} = \frac{\left(s^2 - \frac{s}{C_1 R_1} + \frac{1}{C_1 C_2 R_1 R_2}\right)}{D(s)} = \frac{H_0 \left(s^2 - \frac{\omega_0}{Q_0}s + \omega_0^2\right)}{D(s)} (6)$$

where, D(s) is same as in (4) and $H_0 = 1$.

Thus, the proposed biquad provides all the five standard filter responses from the same structure without requiring any component matching conditions or equality constraints. Also, note that all the CM outputs are available *explicitly* from the high output impedance terminals.

From an inspection of the circuit, it appears that the input impedance of the circuit would be frequency dependent and hence may pose some problem. However, if the finite source resistance of I_{in} be considered as R_s in parallel with the source it is clear that grounded R_s directly comes in parallel with the virtual grounded R_1 . Thus, the effect of this can be visualized in the light of the equation (4) that the term $1/(C_1R_1)$ therein will actually get modified to $(1/R_1 + 1/R_2)/C_1$. Since one can always choose $R_1 \ll R_2$ (which is obviously very large), the effect of R_s on the BW (in case of BP an notch responses) and on the *Q*-factor (in case of LP and HP responses) can be made small.

Although, it is well understood that a 2R-2C minimal passive structure usually cannot provide tunability of the filter parameters namely H_0 , BW or Q_0 and ϖ_0 , it might appear that possibly some limited tunability might be attainable by exploiting electronic controllable nature of the input resistance of CF (R_x) as is possible with some structures using current controlled conveyor (CCCII) or current controlled current differencing transconductance amplifier (CCCDTA) (for instance see [15], [16], [19] and references cited therein). However, a non-ideal analysis of the proposed circuit shows that this is not feasible for the instant case. Nevertheless, the non-ideal analysis with non-ideal current gains taken as (α_{11} and α_{12}), α_{21} , and (α_{31} , α_{32} and α_{33}) for the three CFs respectively shows that (see Appendix-A), tuning of the quoted three parameters looks possible through various α 's as follows:

BP: H_0 by α_{12} and $\overline{\omega}_0$ by α_{11}

HP: H_0 by α_{21} and $\overline{\omega}_0$ by α_{11}

LP: H_0 by α_{32} and $\overline{\omega}_0$ by α_{11}

Actual implementation of this idea, however, could be introduced by incorporating CAs or DACAs like [13], [14], [17] at appropriate locations. This will, however, result in increase of the total number of active elements employed. Since the objective of the proposed paper was to present a minimum component CF-based structure as an alternative to that of Fig. 1, which already had sequential/ independent controllability of various parameters, the above-mentioned alternative of employing additional CAs and DACAs to bring in the tunability property (comparable/ better to that of Fig. 1) was considered to be out of the scope of the present paper and had, therefore, not been attempted.

In order to have a single circuit capable of realizing all the five standard filtered functions it is necessary to have three explicit CM outputs namely LP, Inverting BP and HP so that by adding LP and HP currents, one can create a notch function and by adding all three of them one can create an AP response. Thus, at least three active elements (in present case, three CFs are necessary). On the other hand, it is obvious that the minimum number of passive elements to realize second order filters is at least two resistors and two capacitors. Since the proposed realization employs exactly three active elements, two resistors and two capacitors hence, it is a *minimal* realization from the viewpoint of active elements as well as passive elements.

From (4), all sensitivity coefficients $S_{\rm F}$ of angular frequencies (ω_0), quality factor (Q_0) or bandwidth (ω_0/Q_0) and H_0 of the various realized filters are found to be as follows:

$$S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2},$$
(7)

$$S_{C_1}^{BW} = S_{R_1}^{BW} = -1, \quad S_{C_2}^{BW} = S_{R_2}^{BW} = 0,$$
 (8)

$$S_{C_1}^{\mathcal{Q}_0} = S_{R_1}^{\mathcal{Q}_0} = \frac{1}{2}, \quad S_{C_2}^{\mathcal{Q}_0} = S_{R_2}^{\mathcal{Q}_0} = -\frac{1}{2}, \tag{9}$$

$$S_{R_1}^{H_0} = S_{R_2}^{H_0} = S_{C_1}^{H_0} = S_{C_2}^{H_0} = 0.$$
 (10)

It is thus seen from (7-10) that all sensitivity coefficients are in the range $0 \le |S_F| \le 1$ and the circuit, thus, enjoys low sensitivities properties.

2. SPICE Simulation Results

For the simulations of the biquad of Fig. 2 in SPICE, we have adopted the CF of [6] with some changes^{*}. In the resulting circuit, shown here in Fig. 3, additional current $i_{2^{-}}$ (as needed in CF3 in Fig. 2) can be easily obtained by appropriately adding one more pair of transistors similar to M₈ and M₁₆ to the circuit of Fig. 3.

The aspect ratios (W/L) of the MOSFETs used were 26/0.5 for M₃-M₆, M₉; 26.5/0.5 for M₇; 27.7/0.5 for M₈; 24.7/0.5 for M₁₀; 12/0.5 for M₁, M₂, M₁₁, M₁₃-M₁₅; 24/0.5 for M₁₂ and 11/0.5 for M₁₆. In simulations, BSIM3v3 level 7 CMOS model parameters for 0.35 μ m CMOS technology were used. The power supply voltage of 1.65 volts DC power supplies were used and the applied DC bias currents were $I_{B1} = 100 \ \mu$ A and $I_{B2} = 93.9 \ \mu$ A. The LP, HP and AP functions of the proposed biquad have been verified in SPICE with $I_{in} = 10 \ \mu$ A and the component values as $C_1 = 112.5 \ \text{pF}$,

 $^{^{\}ast}$ In the circuit of Fig. 2 of [6] the gates of NMOS transistors $M_{14}\text{-}M_{15}$ and those of PMOS transistors $M_8\text{-}M_9$ have been inadvertently shown as tied together which should not be the case.

 $C_2 = 225 \text{ pF}$ and $R_1 = R_2 = 1 \text{ k}\Omega$ giving $f_0 = 1 \text{ MHz}$, $Q_0 = 0.707$ and $H_0 = 1$ in all the cases. Fig. 4 shows the current-mode LP and HP responses. All pass magnitude and phase responses are given in Fig. 5 and 6, respectively. In Fig. 7, BP and notch responses are given.



Fig. 3. An exemplary CMOS implementation 6 of the CF.



Fig. 4. Current-mode LP and HP responses.



Fig. 5. Current-mode allpass magnitude response.

Variation of the various node voltages as a function of frequency was also observed and it was found that over the entire frequency range of operation i.e. 1 MHz-100 MHz, the maximum swings of the voltages at nodes a-e although they are varying over two decades but still are all very small as compared to DC bias power supplies of 1.65 V (confined to 0.107, 0.006, 0.024, 0.102, 0.005 volts, respectively) even when current outputs change over a range of nearly two dec-

ades and even when it becomes 100 μ A which is of the same order as the DC bias sources $I_{B1} = 100 \ \mu$ A and $I_{B2} = 93.9 \ \mu$ A. This is indicative of the dominant current-mode nature of the circuit.



Fig. 6. Current-mode allpass phase response.



Fig. 7. Current-mode BP and notch responses.



Fig. 8. Large signal behavior of the currents $I_{\rm HP}$, $I_{\rm BP}$ and $I_{\rm LP}$ with $I_{\rm in} = 100 \ \mu A$.

The large signal behavior (with $I_{in} = 100 \ \mu\text{A}$) of the currents I_{HP} , I_{BP} and I_{LP} is shown in Fig. 8. SPICE simulations have shown that typical power consumption of the circuits of Fig. 2 (9.95 mW) is lesser than that for Fig. 1 (11.1 mW).

The % total harmonic distortion (THD) of $I_{\rm HP}$, $I_{\rm BP}$ and $I_{\rm LP}$ verses input signal is shown in Fig. 9, which show that %THD is confined to the range 0.28-1.6% for all the three outputs in the range of input signal (10 μ A-100 μ A).



Fig. 9. The % THD of I_{HP} , I_{BP} and I_{LP} at $f_0 = 1$ MHz.

The simulation results are thus, found to be very good and confirm the workability of the proposed universal biquad filter configuration.

3. Concluding Remarks

A universal current-mode biquad filter employing a minimum number of active and passive elements was presented. In contrast to the universal biquads known earlier [4-8], which employ five to seven followers, the proposed circuit requires only three CFs. Also, in contrast to the earlier circuits of [4-8], the new circuit provides all the five standard filtering functions in current-mode, *explicitly* and without any component matching constraints/ conditions, from the same structure. The only drawbacks of the proposed circuit appear to be nonavailability of independent controllability of filter parameters, non-availability of non-unity H_0 and employment of current followers with non-identical numbers of output terminals.

The workability of the proposed configuration was established by SPICE simulations based upon a CMOS CF implementation which shows that the proposed biquad can be operated from a DC power supply as low as 1.65 volts DC power supplies with total power consumption of 9.95 mW. Through SPICE simulations based on 0.35 μ m CMOS technology, it is shown that circuit is suitable for implementation in CMOS technology.

An interesting question could be how the proposed structure will fair when implemented with more recent nanotechnologies rather than CMOS technologies employed here. It appears that the performance of the structure of the proposed kind may possibly degrade when channel length is reduced. However, investigation of this aspect or the general problem of searching any universal current-mode biquad suitable for implementation in nano-technology, is open for investigation.

Acknowledgements

Authors thankfully acknowledge the help of Prof. A. K. Singh and Prof. D. R. Bhaskar during preparation of this manuscript. Thanks are also due to anonymous reviewers for their constructive comments.

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Appendix A

Effect of the Parasitics and Non-Ideal Current Gains of the Current Followers (CF)

Considering the finite input resistance of the CFs to be R_{xi} and the non-ideal current gain to be α_{ik} , the output impedance to be composed of R_{zik} and C_{zik} in parallel, where *i*=1-3 and *k*=1,2 for CF1; *k*=1 for CF2 and *k*=1-3 for CF3 and assuming that the load impedance at the three outputs is much smaller as compared to $R_{zik}||1/(s R_{zik}))$, a routine analysis shows that the resulting transfer functions are as under:

$$\begin{split} \frac{I_{BP}}{I_{in}} &= -\frac{\frac{\alpha_{12}}{C_1 R_1^{'}} \left(s + \frac{1}{C_2^{'} R_2^{''}}\right) (1 + s C_1 R_{x2})}{R_{x2} C_{z33} s^3 + s^2 \left\{ 1 + R_{x2} \left(\frac{1}{R_2^{'}} + \frac{1}{R_{z33}}\right) + \right\} + s \left\{ \frac{1}{C_1 R_1^{'}} + \frac{R_{x2}}{C_2^{'}} \left(\frac{1}{R_2^{''} R_{z33}} + \frac{1}{R_1^{'} R_2^{''}} + \frac{\alpha_{11} \alpha_{33}}{R_1^{'} R_2^{'}}\right) + \right\} + s \left\{ \frac{1}{R_2^{''}} \left(\frac{1}{C_2^{'}} + \frac{R_{x2}}{C_2^{'}}\right) + \frac{1}{R_{z33} C_1} + \frac{1}{R_{z33}^{'} C_1 C_2^{'}} \left(\frac{1}{R_1^{'}} + \frac{1}{R_{z33}^{'}}\right) \right\} \right\} \\ & \left\{ \frac{\alpha_{11} \alpha_{33}}{C_1 C_2^{'} R_1^{'} R_2^{'}} + \frac{1}{R_2^{''} C_1 C_2^{'}} \left(\frac{1}{R_1^{'}} + \frac{1}{R_{z33}}\right) \right\} \end{split}$$

$$\frac{I_{HP}}{I_{in}} = \frac{\alpha_{21}s\left(s + \frac{1}{C_{2}'R_{2}''}\right)}{R_{x2}C_{z33}s^{3} + s^{2} \left\{ \begin{aligned} 1 + R_{x2}\left(\frac{1}{R_{2}'} + \frac{1}{R_{z33}}\right) + \\ C_{z33}\left(\frac{1}{C_{1}} + \frac{R_{x2}}{C_{2}'R_{2}''}\right) + \\ \\ + s \left\{ \frac{1}{C_{1}R_{1}'} + \frac{R_{x2}}{C_{2}'}\left(\frac{1}{R_{2}''R_{z33}} + \frac{1}{R_{1}'R_{2}''} + \frac{\alpha_{11}\alpha_{33}}{R_{1}'R_{2}'}\right) + \\ \\ \\ + s \left\{ \frac{1}{R_{2}''}\left(\frac{1}{C_{2}'} + \frac{C_{z33}}{C_{1}C_{2}'}\right) + \frac{1}{R_{z33}C_{1}} + \frac{1}{R_{z33}C_{1}} + \frac{1}{R_{2}''C_{1}C_{2}'}\left(\frac{1}{R_{1}'} + \frac{1}{R_{z33}}\right) \right\}$$

$$\begin{split} \frac{I_{LP}}{I_{in}} = & \frac{\frac{\alpha_{11}\alpha_{32}}{C_{1}C_{2}'R_{1}'R_{2}'}\left(1 + sC_{1}R_{x2}\right)}{R_{x2}C_{z33}s^{3} + s^{2} \left\{ 1 + R_{x2}\left(\frac{1}{R_{2}'} + \frac{1}{R_{z33}}\right) + \right\} + s \left\{ \frac{1}{C_{1}R_{1}'} + \frac{R_{x2}}{C_{2}'}\left(\frac{1}{R_{2}''R_{z33}} + \frac{1}{R_{1}'R_{2}''} + \frac{\alpha_{11}\alpha_{33}}{R_{1}'R_{2}'}\right) + \right\} + s \left\{ \frac{1}{R_{2}''}\left(\frac{1}{C_{2}'} + \frac{R_{x2}}{C_{2}'}\left(\frac{1}{R_{2}''R_{z33}} + \frac{1}{R_{1}'R_{2}''} + \frac{\alpha_{11}\alpha_{33}}{R_{1}'R_{2}'}\right) + \right\} + s \left\{ \frac{1}{R_{2}''}\left(\frac{1}{C_{2}'} + \frac{R_{x2}}{C_{2}'}\left(\frac{1}{R_{2}''R_{z33}} + \frac{1}{R_{1}'R_{2}''} + \frac{\alpha_{11}\alpha_{33}}{R_{1}'R_{2}'}\right) + \frac{1}{R_{2}''C_{2}''R_{2}'''}\right\} + s \left\{ \frac{\alpha_{11}\alpha_{33}}{C_{1}C_{2}'}\left(\frac{1}{R_{1}'} + \frac{1}{R_{2}''C_{2}''}\left(\frac{1}{R_{1}'} + \frac{1}{R_{z33}}\right) \right\} \end{split}$$

where

$$R_{1}^{'} = R_{1} + R_{x1}$$
, $R_{2}^{'} = R_{2} + R_{x3}$, $C_{2}^{'} = C_{2} + C_{z11} + C_{z31}$, $\frac{1}{R_{2}^{'}} = \frac{1}{R_{2}^{'}} - \frac{\alpha_{31}}{R_{2}^{'}} + \frac{1}{R_{z11}} + \frac{1}{R_{z31}}$

Under ideal parasitic conditions i.e. $R_{x1} = R_{x2} = R_{x3} = 0$; $C_{z11} = C_{z31} = C_{z33} = 0$ and also at $\alpha_{31} = 1$ above mentioned transfer functions reduce to:

$$\frac{I_{BP}}{I_{in}} = -\frac{\frac{\alpha_{12}s}{C_1R_1}}{s^2 + \frac{s}{C_1R_1} + \frac{\alpha_{11}\alpha_{33}}{C_1C_2R_1R_2}}$$
$$\frac{I_{HP}}{I_{in}} = \frac{\alpha_{21}s^2}{s^2 + \frac{s}{C_1R_1} + \frac{\alpha_{11}\alpha_{33}}{C_1C_2R_1R_2}}$$
$$\frac{I_{LP}}{I_{in}} = \frac{\frac{\alpha_{32}}{\alpha_{33}} \left(\frac{\alpha_{11}\alpha_{33}}{C_1C_2R_1R_2}\right)}{s^2 + \frac{s}{C_1R_1} + \frac{\alpha_{11}\alpha_{33}}{C_1C_2R_1R_2}}$$

It is seen from above that using the non-unity current gain of the current followers filter parameters can be controlled as under:

For BP filter: ω_0 by α_{11} and / or α_{33} ; H_0 by α_{12}

For HP filter: ω_0 by α_{11} and / or α_{33} ; H_0 by α_{21}

For LP filter: ω_0 by α_{11} ; H_0 by α_{32}