Opportunistic Relaying in Time Division Broadcast Protocol with Incremental Relaying

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Abstract. In this paper, we investigate the performance of time division broadcast protocol (TDBC) with incremental relaying (IR) when there are multiple available relays. Opportunistic relaying (OR), i.e., the “best” relay is selected for transmission to minimize the system’s outage probability, is proposed. Two OR schemes are presented. The first scheme, termed TDBC-OIR-I, selects the “best” relay from the set of relays that can decode both flows of signal from the two sources successfully. The second one, termed TDBC-OIR-II, selects two “best” relays from two respective sets of relays that can decode successfully each flow of signal. The performance, in terms of outage probability, expected rate (ER), and diversity-multiplexing tradeoff (DMT), of the two schemes are analyzed and compared with two TDBC schemes that have no IR but OR (termed TDBC-OR-I and TDBC-OR-II accordingly) and two other benchmark OR schemes that have no direct link transmission between the two sources.

Keywords
Opportunistic relaying, incremental relaying, two-way channel, time division broadcast protocol.

1. Introduction

Two-way relaying improves the spectral efficiency of conventional half-duplex cooperative communications due to two concurrent flows of signal transmission from the two sources [1]. It executes information exchange between the two sources with the aid of the relay. There are two well-known protocols for decode-and-forward (DF) two-way relaying channel, i.e., physical-layer network coding (PNC) and time division broadcast protocol (TDBC) [2], [3]. To fulfill the information exchange, PNC needs two time slots while TDBC entails three time slots. Recently, there were research interests in studying two-way relaying with direct link transmission [2-6], since in many physical environments the direct link does exist between the two sources. The direct link transmission provides one more diversity order than the scenario where there is no direct link transmission [2], but suffers from spectral efficiency loss in the TDBC protocol because of its three-time-slot transmission. To overcome this disadvantage, we proposed to combine incremental relaying (IR) with TDBC to form a new scheme, i.e., TDBC-IR [7], since IR was well known for enhancing spectral efficiency [8], [9]. It has been shown that TDBC-IR possesses improved spectral efficiency performance compared with TDBC and even PNC (except at asymptotic high signal-to-noise ratio (SNR) regime where their spectral efficiency performance meets [7]), though it consumes three time slots in transmission. This is because TDBC-IR utilizes the acknowledgement (ACK) feedback information to reduce the unnecessary retransmission times if the direct link transmission is successful. The limitation of [7] is that it considers only one relay scenario while in practical systems there may be multiple available relays, such as the cellular system and wireless sensor networks.

In this paper we focus on the multiple-relay two-way relaying channel with IR. Particularly, we investigate opportunistic relaying (OR), i.e., the “best” relay is selected based on minimizing the system’s outage probability. It has been revealed in one-way cooperative communications that opportunistic relaying could achieve the same diversity order as the all-relay-participation scheme but has improved multiplexing gain over the latter¹, since OR consumed two orthogonal channels (time slots, frequency bands, or CDMA codes) while the latter entailed multiple orthogonal channels for transmission [10]. For two-way relaying, OR had been widely adopted as well (see [12]-[21] and references therein). Max-min based relay selection criterion was adopted in [12]-[15] for amplify-and-forward (AF) two-way relaying channel. This criterion selects the relay corresponding to the maximum of the worse SNRs of the two sources. [16] chose the relay to maximize the sum ergodic rate of the two sources with AF retransmission. [17] optimized the achievable ergodic sum-rate by choosing the best relay for DF two-way relaying channel. The Max-min criterion was further utilized in [18] to minimize the frame error rate of DF two-way relaying channel. AF two-way

¹ It is noted that there was a distributed space-time-coded protocol [11] which also consumed two orthogonal channels for all relays’ participation. However, it requires rigid synchronization among all relays which is hard to realize in practice and makes it beyond the scope of the paper.
relaying channel with direct link transmission employed
still the max-min criterion to select the best relay to max-
imize the capacity in [19]. In [20], [21], the max-min crite-
rion was used to select the relays to minimize the bit error
rate of the three-time-slot two-way relaying channel with
no direct transmission.

This paper studies the DF retransmission scheme and
adopts the max-min criterion to select the best relay, aim-
ing at minimizing the system’s outage probability. Firstly,
we propose two OR schemes for TDBC-IR, i.e., TDBC-
OIR-I and TDBC-OIR-II (note that the acronym OIR
means opportunistic relaying with incremental relaying). In
the TDBC-OIR-I protocol, the best relay is selected in the
set of relays that can decode successfully both flows of
signal from the two sources according to the max-min
criterion. In the TDBC-OIR-II protocol, the relays are
selected from two sets of relays that can decode success-
sfully each flow of signal. The selection process is identical
to that of [20] and [21] in essence. It is noted that there
may be one or two best relays in TDBC-OIR-II (which will
be shown in section 2). Secondly, the performance of two
proposed protocols is investigated in terms of outage prob-
ability, expected rate (ER), and diversity-multiplexing
tradeoff (DMT). The expected rate is defined as the aver-
sage rate by a system [22]. The DMT provides a more compre-
sensive view on the performance of a system since it takes
into account both the diversity gain and multiplexing gain

The system consists of two sources (denoted by $S_1$ and
$S_2$ respectively) and $N$ relays (denoted by $R_1$, $R_2$, ..., $R_N$), see Fig. 1. The two sources want to exchange informa-
tion to each other with the help of the relays. There is a
direct link between the two sources.

For TDBC-OIR-I (see Fig.1 (a)), $S_1$ broadcasts the infor-
mation-bearing symbol $x_1$ to $S_2$ and the relays in the first
time slot. The received signal at $S_2$, denoted by $y_{S2j}$, is thus
$y_{S2j} = h_{s2j}x_1 + n_{s2j}^i$, where $n_{s2j}^i$ is the complex additive white
Gaussian noise (AWGN) at $S_2$ and $h_{s2j}$ is the complex
channel gain between $S_1$ and $S_2$. Throughout the paper, we
adopt the following notation rules: (1) $y_g$ and $n_g$ denote
the signal received by and the complex AWGN at user $g$
($g \in \{S_1, S_2, R_1, R_2, ..., R_N\}$) at time slot $i$ ($i \in \{1,2,3\}$),
respectively; (2) $h_{ab}$ denotes the complex channel gain
from user $a$ to $b$, with $a,b \in \{S_1, S_2, R_1, R_2, ..., R_N\}$ and $a \neq b$.
Furthermore, $X_{ab} = |h_{ab}|^2$ is the square of the channel gain.
The received signal at the $j$th ($j \in \psi$ and $\psi = \{1,2,3\}$) relay
is expressed as
$y_{abj} = h_{abj}x_1 + n_{abj}$. If $S_2$ decodes the symbol from $S_1$ successfully, it sends back an ACK to $S_1$.
Otherwise, it sends back a non-acknowledgement (NACK).
In the second time slot, $S_2$ broadcasts the information-bear-

2 The transmit power of the users may be different as well
(see [2] or [7]), which makes the performance analysis (e.g. out-
age probability) more tedious but of less interest, since we focus
more on the diversity gain and multiplexing gain (or spectral
efficiency) of the protocols in the paper.
The “best” in the decoding set is selected

![Diagram](image1)

(a) TDBC-OIR-I

![Diagram](image2)

(b) TDBC-OIR-II

Fig. 1. The system models of TDBC-OIR-I and TDBC-OIR-II.

The “best” in the decoding set is selected

![Diagram](image3)

(a) PNC-OIR-I

![Diagram](image4)

(b) PNC-OIR-II

Fig. 2. The system models of PNC-OIR-I and PNC-OIR-II.

The received symbols at $S_1$ and the $j$th ($j \in \{1, 2\}$) relay are written as $y_1^2 = h_{1,1} x_1 + n_1^2$ and $y_j^2 = h_{j,1} x_j + n_j^2$, respectively. If $S_1$ decodes the symbol from $S_2$ successfully, it sends back an ACK, or else, it sends back a NACK. We assume that all the relays and the two sources receive the ACK or NACK feedback correctly and the feedback transmission consumes negligible time (or with no sacrificing of spectral efficiency) [10]. Then if the relays receives two ACKs (which means that the direct link transmission of the two sources is successful)\(^3\), the system enters into the next round of new information transmission process. Otherwise, the system enters into the third time slot transmission. The relays that can decode successfully both flows of signal from the two sources constitute the decoding set, denoted by $D_m$ (the subscript $m$ denotes there are $m$ relays in the set), which is given by

$$D_m = \left\{i : i \in \mathcal{W}, |h_{i,1}| > G \right\}$$

with $G$ being the threshold value the relay can decode the symbols successfully (the value of $G$ is specified in section 3). In the third time slot, the best relay is selected, aiming at minimizing the system’s outage probability, to transmit the received signals. The relay selection is based on the max-min criterion, i.e., the selected relay’s index is

$$k = \arg \max_{m \in \mathcal{W}} \left\{X_{k,1} \times X_{k,2} \right\}.$$ 

The operation of the selected relay is that it first executes bit-level XOR operation (i.e., PNC) on the received information from the two sources, and then broadcasts the remapped symbol to both sources. The received signals at $S_1$ and $S_2$ are given by $y_{S_1}^3 = h_{h,1} x_1 + n_{S_1}^3$ and $y_{S_2}^3 = h_{h,2} x_2 + n_{S_2}^3$, respectively, where $x_1 = x_1 \oplus x_2$. Since the sources know their own transmitted symbols, they can execute backward propagating self-interference cancellation to recover their intended signals [1]. It is noted that if the decoding set is empty, the third time slot is idle.

For TDBC-OIR-II (see Fig. 1 (b)), the transmission process of the first and second time slots is the same as that of TDBC-OIR-I. If ACK is received by the relays, the system enters into the next round of transmission. Otherwise, the third time slot transmission is invoked. There are two decoding sets in the TDBC-OIR-II. The decoding set of $S_1$, denoted by $D_{1,m}$\(^5\), includes relays that can decode successfully the symbol from $S_1$. It can be expressed as $D_{1,m} = \left\{i : i \in \mathcal{W}, |h_{i,1}| > G \right\}$, where $G$ is the threshold value. The decoding set of $S_2$ is defined similarly and expressed as $D_{2,m} = \left\{j : j \in \mathcal{W}, |h_{j,2}| > G \right\}$. For relay selection, the relay in $D_{1,m}$ with the largest channel gain to $S_2$ is selected as the best relay (denoted by $R_1$) for $S_1$. Similarly, the relay in $D_{2,m}$ with the largest channel gain to $S_1$ is selected as the best relay (denoted by $R_2$) for $S_2$. For the latter case, the operation process is identical to that of TDBC-OIR-I in the third time slot. For the former case, $R_1$ and $R_2$ retransmit the received symbols, respectively. Thus, the received signals at $S_1$ and $S_2$ are given by $y_{S_1}^3 = h_{h,1} x_1 + h_{R,1} x_i + n_{S_1}^3$ and $y_{S_2}^3 = h_{h,2} x_2 + h_{R,2} x_i + n_{S_2}^3$, respectively. Since the sources know their own transmitted symbols, the backward propagation self-interference can be cancelled and thus the received signals become $y_{S_1}^3 = h_{h,1} x_1 + n_{S_1}^3$ and $y_{S_2}^3 = h_{h,2} x_2 + n_{S_2}^3$. It is emphasized that the relay selection process can be executed in the same distributed way as that of [10] and is omitted here.

\(^3\) With the assumptions (1), (2), and (3), we know that the transmission of $S_1$ and $S_2$ is both successful (or failed). Therefore only one ACK (or NACK) indicating the success (or failure) of the direct link transmission is needed [7]. In the following description of TDBC-OIR-II, we only say one ACK (or NACK).

\(^5\) The subscript $i$ means that it is the decoding set of $S(i=1,2)$ whereas the subscript $m_i$ ($i=1,2$) denotes that there are $m_i$ relays in the set. This notation rule is also applied for other protocols expected specified otherwise.
For the comparison purpose, we also introduce two protocols, i.e., TDBC-OR-I and TDBC-OR-II. The TDBC-OR-I (TDBC-OR-II) protocol works in the same way as TDBC-OIR-I (TDBC-OIR-II) except that there is no feedback here, i.e., it has no IR. Thus, it always enters into the third time slot’s transmission. This is also the reason why TDBC-OR-I (TDBC-OR-II) has worse spectral efficiency performance than TDBC-OIR-I (TDBC-OIR-II).

### 2.2 PNC-OR-I and PNC-OR-II

For PNC-OR-I (see Fig. 2(a)), there is no direct link between the two sources. The transmission is finished in two time slots. In the first time slot, $S_1$ and $S_2$ transmit simultaneously to the relays. The relays that can decode successfully both sources’ information are collected in the decoding set $D_{12}^{PNC}$. The best relay in the decoding set is selected according to the max-min criterion, i.e., the index of the selected relay is $k = \arg \max \min \{X_{s,k}, X_{r,k}\}$, the same as TDBC-OIR-I. In the second time slot, the selected relay operates in the same way as the third time slot’s transmission of TDBC-OIR-I. After receiving in the second time slot, $S_1$ and $S_2$ execute self-interference cancellation to recover their corresponding signals.

For PNC-OR-II (see Fig. 2(b)), there is no direct link either. However it consumes three time slots for transmission. Its operation process is identical to that of TDBC-OR-II (except that there is direct link transmission in the latter) and thus is omitted here.

### 3. Performance Analysis

Before the analysis, we make the following assumptions: (1) all complex AWGN terms, at different users and time slots, are independent and identically distributed (i.i.d.) with distribution $CN(0,1)$; (2) all channel gains are also i.i.d. complex RVs with distribution $CN(0,1)$, i.e., Rayleigh fading, and independent of the noises; and 3) the channels are slow, flat-block fading, i.e., the channel gains keep the same in one block of transmission and change independently from one block to the next block. The PDF of the square of the channel gain $X_{ab} = |h_{ab}|^2$ is then given as $f_{X_{ab}}(x) = e^{-x}$ when $x \geq 0$ and $f_{X_{ab}}(x) = 0$ otherwise.

To make performance comparison of the abovementioned protocols under a unified framework, we assume that each block of transmission includes $M$ bits information and each time slot has a length of $T$ seconds (s) [7]. The two sources thus have $M/2$ bits information to transmit to each other in one block. Let the transmit power be $\rho$, then the transmit SNR is $\rho$ since the noise’s power is 1. The system’s transmission bandwidth $B$ is assumed to be 1 Hz. Define $c = M/(3TB) = M/(3T)$ bits/s/Hz as the baseline data rate. It is further assumed that the relays know the instantaneous channel state information (CSI) of their corresponding backward and forward channels, and the corresponding receivers have their backwards channels’ CSI. In the following, we first derive the outage probability, then the expected rate, and finally the DMT performance of the abovementioned protocols.

#### 3.1 Outage Probability Performance Analysis

For TDBC-OIR-I (with direct link), the instantaneous channel capacity of the direct link transmission is $I_{s,S} = \frac{1}{2} \log(1 + \rho X_{s,k})$, where the factor $1/2$ accounts for the two-time-slot transmission if the direct transmission is successful. Similarly, the instantaneous channel capacity of the links $S_i \rightarrow R_i$ and $S_j \rightarrow R_i$ ($i \neq j$) are denoted by $I_{s,S} = \frac{1}{3} \log(1 + \rho X_{s,k})$ and $I_{s,S} = \frac{1}{3} \log(1 + \rho X_{s,k})$, respectively. Therefore, the probability that the ith relay $R_i$ can decode successfully both flows of signal from the sources are given by

$$P_i^{(2)} = P_i \left( I_{s,R_i} > \frac{C}{2} \right)$$

$$= P_i \left( X_{s,R_i} > G \right)$$

where $G = \frac{2^{1/c} - 1}{\rho}$.

For any relay in the decoding set (denote its index as $i$, i.e., $i \in D_w$), the conditional CDF and PDF of $X_{s,R_i}$ (or $X_{s,k}$) are written as

$$F_{X_{s,R_i} > G}(z) = P_{X_{s,R_i} > G}$$

and

$$f_{X_{s,R_i} > G}(z) = \begin{cases} 1 - e^{-(z-G)}, & z > G \\ 0, & z \leq G \end{cases}$$

Let $X_{R_i} = \min \{X_{s,R_i}, X_{s,k}\}$, $i \in D_w$, then the conditional CDF and PDF of $X_{R_i}$ are expressed as

$$F_{X_{R_i} > G}(z) = P_{X_{R_i} > G}$$

and

$$f_{X_{R_i} > G}(z) = \begin{cases} 1 - e^{-(z-G)}, & z > G \\ 0, & z \leq G \end{cases}$$

Since the channels are reciprocal, the transmitters know the instantaneous CSI as well.
We define the outage event occurs if either the link from \(S_1\) to \(S_2\) or the link from \(S_2\) to \(S_1\) is in outage in the paper [2]. Therefore, the outage probability of TDBC-OIR-I is written as \((k\) is the index of the selected best relay)

\[
P_r^{TDBC-OIR-I}(C) = P_r(\text{both the direct and selected relay links are in outage})
\]

\[
= P_r(D_m) P_r\left(2t \cdot I_{S_S} < \frac{M}{2} \right)+ \sum_{a=1}^{N} P_r(D_a)
\]

\[
\times P_r\left(2t \cdot I_{S_S} < \frac{M}{2} \right)\left(1-e^{-2G}\right)\left(1-e^{-G}\right)
\]

where

\[
I_{S_S} = \frac{1}{3} \log(1 + \rho(X_{S_S} + X_{S_R}))
\]

\[
I_{S_R} = \frac{1}{3} \log(1 + \rho(X_{S_R} + X_{S_R}))
\]

are the instantaneous capacity at \(S_2\) and \(S_1\) respectively after the third time slot’s transmission in TDBC-OIR-I [2, 3]. Equality (a) is derived according to the total probability formula and equality (b) results from the fact that whenever there is a relay that can simultaneously decode the two flows of signal from the sources the third time slot’s transmission will be successful, i.e., the outage event will never happen.

Based on this fact, we know that the outage event may be possible for TDBC-OIR-II (with direct link) only when there is no relay that can simultaneously decode successfully both flows of signal if the direct link transmission is failed. Thus, we can redefine the decoding sets for the two sources. For \(S_1\), its decoding set, denoted by \(D_{1,m}\), is defined as the relays that can decode successfully the symbol from \(S_1\) but cannot decode the symbol from \(S_2\). Thus we have

\[
D_{1,m} = \{ i : i \in \psi, X_{S_S, i} > G, \text{ and } X_{S_R, i} < G \} \quad (m_i \leq N).
\]

The decoding set of \(S_2\) is defined similarly and denoted by \(D_{2,m}\) as

\[
D_{2,m} = \{ j : j \in \psi, X_{S_S, j} = G, \text{ and } X_{S_R, j} > G \} \quad (m_j \leq N-m_i).
\]

It is obvious that \(D_{1,m} \cap D_{2,m} = \phi\), where \(\phi\) is the empty set. Except the relays in the two decoding sets, there are \(N-m_i\) \(m_j\) relays left that can not decode successfully either flow of signal. Collecting these relay as the set \(D_{2,N-m-m_i}\), we have

\[
D_{2,N-m-m_i} = \{ i : i \in \psi, X_{S_S, i} < G, \text{ and } X_{S_R, i} < G \}.
\]

Let \(R_i^1\) and \(R_i^2\) be the two selected best relays for \(S_1\) and \(S_2\), i.e., \(R_i^1 = \arg \max_{i \in D_m}(X_{S,R,i})\), and \(R_i^2 = \arg \max_{i \in D_m}(X_{R,R,i})\), the outage probability of TDBC-OIR-II is thus written as (according to the total probability formula)

\[
P_r^{TDBC-OIR-II}(C)
\]

\[
= P_r(\text{both the direct and selected relay links are in outage})
\]

\[
= P_r(\Pi_0) P_r\left(2t \cdot I_{S_S} < \frac{3C}{4} \right)+ \sum_{a=1}^{N} P_r(D_a) P_r\left(D_m \bigcap D_{2,N-m-m_i}\right)
\]

\[
\times P_r\left(2t \cdot I_{S_S} < \frac{3C}{4} \right)\left(1-e^{-2G}\right)\left(1-e^{-G}\right)
\]

\[
= \sum_{m_i=1}^{N} \sum_{a=1}^{N} P_r(D_m) P_r(D_a)
\]

\[
\times P_r\left(2t \cdot I_{S_S} < \frac{3C}{4} \right)\left(1-e^{-2G}\right)\left(1-e^{-G}\right)
\]

\[
\times P_r\left(\min(X_{S_S}, X_{S_R}) < G \right) A_{m_i,m_j}
\]

(7)

where \(\Pi_0\) is the event that “either \(D_{1,m}\) or \(D_{2,m}\) is empty”, \(I_{S_S} = \frac{1}{3} \log(1 + \rho(X_{S_S} + X_{S_R}))\) and \(I_{S_R} = \frac{1}{3} \log(1 + \rho(X_{S_R} + X_{S_R}))\) are the instantaneous capacity at \(S_2\) and \(S_1\) respectively after the third time slot’s transmission in TDBC-OIR-II, \(P_r = e^{-G}\) and \(P_r = e^{-G}\) are the probability that one relay can decode successfully the symbols from \(S_1\) and \(S_2\), respectively, and \(A_{m_i,m_j} = P_r\left(\min(X_{S_S}, X_{S_R}) < G \right) A_{m_i,m_j}\).

From the analysis above, we learn that the calculation of \(A_{m_i,m_j}\) is the key step in deriving \(P_r^{TDBC-OIR-II}(C)\). To derive \(A_{m_i,m_j}\), we first have the following conditional CDFs and PDFs.

\[
F_{X_S|X_S < G}(z) = P_r\left(X_{S,R} < z | X_{S,S} < G \right)
\]

\[
= \frac{P_r\left(X_{S,R} < z \text{ and } X_{S,R} < G \right)}{P_r\left(X_{S,R} < G \right)}
\]

\[
= \begin{cases} 0, & z < 0 \\ 1 - e^{-z}, & 0 \leq z \leq G \\ 1, & z > G \\ 
\end{cases}
\]

and thus the conditional CDF of \(X_{S,R} = \max_{i \in D_m}(X_{S,R,i})\) is

\[
F_{X_{S,R}|X_{S,R} < G}(z) = \begin{cases} 0, & z < 0 \\ \frac{1-e^{-z}}{1-e^{-G}}, & 0 \leq z \leq G \\ 1, & z > G \\ 
\end{cases}
\]

(9)
identical to that of $X_{S_i R_i} (i \in D_{n,m})$. The conditional CDF of $X_{R_i} = \max_{i \in D_{n,m}} (X_{S_i R_i})$ is then written as

$$F_{X_{R_i} | X_{S_i R_i}}(z) = \begin{cases} 
0, & z < 0 \\
\left(1 - \left(1 - \frac{1}{1 - e^{-G}}\right)^{m_i} \right), & 0 \leq z \leq G \\
1, & z > G
\end{cases} \quad (10)$$

Therefore, the CDF of $X = \min(X_{R_i}, X_{S_i R_i})$ is

$$F_X(z) = \begin{cases} 
0, & z < 0 \\
\left(1 - \left(1 - \frac{1}{1 - e^{-G}}\right)^{m_i} \right) \times \left(1 - \left(1 - \frac{1}{1 - e^{-G}}\right)^{m_{i+1}} \right), & 0 \leq z \leq G \\
1, & z > G
\end{cases} \quad (11)$$

and its PDF is

$$f_X(z) = \begin{cases} 
0, & z < 0 \text{ or } z > G \\
\frac{m_i \left(1 - e^{-G}\right)^{m_i - 1} + m_{i+1} \left(1 - e^{-G}\right)^{m_{i+1} - 1}}{\left(1 - e^{-G}\right)^{m_i + m_{i+1}}}, & 0 \leq z \leq G
\end{cases} \quad (12)$$

With these (conditional) CDFs and (conditional) PDFs at hand, we thus calculated $A_{m,m_2}$ as in (13). Substituting (13) into (7), we then obtain the outage probability of TDBC-OIR-II.

Remark: The outage probability of TDBC-OIR-I (TDBC-OIR-II) is identical to that of TDBC-OIR-I (TDBC-OIR-II). This is explained as follows. The outage event occurs in TDBC-OIR-I (TDBC-OIR-II) when the two events occur simultaneously, i.e., the direct link transmission is failed and the third time slot’s transmission is unsuccessful either. Since the intersection of the two events is exactly the latter event. Thus the outage probability of TDBC-OIR-I (TDBC-OIR-II) is decided by the latter event, which is exactly the outage event of TDBC-OIR-I (TDBC-OIR-II) [7, 8].

For PNC-OR-I (without direct link), the relay $R_i (i \in \psi)$ must meet three conditions to decode successfully both flows of signal in the first time slot, i.e.,

$$2t \cdot \frac{1}{2} \log \left(1 + \rho X_{S_R} \right) > \frac{M}{2}, \quad 2t \cdot \frac{1}{2} \log \left(1 + \rho X_{S_R} \right) > \frac{M}{2} \quad \text{and} \quad 2t \cdot \frac{1}{2} \log \left(1 + \rho (X_{S_R} + X_{S_R}) \right) > \frac{M}{2} \quad [2].$$

Therefore, the decoding set $D_{12}^{PNC}$ is written as

$$D_{12}^{PNC} = \{ j : j \in \psi, X_{S_j R_j} > G, X_{S_j R_j} > G, \text{ and } X_{S_j R_j} + X_{S_j R_j} > L \},$$

where $L = \frac{2^x - 1}{\rho}$. Since the selected best relay (in the decoding set) is utilized to retransmit in the second time slot, outage event will never happen if the decoding set is not empty. Thus, the outage event of PNC-OR-I is exactly that the decoding set is empty. The outage probability of PNC-OR-I is then calculated as $P_{12}^{PNC-OR-I} (C) = \left(1 - P_{12}^{PNC-OR-I} \right)^N$, where $P_{12}^{PNC-OR-I} (C)$ is the probability that one relay can decode successfully both flows of signal in the first time slot and is expressed in (14).

$$A_{m,m_2} = P_r \left( X_{S_i R_j} + X < G \right) = \int_0^G e^{-x} \left[ \frac{1}{1 - e^{-G}} \int_0^G e^{-x} \left( \sum_{k=1}^{m_1} \frac{m_k}{k!} e^{-x} \right) \right] dx$$

$$= \frac{1}{1 - e^{-G}} \int_0^G e^{-x} \left( \sum_{k=1}^{m_1} \frac{m_k}{k!} e^{-x} \right) \left[ \frac{1}{1 - e^{-G}} \sum_{k=2}^{m_2} \frac{m_k}{k!} e^{-x} \right] dx$$

$$= \frac{1}{1 - e^{-G}} \int_0^G e^{-x} \left( \sum_{k=1}^{m_1} \frac{m_k}{k!} e^{-x} \right) \left[ \frac{1}{1 - e^{-G}} \sum_{k=2}^{m_2} \frac{m_k}{k!} e^{-x} \right] dx \quad (13)$$

With the same way as TDBC-OIR-II, we obtain the outage probability of PNC-OR-II (without direct link) according to the total probability formula as follows
The expected rates of TDBC-OIR-I, TDBC-OIR-II, TDBC-OR-I, TDBC-OR-II, PNC-OR-I, and PNC-OR-II are written as follows respectively.

\[ R_{TDBC-OIR-I} = \frac{3C}{2} \left( 1 - P_r^{direct} \right) \]
\[ + C \cdot P_r^{direct} \left( 1 - P_r^{TDBC-OIR-I} (C) \right) \]
\[ R_{TDBC-OIR-II} = \frac{3C}{2} \left( 1 - P_r^{direct} \right) \]
\[ + C \cdot P_r^{direct} \left( 1 - P_r^{TDBC-OIR-II} (C) \right) \]
\[ R_{TDBC-OR-I} = C \cdot \left( 1 - P_r^{TDBC-OR-I} (C) \right) \]
\[ R_{TDBC-OR-II} = C \cdot \left( 1 - P_r^{TDBC-OR-II} (C) \right) \]
\[ R_{PNC-OR-I} = \frac{3C}{2} \left( 1 - P_r^{PNC-OR-I} (C) \right) \]
\[ R_{PNC-OR-II} = C \cdot \left( 1 - P_r^{PNC-OR-II} (C) \right) \]

where \( P_r^{direct} (C) \) is the outage probability of the direct link transmission, \( P_r^{TDBC-OIR-I} (C) \) and \( P_r^{TDBC-OIR-II} (C) \) are the outage probability of TDBC-OR-I and TDBC-OR-II and are equal to \( R_{TDBC-OIR-I} (C) \) and \( R_{TDBC-OIR-II} (C) \) respectively. Substituting the corresponding outage probability expressions in subsection 3.1 into these expressions, we readily obtain the expected rates of the six protocols. The expected rate's physical meaning is the maximum data rate or spectral efficiency that can be supported without outage. Note that the expected rate is defined on the system’s basis not on each source’s.

### 3.3 DMT Performance Analysis

To derive the DMT of TDBC-OIR-I, let \( \rho \to +\infty \), then the expected rate becomes \( R_{TDBC-OIR-I} = \lim_{\rho \to +\infty} R_{TDBC-OIR-I} = \frac{3C}{2} \) since \( P_r^{direct} \) and \( P_r^{TDBC-OIR-I} (C) \) tends to zero. Furthermore, let \( R_{TDBC-OIR-I} = \rho \log \rho \), where \( \rho \) is the multiplexing gain, we have \( G = \left( \frac{3C}{2} - 1 \right) / \rho \). According to the DMT definition [10, Definition 3], the DMT of TDBC-OIR-I is given as follows

\[ d_{TDBC-OIR-I} (r) \equiv \lim_{\rho \to +\infty} \log P_r^{TDBC-OIR-I} \left( C = \frac{2}{3} \log \rho \right) \]
\[ = \lim_{\rho \to +\infty} \log \left( 1 - e^{-\rho r} \right)^{N} \left( 1 - e^{-\rho r} \right) \]
\[ = \lim_{\rho \to +\infty} \log \left( 2^N \rho^{N(1+r)} + o(\rho^{N(1+r)}) \right) (\rho^{r^2} + o(\rho^{r^2})) \]
\[ = \lim_{\rho \to +\infty} \log \left( 2^N \rho^{N(1+r)} + o(\rho^{N(1+r)}) \right) \]
\[ = (N + 1)(1 - r) \]

For TDBC-OR-II, its DMT is given in the following theorem.

**Theorem:** The DMT of TDBC-OR-II is expressed as \( d_{TDBC-OR-II} (r) = (N + 1)(1 - r) \) with \( 0 \leq r \leq 1 \), the same as TDBC-OIR-I.

**Proof:** Let \( \rho \to +\infty \), then the expected rate becomes \( R_{TDBC-OR-II} = \lim_{\rho \to +\infty} \log \rho \), considering the outage probability expression (7) and let \( R_{TDBC-OR-II} = \rho \log \rho \), the first two terms can be expressed as

\[ 2(1 - e^{-\rho r})^{N+1} (1 - 1 - e^{-\rho r})^{N} \approx \rho^{N+1}(1 - r) \]
\[ (1 - e^{-\rho r})^{2N+1} \approx \rho^{2N+1}(1 - r) \]

In the following we calculate the third term of (7). Denoting \( X_{S_1} = \rho^{-n} \), \( X = \rho^{-n} \), and \( B_{n,n} = \sum_{m_1=0}^{N-n} \binom{N-n}{m_1} \left( 1 - r \right)^{m_1} \left( 1 - r \right)^{(N-n) - m_1} \right) \), it is shown from [24, eq. (5)] that the PDF of \( u \) is given by

\[ f_u (u) = \begin{cases} \rho^{-u}, & u > 0 \\ 0, & u < 0 \end{cases} \]  

(23)

The PDF of \( v \) can be derived through variable change based on (12), and is written as
Following the same way, we obtain the DMTs of PNC-OR-I and TDBC-OR-II are same and expressed as
\[
\rho \approx \min (m, m, v + (r - 1) + u) = 1 - r.
\]
Therefore, the outage probability can be rewritten as
\[
P_r^{\text{TDBC-OR-II}} (C) \approx \rho^{-N(1-r)} + \rho^{-N(1-r)} + \rho^{-N(1-r)}.
\]
\[
d_{\text{TDBC-OR-I}} (r) = N(1 - 3r/2), \quad \text{with } 0 \leq r \leq 2/3.
\]
\textbf{Proof:} Let \( \rho \to +\infty \), then the expected rates of the two protocols, i.e., PNC-OIR-I and PNC-OIR-II become
\[
R_{\text{PNC-OIR-I}} = \frac{3C}{2} \quad \text{and} \quad R_{\text{PNC-OIR-II}} = C.
\]
We first derive the DMT of the former protocol then the latter.
\[
1 - R_{\text{PNC-OIR-I}} = \left( 1 - e^{-(1-r)} \left( \rho^{-1} - 2\rho^{-1} + 1 \right) \right) \approx 1 - e^{-(1-r)},
\]
\[
\rho = e^{-\rho} \approx \rho^{-1} + \rho^{-2}.
\]
The DMT of PNC-OIR-II is thus written as
\[
d_{\text{PNC-OIR-II}} (r) = N(1 - 3r/2).
\]
For PNC-OIR-II, let \( R_{\text{PNC-OIR-II}} = r \log \rho \), where \( r \) is the multiplexing gain, then \( G = (\frac{2}{3} - 1)/\rho \Rightarrow \rho \approx \rho^{-1 - 2} \). Therefore, we have
\[
1 - R_{\text{PNC-OIR-II}} = \left( 1 - e^{-\rho} - 2\rho^{-1} + 1 \right) \approx 1 - e^{-\rho} + \rho^{-1} - \rho^{-2}.
\]
\[
\rho = \rho^{-1} + o(\rho^{-1}) \approx 2\rho^{-1} - \rho^{-2}.
\]
The DMT of PNC-OIR-II is readily obtained as
\[
d_{\text{PNC-OIR-II}} (r) = N(1 - 3r/2).
\]
\subsection{4. Simulation Results and Discussions}
In this section, we provide simulation results to validate the analytical results in the above section. The baseline data rate is set to be \( C = 1 \text{ bits/s/Hz} \). The transmit power is equal to the transmit SNR \( \rho \) since the AWGNs are assumed to be with unit power. All channel gains are independent complex Gaussian distributed RVs with distribution \( CN(0,1) \). The number of relays is denoted by \( N \).

Fig. 3 presents the DMTs of the six protocols, i.e., TDBC-OIR-I, TDBC-OIR-II, TDBC-OR-I, TDBC-OR-II, PNC-OIR-I, and PNC-OIR-II. It is shown that all the protocols achieve the full diversity order (i.e., the maximum achievable diversity gain, corresponding to \( r = 0 \)). For the former four protocols, the diversity order is \( N+1 \). The diversity order of the latter two protocols is \( N \) because of lack of direct link transmission (leading to one diversity order loss). As for multiplexing gain, TDBC-OIR-I (TDBC-OR-II) has greater multiplexing gain than TDBC-
OR-I (TDBC-OR-II) for given diversity gain. This is because the former utilize feedback to reduce the number of time slots for transmission. Since multiplexing gain reflects the increase of spectral efficiency (or data rate) [23], the spectral efficiency performance of the former is better than the latter (see from Fig. 5 and Fig. 7). Though TDBC-OIR-I and TDBC-OIR-II may consume three time slots for transmission, its maximum achievable multiplexing gain is the same as PNC-OR-I (with two time slots for transmission). The PNC-OR-II protocol has the worst DMT performance (its maximum achievable diversity and multiplexing gains are $N$ and $2/3$ respectively) since it has no direct link transmission and consumes three time slots for transmission.

Fig. 3. The DMTs of TDBC-OIR-I, TDBC-OIR-II, TDBC-OR-I, TDBC-OR-II, PNC-OR-I, and PNC-OR-II.

Fig. 4. Outage performance comparison between TDBC-OIR-I and TDBC-OIR-II for various numbers of relays.

Fig. 5. ER performance comparison between TDBC-OIR-I and TDBC-OIR-II for various numbers of relays.

Fig. 6. The outage performance comparison of TDBC-OIR-I, TDBC-OIR-II, PNC-OR-I, and PNC-OR-II is presented in Fig. 6 when there are two relays, i.e., $N=2$. From the figure, we learn that (1) the simulated results coincide with the theoretical results exactly; (2) the outage performance of PNC-OR-II is better than PNC-OR-I with the same reason as in Fig. 4, i.e., there may be two best relays for transmission when there is no relay that can successfully decode both flows of signal; and (3) both PNC-OR-I and PNC-OR-II have worse outage performance than TDBC-OIR-I or TDBC-OIR-II resulting from the lack of direct link transmission (i.e., with one diversity order loss).

The outage performance comparison of TDBC-OIR-I, TDBC-OIR-II, PNC-OR-I, and PNC-OR-II is presented in Fig. 6 when there are two relays, i.e., $N=2$. From the figure, we learn that (1) the simulated results coincide with the theoretical results exactly; (2) the outage performance of PNC-OR-II is better than PNC-OR-I with the same reason as in Fig. 4, i.e., there may be two best relays for transmission when there is no relay that can successfully decode both flows of signal; and (3) both PNC-OR-I and PNC-OR-II have worse outage performance than TDBC-OIR-I or TDBC-OIR-II resulting from the lack of direct link transmission (i.e., with one diversity order loss).

The ER performance of the six protocols when $N=2$ is provided in Fig. 7. From the figure, it is revealed that (1) TDBC-OIR-I (or TDBC-OIR-II) has better expected rate performance than PNC-OR-I at low to medium SNR conditions.
regime, though it may consume three time slots. This is because it has much better outage performance than PNC-OR-I; (2) TDBC-OR-I (or TDBC-OR-II) has improved ER performance over PNC-OR-II at low to medium SNR regime because of its better outage performance; and (3) the ER performance of PNC-OR-I is better than PNC-OR-II at medium to high SNR regime since it consumes less time slots for transmission and time slot plays the dominant role in ER performance. At low SNR regime, the situation reverses because at this regime the outage probability is the dominant role in ER performance.

5. Conclusions

In this paper, we propose two relay selection schemes for time division broadcast protocol with direct link transmission and incremental relaying in two-way relaying channel when there are multiple available relays, i.e., TDBC-OR-I and TDBC-OR-II. The two schemes utilize feedback to improve the spectral efficiency of the relay selection schemes of TDBC without feedback. As for performance comparison, we also introduce two benchmark protocols without direct link transmission, i.e., PNC-OR-I and PNC-OR-II. The performance of the abovementioned protocols are analyzed and compared in terms of outage probability, expected rate, and diversity-multiplexing tradeoff. It is revealed that the proposed schemes have improved spectral efficiency performance over all the other protocols, even PNC-OR-I that consumes two time slots for transmission.

References

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