

Two-port Network Transfer Function for Power Line Topology Modeling

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Abstract. *This paper deals with modeling of power line communication. A two-port network model is theoretically described and compared with measurement. A substantial part is focused on the mathematical description of distribution network using the method, which uses chain parameter matrices describing the relation between input and output voltage and current of the two-port network. This method is used for modeling sample power line topology. Furthermore, taps length and taps impedance influence on the transfer functions for different topology are examined.*

Keywords

Power line communication, two-port network, distribution network topology, transfer function.

1. Introduction

Systems for communication over power lines are referred to as Power Line Communication (PLC). PLC technology takes the advantage of no additional wiring required.

PLC systems can be divided into two areas: broadband PLC and narrowband PLC. Broadband PLC achieves the characteristics of broadband communication, enabling, for example, fast Internet access or implementation of small LAN networks. The broadband technology works in frequency range 150 kHz to 34 MHz and its theoretical maximum speed is 200 Mbit/s [1], [2].

Narrowband PLC is used mainly for specific services including central management of power consumption, tariffing, remote meter reading, controlling, etc. The narrowband technology works up to a maximum frequency of 150 kHz and its theoretical bit rate is of the order of kilobits up to 2 Mbit/s [1].

As a result of recent developments, the electrical power supply system is on the way to migrate from a pure energy distribution network to a multipurpose medium delivering energy, voice, and various data services. Nowadays, most technical effort is concentrated on low-voltage

power line channels owing to the huge development of PLC in smart metering and remote data acquisition [3].

The power line network differs considerably in topology, structure, and physical properties from conventional media such as twisted pair, coaxial, or fiber-optic cables. Therefore PLC systems have to encounter rather hostile properties [4]. For computer simulations oriented to appropriate system design, models of the transfer characteristics of the mains network are of major interest.

Although some model proposals can be found in literature, their practical value is generally very limited, because most of them represent only PLC model for known topology. The structure and topology of power line is in the most case unknown. The unplugged and plugged appliance may involve the topology. Many approach in literature [5], [6], [8], [10] works only with known topology. Therefore, this article describes the approach for modeling power line for different topology and prepares the basic apparatus for modeling of unknown topology. The PLC model enables comparison of the performance of different modulation and coding schemes for future standardization.

In this paper, the first part describes power line model, comparison of model results with measurement and mathematical description of two-port network modeling. Secondly, some theoretical observations are made on the parameters of PLC cables, and relevant input parameters for power line model were calculated and measured for different cables. Thirdly, a simplified topology model of transmission line is proposed. Finally, a power line model is designed and simulation results are reported.

2. Transmission Line Model

In literature the methods used to simulate and study the transmission line behavior are different [5] – [7]. Most of them are obtained from the time dependent telegrapher's equations which are for the elementary line transmission cell, shown in Fig. 1, the following:

$$\frac{\partial v(x, t)}{\partial x} + R' i(x, t) + L' \frac{\partial i(x, t)}{\partial t} = 0, \quad (1)$$

$$\frac{\partial i(x,t)}{\partial x} + G'v(x,t) + C' \frac{\partial v(x,t)}{\partial t} = 0 . \quad (2)$$

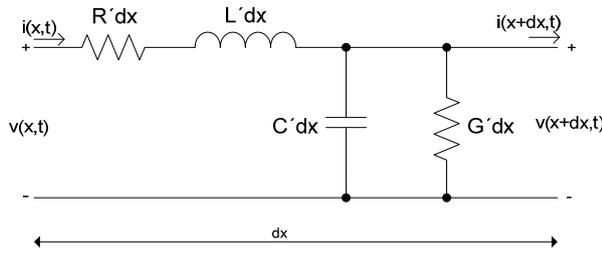


Fig. 1. Elementary cell of a transmission line.

In these equations x denotes the longitudinal direction of the line and R', L', G' and C' are the per unit length resistance (Ω/m), inductance (H/m), conductance (S/m) and capacitance (F/m) respectively. The electric quantities are dependent on the geometric and constitutive parameters. The parameters to describe a transmission line are the characteristic impedance Z_c and the propagation constant γ :

$$Z_c = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} , \quad (3)$$

$$\gamma = \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')} . \quad (4)$$

3. Modeling of the Power Line Channel

The power line model is considered as a black box described by transfer function, the method for modeling the transfer function of a power line channel uses the chain parameter matrices describing the relation between input and output voltage and current of two-port network.

In Fig. 2, the relation between input voltage and current and output voltage and current of a two port network can be represented as:

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \quad (5)$$

where A, B, C and D are frequency dependent coefficients.

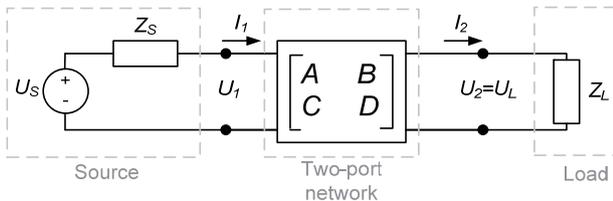


Fig.2. Two-port network connected to a source and load.

The transfer function of two-port network is given by the equation [8], [9]:

$$H = \frac{U_L}{U_S} = \frac{Z_c}{AZ_c + B + CZ_cZ_S + DZ_S} . \quad (6)$$

The ABCD matrix for the transmission line with characteristic impedance Z_c , propagation constant γ and length l can be calculated as [8]:

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_c \sinh(\gamma l) \\ \frac{1}{Z_c} \sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} . \quad (7)$$

3.1 Model Verification

The model verification was performed on topology in Fig. 3. The equivalent circuit for transmission line with one bridge tap connection is shown in Fig. 4.

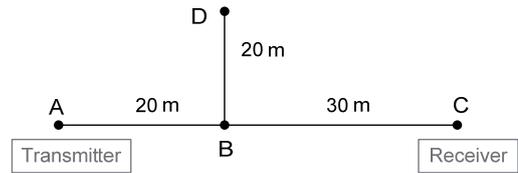


Fig. 3. Sample topology.

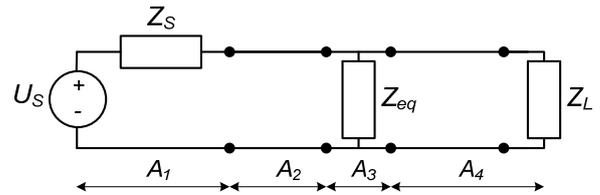


Fig. 4. Equivalent circuit for transmission line with one bridge tap.

Fig. 5 shows the simulation results of the sample network topology based on transfer function of two-port network model.

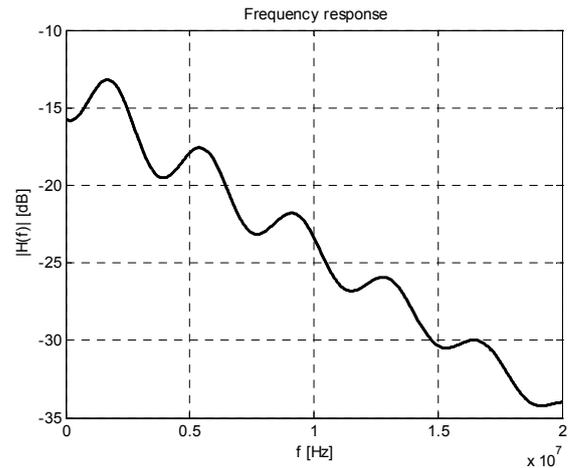


Fig. 5. Simulation of the sample network.

The results of measurement were gained with an experimental network with one tap in Fig. 6. During the measurements, the cable was matched at the right end with characteristic impedance Z_L and 20 m tap remained open. At the left side in Fig. 6 there is the RF generator, which

serves as signal injection. A sweep with a sinusoidal signal was led through the frequency range for the transfer function. The measurements were then taken each at the right line end terminated by Z_L .

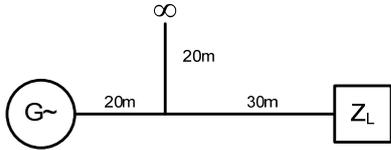


Fig. 6. Experimental network for two-port network model validation.

Fig. 7 shows the measurement results of the transfer function at the experimental setup.

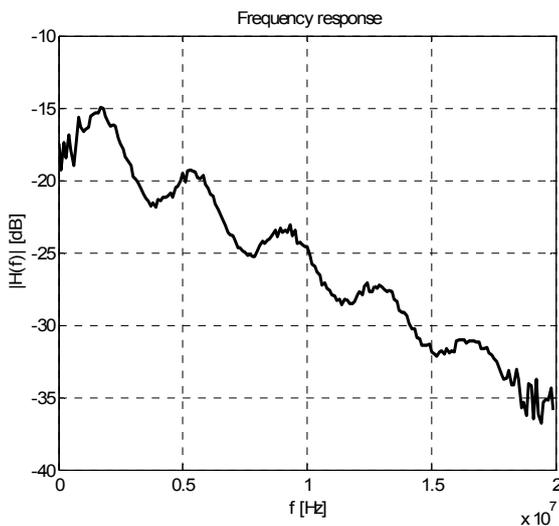


Fig. 7. Measurement of the transfer function at the experimental setup.

The effects of reflection in tap appear in the transfer function in the form of notches with fixed frequency spacing. The first notch occurs where the direct and the reflected waves are shifted exactly a half wavelength against each other, that causes the subtraction. The first frequency $f_1 = 3.86$ MHz belonging to the first notch, that means we have a period of 259 ns. The repetition of notches occurs multiples of f_1 , therefore we have $f_2 = 7.72$ MHz, $f_3 = 11.55$ MHz and $f_4 = 15.44$ MHz obtained from Fig. 5 and Fig. 7. Due to the insulating material with a dielectric constant $\epsilon_r = 4$, the phase speed on the cable is approximately 150 m/ μ s. The first wave from the generator, which has traversed the entire cable length of 50 m, therefore appears after $50/(150 \cdot 10^6) = 333$ ns. Because of the fact that the signal entering the tap is reflected at the tap's open end, the second wave appears after $90/(150 \cdot 10^6) = 600$ ns. The difference of these values (267 ns) should correspond to the measured period 259 ns.

In Fig. 8, the modeling and measurement comparison is shown. Obviously, the agreement of measurement and simulation is visible. The example has shown the performance and efficiency of two-port network model.

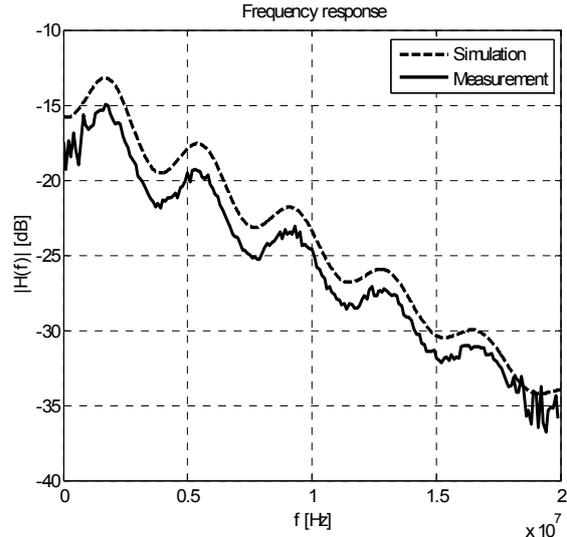


Fig. 8. Model and measurement comparison.

3.2 Sample Network for Modeling of Distribution Network

In Fig. 9, the topology with two bridge taps is shown.

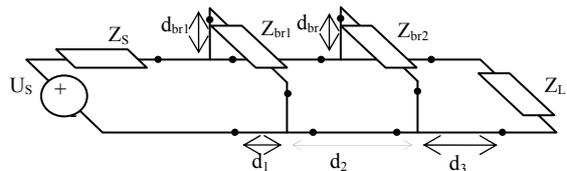


Fig. 9. Transmission line with two bridge taps connection.

We replace the bridge taps with the equivalent impedance (see Fig. 10). The branch cable terminated by the load impedance Z_{br} can be considered to be equivalent load impedance Z_{eq} [8]:

$$Z_{eq} = Z_C \frac{Z_{br} + Z_C \tanh(\gamma_{br} d_{br})}{Z_C + Z_{br} \tanh(\gamma_{br} d_{br})} \tag{8}$$

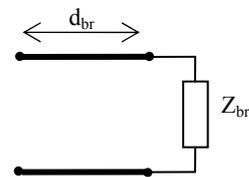


Fig. 10. One bridge tap.

The equivalent circuit for transmission line with two bridge tap connections and the section dividing is shown in Fig. 11. The channel from a source to a load consists of several network sections. Each section can be described with a single transmission matrix. The transmission matrix \mathbf{A} from the source to the load can be formed applying the chain rule:

$$\mathbf{A} = \prod_{i=1}^n \mathbf{A}_i \tag{9}$$

where n represents the number of the network sections.

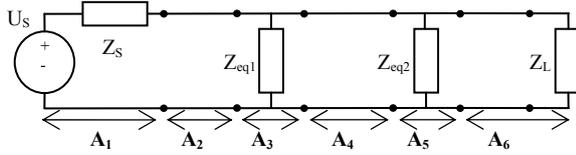


Fig. 11. Equivalent circuit for transmission line with two bridge tap connections.

The matrices of the different sub circuits (sections) for two bridge taps network are [10]:

$$\mathbf{A}_1 = \begin{bmatrix} 1 & Z_s \\ 0 & 1 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} \cosh(\gamma_1 d_1) & Z_1 \sinh(\gamma_1 d_1) \\ \frac{1}{Z_1} \sinh(\gamma_1 d_1) & \cosh(\gamma_1 d_1) \end{bmatrix}, \quad (10)$$

$$\mathbf{A}_3 = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_{eq1}} & 1 \end{bmatrix}, \mathbf{A}_4 = \begin{bmatrix} \cosh(\gamma_2 d_2) & Z_2 \sinh(\gamma_2 d_2) \\ \frac{1}{Z_2} \sinh(\gamma_2 d_2) & \cosh(\gamma_2 d_2) \end{bmatrix}, \quad (11)$$

$$\mathbf{A}_5 = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_{eq2}} & 1 \end{bmatrix}, \mathbf{A}_6 = \begin{bmatrix} \cosh(\gamma_3 d_3) & Z_3 \sinh(\gamma_3 d_3) \\ \frac{1}{Z_3} \sinh(\gamma_3 d_3) & \cosh(\gamma_3 d_3) \end{bmatrix}. \quad (12)$$

Product of all elements of the matrices \mathbf{A}_i gives the resulting cascade ABCD matrix:

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \quad (13)$$

where A , B , C and D are frequency dependent coefficients:

$$A = \cosh(\gamma_3 d_3) \theta + \frac{\sinh(\gamma_3 d_3)}{Z_3} \psi, \quad (14)$$

$$B = Z_3 \sinh(\gamma_3 d_3) \theta + \cosh(\gamma_3 d_3) \psi, \quad (15)$$

$$C = \cosh(\gamma_3 d_3) \chi + \frac{\sinh(\gamma_3 d_3)}{Z_3} \zeta, \quad (16)$$

$$D = Z_3 \sinh(\gamma_3 d_3) \chi + \cosh(\gamma_3 d_3) \zeta, \quad (17)$$

where θ , ψ , χ and ζ are

$$\theta = \alpha \left(\cosh(\gamma_2 d_2) \frac{Z_2}{Z_{eq2}} + \cosh(\gamma_2 d_2) \right) + \beta \left(\frac{\sinh(\gamma_2 d_2)}{Z_2} + \frac{\cosh(\gamma_2 d_2)}{Z_{eq2}} \right) \quad (18)$$

$$\psi = \alpha Z_2 \cosh(\gamma_2 d_2) + \beta \cosh(\gamma_2 d_2), \quad (19)$$

$$\chi = \xi \left(\cosh(\gamma_2 d_2) \frac{Z_1}{Z_{eq2}} + \sinh(\gamma_1 d_1) \right) + \vartheta \left(\frac{\sinh(\gamma_2 d_2)}{Z_2} + \frac{\cosh(\gamma_2 d_2)}{Z_{eq2}} \right) \quad (20)$$

$$\zeta = \xi Z_1 \sinh(\gamma_1 d_1) + \vartheta \cosh(\gamma_2 d_2), \quad (21)$$

where α , β , ζ and ϑ are

$$\alpha = \cosh(\gamma_1 d_1) + \frac{Z_s}{Z_1} \sinh(\gamma_1 d_1), \quad (22)$$

$$\beta = Z_1 \sinh(\gamma_1 d_1) + Z_s \cosh(\gamma_1 d_1), \quad (23)$$

$$\xi = \frac{(Z_1 \cosh(\gamma_1 d_1) + Z_s \sinh(\gamma_1 d_1) + Z_{eq1} \sinh(\gamma_1 d_1))}{Z_1 Z_{eq1}}, \quad (24)$$

$$\vartheta = \frac{(Z_1 \sinh(\gamma_1 d_1) + Z_s \cosh(\gamma_1 d_1))}{Z_{eq1}} + \cosh(\gamma_1 d_1). \quad (25)$$

3.3 Measurement Results of Power Line Primary Parameters

The measurements were carried out with a HP analyzer 4192A, operating in the frequency range 5 Hz–13 MHz. The frequency field investigated was from 10 kHz to 10 MHz according to the EN 50065-1 [11]. This frequency range includes the fixed CENELEC band 95 kHz–148.5 kHz.

The cable CYKY 3x1.5, 3x2.5 a 3x4 were examined. These cables are most commonly used for home and outdoor applications in the Czech Republic. Fig. 12 shows the measurements of the longitudinal parameters $R'(f)$ and $L'(f)$ and cross parameters $C'(f)$ and $G'(f)$, which are used for modeling. More information about measurement can be found in [3].

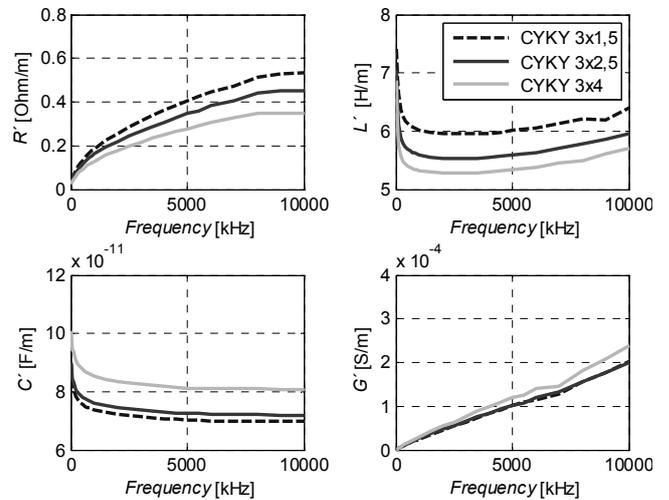


Fig. 12. Measurement values of longitudinal parameters $R'(f)$ and $L'(f)$ and cross parameters $C'(f)$ and $G'(f)$.

4. Distribution Network Topology

The distribution network can be realized in different topologies independent of applied communications technology (bus, star, ring). Bus topology (see Fig. 13) is one of the possible solutions that can be realized at low costs with adequate application areas.

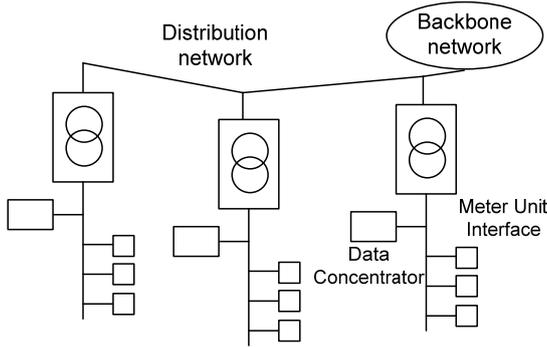


Fig. 13. PLC distribution network topology.

Fig. 14 shows a possible structure of a PLC network. There are several branches (network section) connecting the transformer station with the end users. Each branch can have a different topology connecting a variable number of users.

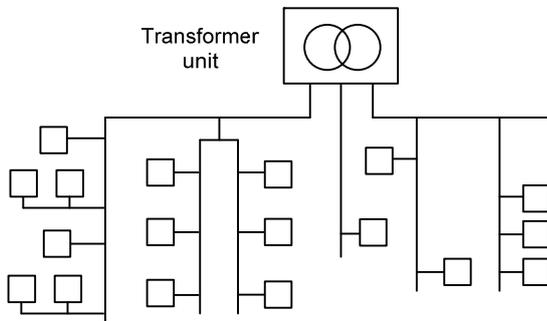


Fig. 14. Possible topology of a low-voltage PLC network.

Low-voltage supply networks differ from each other and it is not possible to specify a typical network structure. However, it is possible to define some characteristic value and describe an average structure of a typical PLC network in accordance with [12] as follows:

- Number of user in the network: ~250 to 400
- Number of network sections: ~5
- Number of user in a network section: ~50 to 80
- Network length: ~500 m (Section length: ~100 m)

On this basis we designed the distribution bus topology in Fig. 15 for simulation issue. This topology is one network section from Fig. 14 with 147 m length.

The distance between end users also differs from place to place. There is significant network length difference between the urban and rural application areas. Our simulation focuses on urban application area, because we deal with narrowband PLC used for smart grid and remote data acquisition and power line is only one from possible communication channel. The lengths of sections are designed for urban area. The values of terminal impedances are shown in Tab. 1.

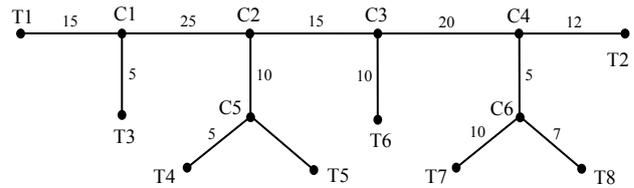
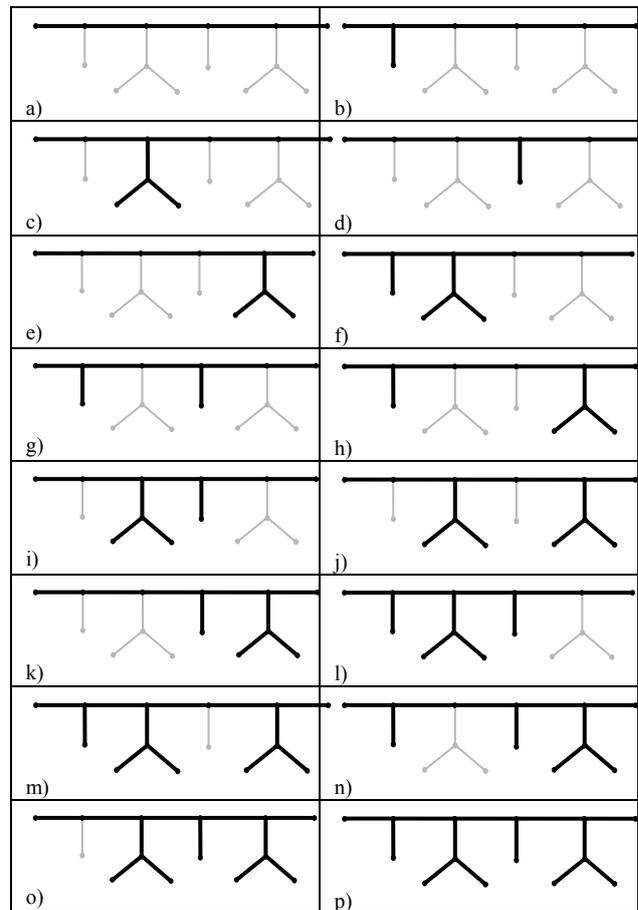


Fig. 15. Sample distribution network topology.

Node	T1	T2	T3	T4	T5	T6	T7	T8
Z[Ω]	100	300	200	200	250	250	150	200

Tab. 1. Terminal impedances.

Tab. 2 shows all topologies, where individual taps and their combinations are gradually connected to the network. Tab. 2 shows sixteen different topologies.



Tab. 2. The combination of distribution topologies.

5. Results of Distribution Network Modeling

On the basis of PLC transmission cable CYKY 3x2.5 measurement in chapter 3.3, we can make a calculation of characteristic impedance and propagation constant for each network segment.

At first, this network was considered without taps to avoid the spread of signal reflections. This is an ideal case where the signal is spread directly from the transmitter to the receiver.

5.1 The Influence of Connected Taps on Transfer Function

For the topologies in Tab. 2 transfer functions were calculated by the help of two-port network modeling using Matlab/Simulink. In Fig. 16 the transfer functions for topology without taps and with one tap are shown. The blue curve shows the transfer function of the topology without taps. Other waveforms show ripples and greater attenuation due to the connected taps in the network. The impact of individual taps on the resulting transfer function is significant. Therefore, the influence of one open tap on the transfer function is obvious.

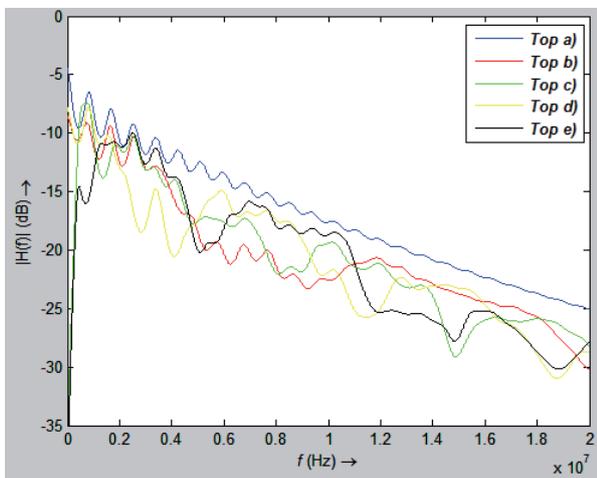


Fig. 16. Simulation of the transfer function for topology without taps and with one tap.

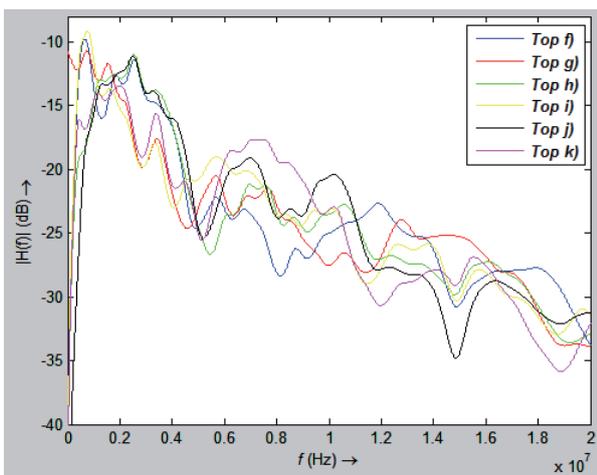


Fig. 17. Simulation of the transfer function for topology with two taps.

In Fig. 17, the transfer functions of the topology with two taps are shown. The figure shows that the additional branches will increase the attenuation, because the result-

ing curves are a combination of transfer functions of individual taps.

In Fig. 18, the transfer functions of the topology with three and four taps are shown. The additional taps increase attenuation. The transfer function attenuation is the largest for the topology with four taps (black curve).

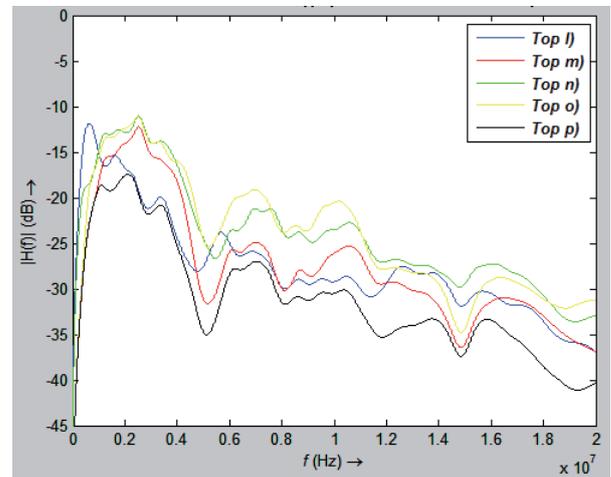


Fig. 18. Simulation of the transfer function for the topology with three and four taps.

5.2 The Influence of Tap Length on Transfer Function

The shape of the transfer function is strongly influenced by the length of the connected tap.

In Fig. 19 and Fig. 20, transfer functions for different taps length were calculated. The topology b) and c) from Tab. 2. were considered. The lengths ranges were set up from 5 to 14 meters. These values are derived for urban application. In Fig. 19, the influence of the tap length on the transfer function for the topology b) is shown. The increasing length of the tap causes increase of the periodical notches in the transfer function waveform.

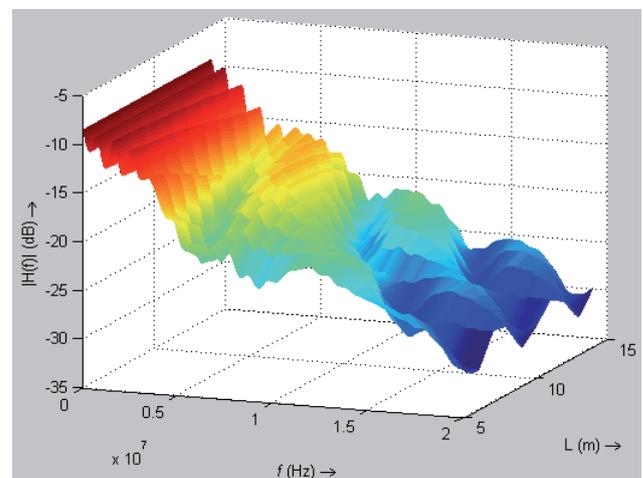


Fig. 19. The influence of the tap length on the transfer function for topology b).

Fig. 20 shows the influence of the length on transfer function for topology c), which has two taps. The length was changed between C2 and C5 nodes (see Fig. 15).

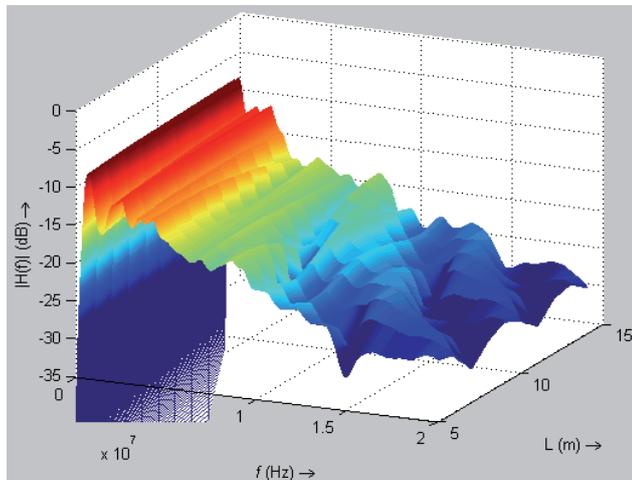


Fig. 20. The influence of the tap length on the transfer function for topology c).

5.3 The Influence of Connected Tap Impedance on Transfer Function

The impedance mismatch causes reflections on line and therefore the impedance values influence the transfer function. The topology d) in Tab. 2 was chosen for calculation the influence of impedance value on the transfer function. The impedance values were set up from 50 to 500 Ω . Fig. 21 shows the attenuation increasing in local minima of periodical notches with impedance increasing.

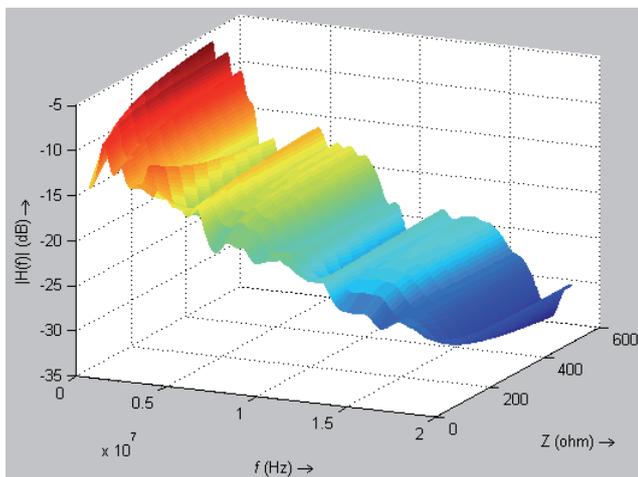


Fig. 21. The influence of the connected tap impedance on the transfer function for topology d).

6. Conclusion

In most cases we cannot know the topology of the distribution network, because the topology is changing due to appliances connection or disconnection and therefore we

were calculated transfer functions of power line modeling for different topologies. From the mentioned results of influence of connected taps on the transfer function the influence of individual branches on the resulting transfer function is evident.

If we wanted to use these models for the time changing topology of distribution network; the transfer function calculation would be complicated. The solution of this problem may be the algorithms, which could determine the structure of the distribution network at any given time.

The waveform of the transfer function is also influenced by the size of the connected impedance to the tap and the tap length. Therefore transfer functions for these quantities were calculated. The waveform shows the increase number of ripple with tap length increasing and attenuation increase of local minima in periodic ripple with impedance increasing.

For the sample network topology power lines were modeled and the resulting channel model was constructed. Power lines are modeled through chain parameter describing the relation between input and output voltage and current of two-port network. The model of two-port network is characterized by its simplicity and does not require so much computing demands.

The work provides a computer apparatus for creating models and modeling power lines for the simulation of data transmission over power lines. The constructed power line model offers the possibility to carry out investigations in different network topologies and study their effect on the communication system.

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