Improving Link Reliability through Network Coding in Cooperative Cellular Networks

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Abstract. *The paper proposes a XOR-based network coded cooperation protocol for the uplink transmission of relay assisted cellular networks and an algorithm for selection and assignment of the relay nodes. The performances of the cooperation protocol are expressed in terms of network decoder outage probability and Block Error Rate of the cooperating users. These performance indicators are analyzed theoretically and by computer simulations. The relay nodes assignment is based on the optimization, according to several criteria, of the graph that describes the cooperation cluster formed after an initial selection of the relay nodes. The graph optimization is performed using Genetic Algorithms adapted to the topology of the cooperation cluster and the optimization criteria considered.*

Keywords

Cooperative transmissions, genetic algorithms, graph optimization, network coding, relay assignment.

1. Introduction

The unprecedented development of the mobile multimedia applications witnessed in the last years and the significant increase of the number of smart user equipments generate an important overload of the deployed mobile communications networks. Even new systems, like Long Term Evolution (LTE), will face overloads in the near future, if the current trend of development of mobile applications will continue. In order to cope with this increased demand of traffic, new communication technologies, like cooperative transmis-sion techniques, will have to be implemented in future wireless networks.

The basic idea of relaying and cooperation in telecommunications was introduced by Van der Meulen [1] and later, the information theoretic properties of the relay channel was studied by Cover and Gamal [2]. The concept of cooperative relaying was further extended by Laneman and Wornell [3] by designing energy efficient protocols based on Decode and Forward (DF) and Amplify and Forward (AF) relaying. The channel-coded cooper-ation paradigm integrated in schemes where a relay node (RN) serves only one user equipment (UE), connected to the base station (BS), is one of the techniques proposed in literature [4], [5]. Though, this approach is shown to bring performance improvements for the served UE, in terms of Bit Error Rate (BER), Block Error Rate (BLER) and coverage, the additional resources required by the RN to serve only one UE leads to low spectral efficiency.

In order to decrease the effect of the additional resources required by the RN upon the spectral efficiency, cooperation algorithms that use network coding (NC) techniques [6] were proposed. These techniques allow cooperative structures within which an RN serves more UEs, and this approach leads to a more efficient usage of the additional resources used by the RN, but involves dynamic structuring of the cooperation cluster.

Hausl and Dupraz [7] studied the use of distributed turbo codes for the Multiple Access Relay Channel and concluded that joint decoding of the network and channel codes performs better than their separate decoding. Another approach proposed in [8] employs joint network and channel decoding for all users served by relays and uses product codes as network codes. In order to obtain a better spectral efficiency without affecting the diversity gain, a solution simple to NC, consisting in XOR-based coding, is proposed in [9]. The graph representation of a cooperative network was employed in [10], in order to implement coded cooperation in Multi-Source Multi-Relay (MSMR) topologies. This approach uses codes on graph, like the Low Density Parity Check Codes, aiming to decrease the cluster's outage probability.

The theoretical analyses performed in several studies, e.g., [2], show that the choice of proper RN(s) to serve a certain UE, or a group of UEs, is of critical importance for achieving the expected performances of cooperative transmissions. In [11] it is proposed an optimal relay assignment algorithm for ad-hoc networks based on the maximization of the minimum rate for all source-destination node pairs. The algorithm does not have a unique solution and it optimizes only the performance of the worst user. Thus, a modified algorithm is proposed in [12] that offers uniqueness of the relay selection and takes into account the SNR condition of all users in addition to the user with the worst SNR. In [13] is proposed a method for optimizing an initial relay selection stage, which aims to maximize the sum rate of all users and the minimum of all user rates.

An interesting solution for generating NC-capable clusters (topologies) is proposed by Kim and Medard [14] who employ genetic algorithms (GA) to this end. These algorithms, inspired by natural evolution, are based on the mathematical framework set up by J. Holland [15] and are used in [14] to provide solutions for the selection and optimization issues raised by NC-capable cooperative topologies. In [16] also a graph-based optimization method is used for structuring the cooperation cluster. This method maximizes the flow rate between a source and a destination and is based on the selection of the graph cuts, containing the source node and relays and re-spectively the destination node and relays, which ensure a maximal source-destination rate.

This paper proposes a XOR-based coded cooperation protocol in MSMR topologies and analyzes theoretically its performances under the assumption of Block Erasure Channels (BLECs). The formulae needed to compute the cluster's outage probability and the BLER ensured to individual UEs are derived. The performance analysis relies on the theory of graph-based codes and graph related parameters, like outage sets, which are defined. The paper proposes also a method, which uses genetic algorithms, for configuration and optimization of the MSMR cooperation cluster within which the NC-based cooperation protocol will be used. Just like the approach of Kim and Medard [14], our method aims to optimize the cooperation graphs, but the optimization criteria and the protocol governing the cooperation process are different.

The paper is structured as follows: Section 2 presents the system and channel models employed and proposes a graph-based description of the cooperation cluster. Section 3 describes the proposed network coded protocol and analyzes theoretically its performances in terms of outage probability and user's BLER. Section 4 proposes a method for structuring and optimization of the cooperation cluster according to several criteria. The optimization is performed using a genetic algorithm adapted to the network topology and the optimization criteria considered. Section 5 validates the theoretical analysis of the cooperation protocol and that of the cooperation graph structuring and optimization algorithm, while Section 6 concludes the paper.

2. System and Channel Models

The network architecture considered is a relay-enhanced cellular one, which implements MSMR cooperation, as presented in Fig. 1. The RNs, assigned to one or several UEs, implement network-coded cooperation in the uplink transmission. We assume that all UEs have direct links (not necessarily line of sight, LOS) to the BS.

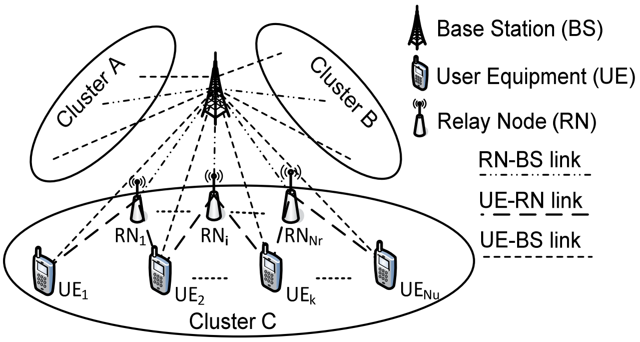

Fig. 1. MSMR cooperation cluster topology.

The RN_i-UE_j orthogonal links established within the cluster are defined by the connection graph, see Fig. 1, which contains two sets of nodes: the U-nodes set, $\boldsymbol{U} = \{\boldsymbol{UE_1}, \ldots, \boldsymbol{UE_{Nu}}\}$ and the R-nodes set, $\boldsymbol{R} = \{\boldsymbol{RN_1}, \ldots, \boldsymbol{RN_{Nr}}\}$, where N_u is the number of UEs and N_r is the number of RNs of the cluster. We consider that two nodes are connected if the BLER on the link between them is below a certain threshold. The links between the U-nodes and R-nodes are represented by the adjacency matrix associated to the connection graph, $\boldsymbol{A[N_r \times N_u]}$, whose entries are defined as:

$$A[j,i] = \begin{cases} 1, & \text{if } RN_j \text{ and } UE_i \text{ are connected,} \\ 0, & \text{if } RN_j \text{ and } UE_i \text{ are not connected.} \end{cases} \tag{1}$$

For each RN belonging to the set R, we define a set of UE neighbors from the set U, denoted by nR_j, indicating the UEs that are served by the j^{th} RN. In addition, for each UE we define the set of RN neighbors from the set R, nU_i, indicating the RNs which assist the i^{th} UE.

$$\begin{aligned} nR_j &= \{UE_i \mid A[j,i] = 1, i = 1,2,\ldots,N_u\}, \\ nU_i &= \{RN_j \mid A[j,i] = 1, j = 1,2,\ldots,N_r\}. \end{aligned} \tag{2}$$

The UE_i-BS and RN_j-BS communications are modeled by the cooperation graph, shown in Fig. 2. The T-nodes set, $\boldsymbol{T} = \{\boldsymbol{T_1}, \ldots, \boldsymbol{T_i}, \ldots, \boldsymbol{T_{Nu+Nr}}\}$, of the cooperation graph includes all nodes that have direct link to the BS, $T = U \cup R$, in the assumption that all UEs of the cluster are connected to at least one RN. The B-nodes, $\boldsymbol{B} = \{\boldsymbol{B_1}, \ldots, \boldsymbol{B_{Nr}}\}$, of the cooperation graph indicate the T-nodes groups that are processed jointly, as well as the processing which has to be performed by the network decoder, hosted by the BS, on these groups of T-nodes. Opposed to usual network-on-graph representations [10], where a B-node represents a check equation, in our approach a B-node could represent a system of check equations involving the information blocks generated by the UEs (one block/ UE/cooperation period) and the check blocks generated by the RN(s). The cooperation graph's adjacency matrix is $\boldsymbol{A'[A \mid I]}$, $\boldsymbol{I}$ being the $\boldsymbol{[N_r \times N_r]}$ identity matrix. We define the nB_j and nT_i neighbor sets as:

$$\begin{aligned} nB_j &= \{T_i \mid A'[j,i] = 1, i = 1,\ldots,N_u + N_r\}, \\ nT_i &= \{B_j \mid A'[j,i] = 1, j = 1,\ldots,N_r\}. \end{aligned} \tag{3}$$

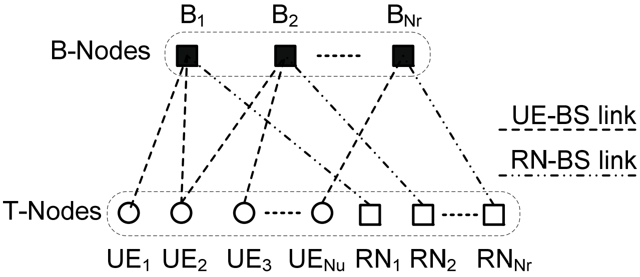

Fig. 2. MSMR cooperation graph.

We assume that all UE_i-BS and RN_j-BS links are affected by quasi-static Rayleigh fading and AWGN. For each of these links the channel output for the i^{th} input sample, y_i, is given by: $y_i = h_i \cdot x_i + n_i$, where $x_i \in \{-1, 1\}$ represents the channel input, h_i is the Rayleigh-distributed channel coefficient, which is constant during a data block and is constrained by the condition: $E\{h_i^2\} = 1$, while n_i is the normally distributed noise with zero mean and variance σ. The UE_i-RN_j links are considered quasi-error free. We assume that each UE is characterized by a full-queue traffic model, transmits the same bit rate and uses the same type of Medium Access Control (MAC) frame.

The total transmitted power in the cooperation cluster is limited to the one of the non-cooperative transmissions.

$$\underbrace{\sum_{UE\in U} P^{c}_{t,UE} + \sum_{RN\in R} P^{c}_{t,RN}}_{cooperation} = \underbrace{\sum_{UE\in U} P^{n}_{t,UE}}_{no\ cooperation} \qquad (4)$$

where $P^c_{t,UE}$ and $P^n_{t,UE}$ represent the transmission powers of an UE in the cooperative respectively the non-cooperative transmissions; $P^c_{t,RN}$ represents the transmission power of an RN of the cluster.

3. Network Coded Cooperation

3.1 Protocol Description

The principle of the NC-based cooperation protocols for MSMR clusters consists in making the RNs generate and send to the BS additional check blocks that will be used to recover the erroneous data blocks received by the BS from the UEs over their direct links. The protocol is supported by the classical *two-phase* cooperation scheme, i.e., a *broadcast phase*, when the UEs send their blocks to the BS and RNs, and a *relaying phase*, during which the RNs transmit additional blocks to the BS.

The coded cooperation scheme proposed for MSMR cooperation topologies employs two levels of coding:

1. The first level of coding is represented by the channel code employed on each link of the cluster. The channel code employed by the UEs has to ensure quasi error-free UE_i-RN_j links. This coding layer must provide an error detection mechanism, i.e. Cyclic Redundancy Check (CRC), which transforms the UE_i-BS and RN_j-BS links in Block Erasure Channels (BLECs). The BLECs erasure probabilities are equal with the BLERs ensured by the channel code and are considered to be lower than a certain threshold, $BLER_{th}$. In addition, we assume that all direct links have the same BLER values, as it is expressed in (5).

$$\begin{aligned} &BLER_{UE_1-BS} = \ldots = BLER_{UE_{N_u}-BS} = BLER_{RN_1-BS} = \ldots \\ &\ldots = BLER_{RN_{N_r}-BS} \le BLER_{th} \end{aligned} \qquad (5)$$

2. The second level of coding is represented by the network code and it is performed by the RNs (see Fig. 3). This coding process is carried out only on the information bits received by the RNs from the UEs connected to these relay nodes. We assume that the lengths of the UEs information blocks are equal.

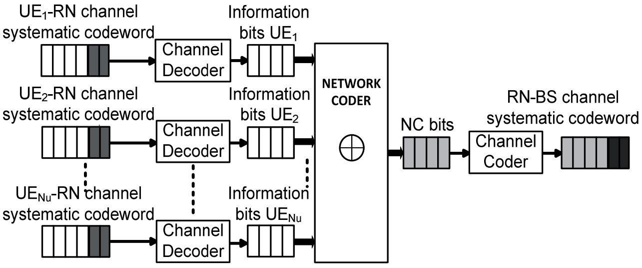

Fig. 3. Network encoding process performed by an RN.

Each RN generates one or several check blocks by encoding with the network code one symbol from all user data blocks available to that RN. These check blocks are channel-encoded and sent over the RN_j-BS links. In this paper we propose the utilization of XOR-based network codes and therefore each RN computes its check block (one check block per data block period) by simply XOR-ing the information bits of its neighbor UEs, $UE_i \in nR_j$.

$$RN_j \text{ check block} = \underset{UE_i \in nR_j}{XOR}(UE_i \text{ data block}). \qquad (6)$$

At the BS, see Fig. 4, the network decoder tries to recover the erroneous data blocks received from the UEs.

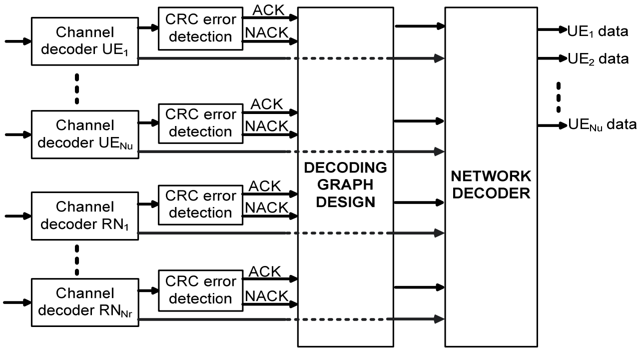

Fig. 4. Cluster-code decoding performed by the BS.

The network decoder uses the belief propagation algorithm, in a manner similar to the one used for Binary Erasure Channels (BEC) [17]. The decoder acts on the cooperation graph which characterizes the cooperation cluster (see Fig. 2). The erroneous data blocks received on the edges between the T-nodes and the B-nodes are marked as erasures by the error detection mechanism used at the first coding level. The decoding algorithm tries to recover the lost data blocks from the correctly received ones (check blocks and other UEs data blocks). This process repeats until there is no lost data block that can be recovered or all data blocks have been recovered.

3.2 Analysis of the Cooperation Protocol

Three performance metrics are used to evaluate the proposed NC-coded cooperation protocol: P_{outage}, the outage probability of the network decoder, $BLER^{av}_{NC\text{-}UE}$, the average BLER of the UEs after NC decoding and $BLER_{NC\text{-}UEi}$, the BLER of each UE_i, $i = 1,...,N_u$, after NC decoding.

Starting from the definition of stopping sets [9], [18] which are structures that cannot be recovered over BECs by a decoding algorithm using the belief propagation principle, we define similar structures for the decoding algorithm of the network code, which operates on the same principle, over independent BLECs. These structures, called *outage sets* (*os*), are defined as follows:

Definition: A subset S of the T-nodes set, is an outage set if all the B-nodes neighbors of the T-nodes belonging to S are connected to S at least twice.

According to this definition, the family of outage sets, OS, is expressed by:

$$OS = \left\{ os \in P(T) \left| \left| \left\{ B_k \in B \mid \sum_{t_q \in os} A'[k,q] = 1 \right\} \right| = 0 \right. \right\} \quad (7)$$

where $P(T)$ represents the power set of T and $t_q \in T$.

Since the outage events are generated by the outage sets, the outage probability may be computed by summing the occurrence probabilities of block errors on all links of the T-nodes that are part of outage sets, as it is expressed by (8):

$$P_{outage} = \sum_{os \in OS} \left(\prod_{t_q \in os} BLER_{t_q-BS} \cdot \prod_{t_q \in T \setminus os} \left(1 - BLER_{t_q-BS}\right) \right). \quad (8)$$

Assuming that the channel code ensures the same BLER, not exceeding $BLER_{th}$, on every BS-terminated link, the outage probability is upper-bounded by:

$$P_{outage} \le \sum_{k=2}^{N_u+N_r} \left(\left| OS^{(k)} \right| \cdot BLER_{th}^{k} \cdot \left(1 - BLER_{th}\right)^{N_u+N_r-k} \right) \quad (9)$$

where $OS^{(k)}$ represents the family of k-length outage sets.

Alternatively, the outage probability can be computed as the probability to have k erroneous blocks which form an *outage set*, as it is expressed by (10). Such an approach would ensure a correct computation of P_{outage} only if all UE_i-BS and RN_j-BS links have the same BLER:

$$P_{outage} = \sum_{k=2}^{N_u+N_r} P_{OS}^{k} \cdot p_{k-err} \quad (10)$$

where P^{k}_{OS} is the probability to have k-long *outage sets* according to the cooperation graph and $p_{k\text{-}err}$, given by (11), is the probability to have k erroneous blocks.

$$p_{k-err} = \sum_{T' \in T^{(k)}} \left(\prod_{t_q \in T'} BLER_{t_q-BS} \cdot \prod_{t_q \in T \setminus T'} \left(1 - BLER_{t_q-BS}\right) \right) \quad (11)$$

where $T^{(k)}$ *r*epresents the family of k-size subsets of set T.

The occurrence probability of the k-size *outage sets* is not the same for all the *outage sets* having this size, but for simplification we consider that the probabilities of all k-size *outage sets* are equal and can be computed as:

$$P_{OS}^{k} = \left| OS^{(k)} \right| \Big/ \binom{N_u + N_r}{k}. \quad (12)$$

In order to determine the BLER of an UE, we need to find the block error patterns that make the network decoder to fail in recovering the data block sent by that UE. We denote such a structure as an *extended outage set* (*eos*) and the family of this structures for a given UE_i by EOS_{UEi}, which is defined as:

$$EOS_{UE_i} = \left\{ eos \in P(T) \middle| UE_i \in eos, \exists os \subseteq eos, os \in OS,\ UE_i \in os \right\}. \quad (13)$$

The block error rate obtained for each UE_i after the network decoder, $BLER_{NC\text{-}UEi}$, can be computed as the occurrence probability of the block error pattern that is an *extended outage set* for that UE_i, and has the expression:

$$BLER_{NC-UE_i} = \sum_{eos \in EOS_{UE_i}} \left(\prod_{t_q \in eos} BLER_{t_q-BS} \cdot \prod_{t_q \in T \setminus eos} \left(1 - BLER_{t_q-BS}\right) \right). \quad (14)$$

The average BLER per UE ensured by the cooperation protocol, $BLER^{av}_{NC\text{-}UE}$, is computed as:

$$BLER_{NC-UE}^{av} = \frac{1}{N_u} \sum_{i=1}^{N_u} BLER_{NC-UE}\ . \quad (15)$$

4. Cooperation Cluster Structuring

This section proposes a two-phase algorithm for structuring of the cooperation cluster, i.e. the *initial selection* and the *optimization* phase.

The first phase consists of an initial selection, which assigns to each UE the RNs fulfilling some conditions and provides an initial connection graph $G'(V', E')$. V' denotes the set of vertices, $V' = T$, and E' is the set of edges composed of all selected $UE_i - RN_j$ links. The second phase modifies the initial connection graph according to some criteria, aiming to improve the performances of the cooperative transmission. It is expected that the graph G' provided by the first phase will be a dense one, since the set of edges, E', includes all UE_i-RN_j links initially selected. The theory of the graph-based codes [10], [18] shows that dense graphs generate codes with poor performances, meaning that the performances of the network code could be improved by making the graph sparser through an optimization process, as depicted in Fig. 5. This process generates a sub-graph $G(V, E)$ of the initial connection graph $G'(V', E')$, having the same number of vertices, $V = V'$, but a smaller number of edges.

Remark: by making the connection graph sparser, the cooperation graph becomes also sparser.

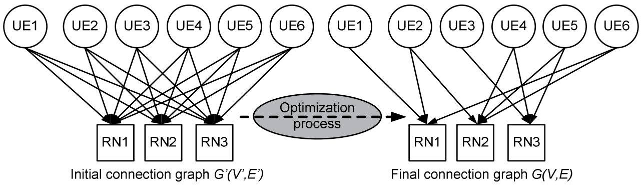

Fig. 5. The cooperation cluster optimization process.

The steps performed within the two phases of the cooperation cluster structuring are the following:

1. *The initial selection phase:*

a. Let R'' be the set of RNs available in the cell, or in a sector of the cell. Select a subset of RNs $\boldsymbol{R'} = \{\boldsymbol{RN_1,\ldots,RN_{N'r}}\} \subset \boldsymbol{R''}$ whose elements fulfill the condition: $BLER_{RNj\text{-}BS} \le BLER_{th\text{-}r}$, $j = 1,\ldots,N'_r$.

b. Let U' be the set of UEs located in a cell, or a sector of the cell. Select a subset of UEs $\boldsymbol{U} = \{\boldsymbol{UE_1,\ldots,UE_{Nu}}\} \subset \boldsymbol{U'}$ whose elements fulfill the conditions: $BLER_{UEi\text{-}RNk\text{-}i} \le BLER_{th\text{-}u}$, $i = 1,\ldots,N_u$, $RN_{k\text{-}i} \in R'$, $k\text{-}i \in \{a_{k\text{-}i}\}$, where $\{a_{k\text{-}i}\}$ is a non-empty set of $N_{k\text{-}i}$ indexes, $1 \le N_{k\text{-}i} \le N'_r$. By imposing $BLER_{th\text{-}u}\to 0$, this condition selects for cooperation all UEs which have quasi error-free connections to at least one of the initially selected RNs and assigns to each UE only the RNs which fulfill the above mentioned condition. If all $\{a_{k\text{-}i}\}$ sets are void then the initial selection process cannot be completed.

c. Select the final set of cooperating RNs, $\boldsymbol{R} = \{\boldsymbol{RN_{b1},\ldots,RN_{bNr}}\} \subset \boldsymbol{R'}$ whose elements fulfill the additional condition: $|nR_j| \ge 2$, $RN_j \in R'$, $j \in \{b_k\}$, where $\{b_k\}$ is a non-empty set of N_r indexes, $1 \le N_r \le N'_r$. This condition imposes a minimum number of UEs to which an RN is assigned. If $\{b_k\}$ is void then the initial selection process cannot be completed and the algorithm ends. The $\mathrm{RN_j}$ relay nodes having $|nR_j| = 1$ will not be included in the cooperation cluster because they would not allow the use of network code.

2. *The cluster optimization phase*:

This phase removes some of the $\mathrm{UE_i}$-$\mathrm{RN_j}$ edges from the initial connection graph, to make this graph sparser, thus aiming to decrease the maximum BLER per UE or the cluster's outage probability. The edges are removed according to the following optimization criteria:

a. *Criterion a* (*C-a*): targets the minimization of the BLER ensured to any $\mathrm{UE_i}$-BS connection after the network decoding, $BLER_{NC\text{-}UEi}$, while imposing that all $BLER_{NC\text{-}UEi}$ should be smaller or equal to the BLER values, $BLER'_{NC\text{-}UEi}$, ensured by the initial connection graph. This optimization criterion is expressed by:

$$\begin{aligned} &\text{minimize}\max_{i=1,\ldots,N_u}\left(BLER_{NC-UE_i}\right),\\ &\text{subject to}:|V| = N_u + N_r,\ th_i \le |nR_j| \le th_s,\\ &\qquad BLER_{NC-UE_i} \le BLER'_{NC-UE_i},\ i = 1,\ldots,N_u. \end{aligned} \tag{16}$$

The lower threshold, $th_i \ge 2$, is imposed to involve each RN in the NC-based cooperation scheme, while the upper threshold, $th_r < 5$, is imposed to generate a sparse graph.

b. *Criterion b* (*C-b*): is similar to the previous one, targeting the decrease of the maximum BLER over all UEs, but imposes no conditions on the final BLER value of each UE. Thus, it does not guarantee the decrease of $BLER_{NC\text{-}UEi}$ for all UEs after the cluster optimization step. This criterion is expressed by:

$$\begin{aligned} &\text{minimize}\max_{i=1,\ldots,N_u}\left(BLER_{NC-UE_i}\right),\\ &\text{subject to}:|V| = N_u + N_r,\ th_i \le |nR_j| \le th_s. \end{aligned} \tag{17}$$

c. *Criterion c* (*C-c*): targets the minimization of the outage probability of the network decoder, as defined by (10), when imposing the RNs degrees $|nR_j|$ to belong to a predefined interval. This criterion is expressed by:

$$\begin{aligned} &\text{minimize}\,P_{\text{outage}},\\ &\text{subject to}:|V| = N_u + N_r,\ th_i \le |nR_j| \le th_s. \end{aligned} \tag{18}$$

5. Cluster Optimization using GA

This section presents the optimization, according to the described criteria, of the NC-based cooperation cluster by using genetic algorithms. The flow chart of the GA-based optimization process is presented in Fig. 6.

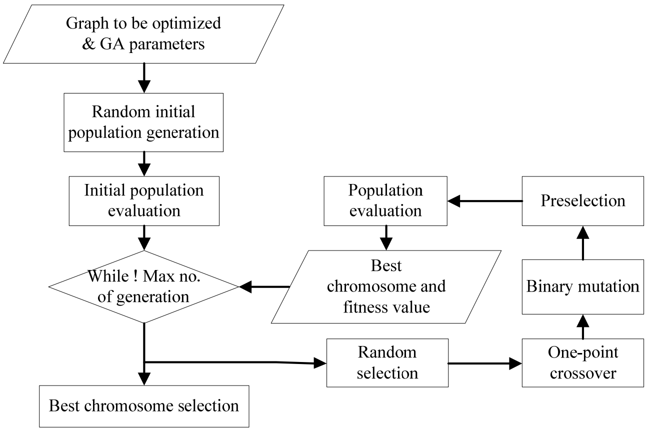

Fig. 6. Flow chart of the GA-based graph optimization.

The steps of the algorithm are briefly the following:

1. *Chromosome representation*: each chromosome represents a sub-graph $G(V, E)$ of the initial connection graph $G'(V', E')$, all sub-graphs having the same number of vertices, $V = V' = |T|$. A chromosome is represented by a string of 0 and 1 symbols obtained by concatenating the rows of the $A[j, i]$ adjacency matrix associated to the initial graph G' or to a sub-graph G. The length of a chromosome, m, is defined by $m = |U|\cdot|R| = N_u \cdot N_r$.

2. *Population initialization*: the initial population is formed of N chromosomes. The first chromosome of the initial population represents the initial graph, $G'(V', E')$, the rest of the chromosomes represent the $G(V, E)$ sub-graphs and are generated randomly.

3. *Pre-selection*: only the sub-graphs fulfilling the condition: $th_i \leq |nR_j| \leq th_s$ are evaluated. This condition ensures the sparseness of the optimized connection graph $G(V, E)$ and reduces to some extent the amount of computations required by the optimization process.

4. *Chromosome feasibility*: the chromosomes satisfying the pre-selection condition and other additional conditions, if any, are considered feasible. Additional conditions, defined by (19), are imposed when criterion *C-a* is used:

$$BLER_{x-NC-UE_i} \leq BLER_{X-NC-UE_i}\ , \forall i = 1, ..., N_u \qquad (19)$$

where $BLER_{x\text{-}NC\text{-}UEi}$ is the block error rate of UE_i after the NC decoding based on the connection sub-graph associated to chromosome x, while $BLER_{X\text{-}NC\text{-}UEi}$ is the block error rate of UE_i obtained after NC decoding based on the initial connection graph $G'(V', E')$.

5. *Population evaluation*: the value of the fitness function is computed for each feasible chromosome x. The specific fitness functions $f(x)$ adapted to the optimization criteria are the following:

a. For the *C-a* and *C-b* criteria the fitness function is defined by:

$$f_{alb}(x) = \begin{cases} \max(BLER_{x-NC-UE_i}),\ i = 1, ..., N_u, & \text{if } x \text{ is feasible} \\ \infty, & \text{otherwise} \end{cases} \qquad (20)$$

where $BLER_{x\text{-}NC\text{-}UEi}$ has the same meaning as in (19).

b. For the third optimization criterion, *C-c*, the fitness function is defined by:

$$f_c(x) = \begin{cases} \max P_{outage-x}, & \text{if } x \text{ is feasible} \\ \infty, & \text{otherwise} \end{cases} \qquad (21)$$

where $P_{outage\text{-}x}$ represents the outage probability of the NC decoder that operates over the connection sub-graph associated to chromosome x.

6. *Selection*: involves the random selection of P chromosomes out of the current population, where P is the *selection pressure*. Out of the P chromosomes, the one with the best (smallest) fitness value is kept for the new generation. The process is repeated until the new generation of N chromosomes is completed.

7. *Crossover*: groups of two chromosomes, selected randomly from the new population, undergo *one-point crossover*, by interchanging a number of their genes [19]. The number of the genes involved in this process is selected randomly and the total number of crossovers in the population depends on the *crossover probability* [19].

8. *Mutation*: after the crossover step, the chromosomes undergo a *binary mutation*. The chromosomes change the values of some of their genes which are selected randomly. The number of the genes involved in this process is given by the *mutation probability* [19].

9. *Termination criterion*: the GA stops when the preset maximum number of generations, N_g, is reached.

10. *Best chromosome selection*: the best chromosomes provided by each of the iterations are stored together with their fitness values. This operation provides the set of chromosomes having the best fitness value from all generations, out of which the one having the smallest fitness value is selected. If, after the maximum number of generation, N_g, several chromosomes have the same best fitness value, they are subject of a post-selection process, which chooses only one out of them. The post-selection criterion is: *minimize mean*$(BLER_{NC\text{-}UEi})_{i = 1,...,Nu}$.

6. Evaluation of the Proposed Solution

6.1 Validation of the Protocol Analysis

In this section we validate by computer simulations the theoretical analysis of the network coded cooperation protocol, analysis presented in Section 3.2. For validation, we consider an MSMR cooperation cluster (see Fig. 1) formed inside a cell (or a sector of the cell) and composed of 8 UEs and 4 RNs. The cooperation process is localized inside the cluster, all UEs and RNs have direct, but not necessarily LOS, links to the BS of the cell and the UEs and RNs of the cluster are not interacting with other nodes (other UEs or RNs) of the cell, nodes which may be part of other cooperation clusters. All UE_i-BS and RN_j-BS links are characterized by the same *BLER* values, equaling $BLER_{th}$, while the UE_i-RN_j links are supposed to be quasi error-free. The connection graph of the considered cooperation cluster is described by its adjacency matrix A:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}. \qquad (22)$$

Fig. 7 compares the values of the outage probability vs. $BLER_{th}$ obtained by theoretical computations, using equation (7) – (12), to the values obtained by computer simulations. The results show a very good match between the values provided by simulations and those computed theoretically, thus validating the theoretical analysis. As expected, P_{outage} decreases with the decrease of $BLER_{th}$.

The comparisons between the BLER performances of UE_1, selected as the representative UE, after the cluster decoder, provided by simulations and those computed theoretically using (7), (13) – (14), is presented in Fig. 8. The results show a very good match between the two sets of values, validating the theoretical analysis. Similar good results are obtained for the rest of the UEs.

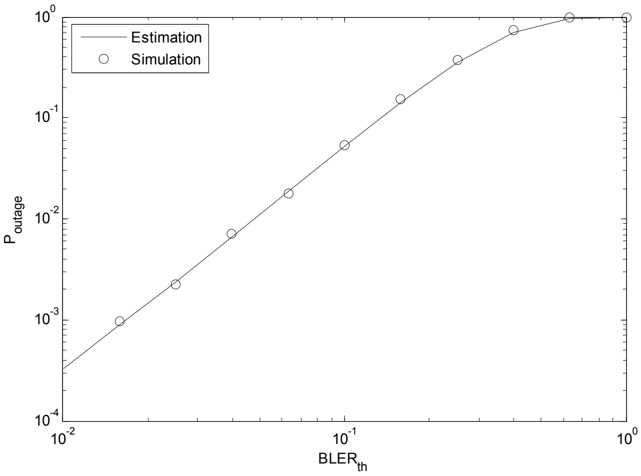

Fig. 7. P_{outage} vs. $BLER_{th}$ performances obtained by thoretical evaluations and computer simulations.

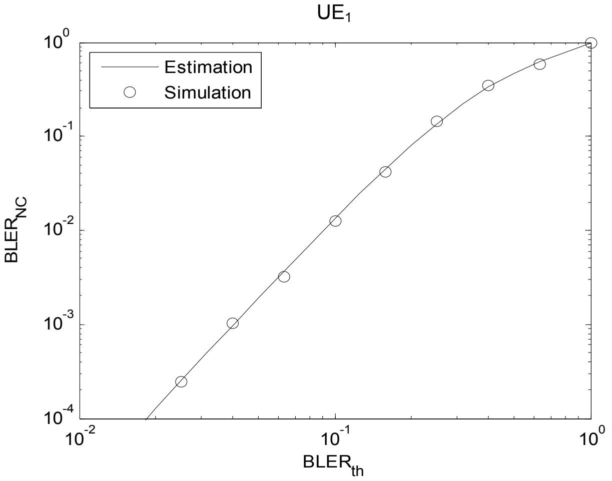

Fig. 8. $BLER_{NC\text{-}UE1}$ vs. $BLER_{th}$ performances obtained by theoretical evaluations and computer simulations.

6.2 Evaluation the Optimization Algorithms

The effectiveness of the GA-based optimization algorithms that use the three optimization criteria, defined by (16), (17) and (18) and the fitness functions defined by (20) and (21), were evaluated by comparing the P_{outage} and $BLER_{NC\text{-}UEi}$ performances ensured by the NC-based cooperation protocol over the optimized connection graphs with the performances ensured over the non-optimized connection graphs.

The performance evaluation was performed in the following scenario: $BLER_{UEi\text{-}BS} = BLER_{RNj\text{-}BS} = BLER_{th}$, $BLER_{UEi\text{-}RNj} = BLER_{th\text{-}u}$, $\forall i = 1,\dots,N_u$, $\forall j = 1,\dots,N_r$; $BLER_{th} = 0.1$ (a value used in several wireless communi-cation standards as the maximum allowed MAC frame error rate); $BLER_{th\text{-}u} \approx 0$; $N_g = 20$; $P = 4$ and $N = 100$. The initial connection graph was either the "complete graph", see (23), or the graph defined in (24), both of them including 8 UEs and 4 RNs.

The results provided by all three optimization criteria are presented in (23) and Fig. 9, for the initial "complete graph", respectively in (24) and Fig. 10, for the initial connection graph defined also by (24). The relations (23) and (24) present the initial and final connection graphs, the values of P_{outage} and $BLER^{av}{}_{NC\text{-}UE}$, while Fig. 9 and Fig. 10 show the users individual $BLER_{NC\text{-}UEi}$ values.

$$\begin{array}{cc} \textit{initial} & \textit{optimized}\, C-a \\ A=\begin{bmatrix} 1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1 \end{bmatrix} & A=\begin{bmatrix} 0&1&1&1&0&1&0&0 \\ 0&1&0&0&0&1&0&1 \\ 1&1&0&1&0&0&1&0 \\ 0&0&0&1&1&1&1&0 \end{bmatrix} \\ P_{outage}=0.1869 & P_{outage}=0.0217 \\ BLER^{av}_{NC-UE}=0.0522 & BLER^{av}_{NC-UE}=0.0083 \end{array} \tag{23}$$

$$\begin{array}{cc} \textit{optimized}\, C-b & \textit{optimized}\, C-c \\ A=\begin{bmatrix} 1&1&0&1&1&0&0&0 \\ 1&1&0&0&1&0&1&0 \\ 0&1&0&0&0&1&0&1 \\ 0&1&1&1&0&0&0&1 \end{bmatrix} & A=\begin{bmatrix} 1&0&0&1&0&0&1&1 \\ 0&1&1&1&0&0&1&0 \\ 0&1&0&0&1&1&0&1 \\ 0&0&1&1&0&1&0&0 \end{bmatrix} \\ P_{outage}=0.0239 & P_{outage}=0.0175 \\ BLER^{av}_{NC-UE}=0.0098 & BLER^{av}_{NC-UE}=0.0073 \end{array}$$

$$\begin{array}{cc} \textit{initial} & \textit{optimized}\, C-a \\ A=\begin{bmatrix} 1&1&1&1&1&0&1&1 \\ 0&1&1&1&1&1&1&0 \\ 1&0&1&1&1&1&1&0 \\ 1&0&0&1&1&1&1&1 \end{bmatrix} & A=\begin{bmatrix} 1&1&0&0&0&0&0&1 \\ 0&1&1&0&1&0&1&0 \\ 1&0&0&1&0&1&1&0 \\ 0&0&0&1&1&0&1&1 \end{bmatrix} \\ P_{outage}=0.0526 & P_{outage}=0.0173 \\ BLER^{av}_{NC-UE}=0.0177 & BLER^{av}_{NC-UE}=0.0072 \end{array} \tag{24}$$

$$\begin{array}{cc} \textit{optimized}\, C-b & \textit{optimized}\, C-c \\ A=\begin{bmatrix} 1&1&0&1&0&0&0&1 \\ 0&1&0&1&1&0&1&0 \\ 1&0&1&0&1&0&1&0 \\ 0&0&0&1&1&1&0&1 \end{bmatrix} & A=\begin{bmatrix} 0&1&0&1&1&0&1&0 \\ 0&1&1&0&0&0&1&0 \\ 1&0&1&1&0&1&0&0 \\ 1&0&0&0&1&0&1&1 \end{bmatrix} \\ P_{outage}=0.0176 & P_{outage}=0.0175 \\ BLER^{av}_{NC-UE}=0.0073 & BLER^{av}_{NC-UE}=0.0073 \end{array}$$

The simulations performed show that criterion *C-a* does not provide always an optimized graph which would ensure $BLER_{NC\text{-}UEi}$ smaller than the values provided by the initial (non-optimized) graph for all UEs of the cluster. This behavior was noticed especially for sparse A matrices, i.e. for sparse connection/cooperation graphs.

The results of (23) and (24) show that all optimization criteria, when they provide solutions, decrease the values of P_{outage} and $BLER^{av}_{NC\text{-}UE}$. The results of Fig. 9 and Fig. 10 show that *C-b* and *C-c* optimization criteria, opposite to criterion *C-a*, ensure the condition $BLER_{NC\text{-}UEi} < BLER'_{NC\text{-}UEi}$ for most of the UEs, but not for all of them, depending on the initial graph's structure.

For all optimization criteria, the number of UE_i-RN_j links included in the optimized connection graph decreases significantly, even with 40-50 %, making the optimized connection graph significantly sparser than the initial one. Thus, the management of the cluster's time-frequency and power resources is also simplified.

Regarding the parameters of the GA, the simulations performed show that increasing the maximum number of generations, N_g, over 25 and the initial population size, N, above 100 chromosomes would not bring significant performance improvements, in the case of the considered test cooperation cluster with 8 UEs and 4 RNs.

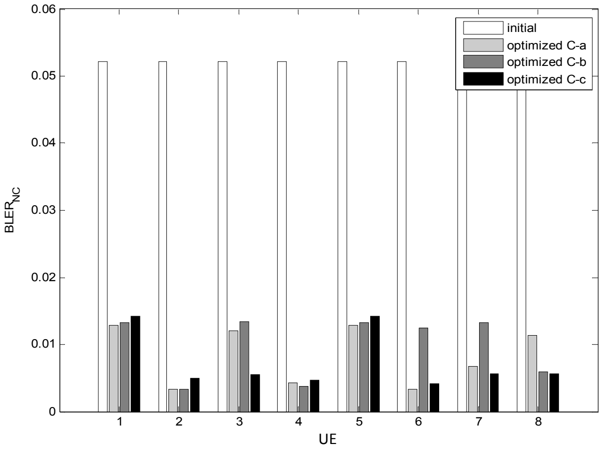

Fig. 9. $BLER_{NC\text{-}UEi}$ values obtained for the connection graphs given by (23) and $BLER_{th} = 0.1$.

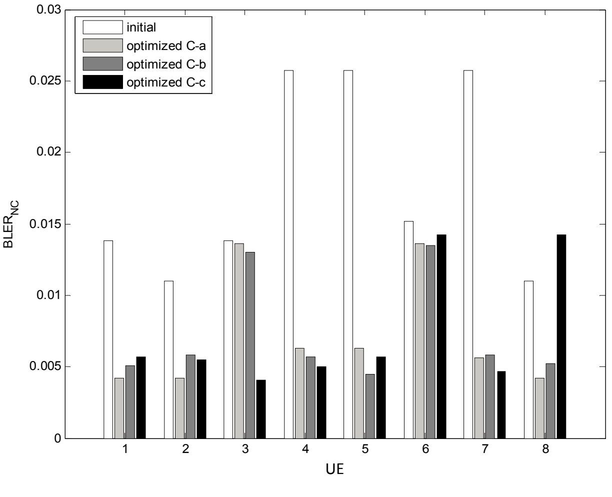

Fig. 10. $BLER_{NC\text{-}UEi}$ values obtained for the connection graphs given by (24) and $BLER_{th} = 0.1$.

6.3 Evaluation of the NC-Based Cooperation Protocol over Block Faded Radio Channels

In this section we evaluate by computer simulation the $BLER_{NC\text{-}UEi}$ vs. $SNR_{UEi\text{-}BS}$ performances provided by the proposed NC-based cooperation protocol over block faded channels. The protocol operates on the optimized connection graphs of (23) and (24) and the obtained results are presented in Fig. 11 for the graph given by (23) and in Fig. 12 for the graph given by (24). The performances were evaluated separately for each UE_i and each optimization criterion. The channel code employed for the first level of coding is the LTE turbo code of rate 1/3 [20], and the modulation employed is BPSK.

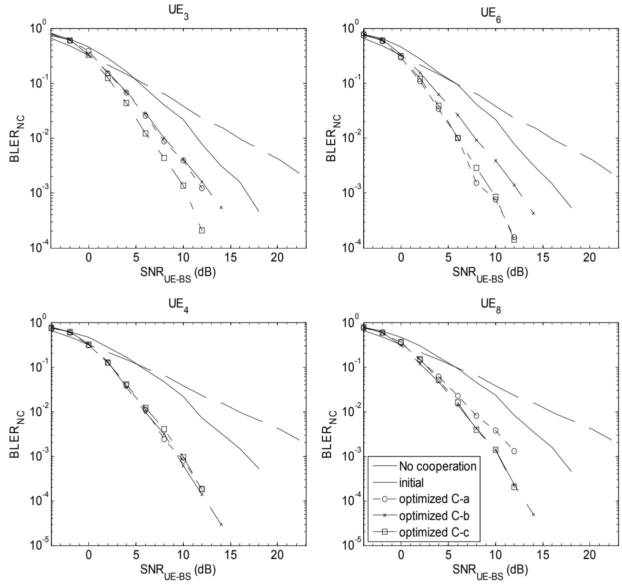

Fig. 11. $BLER_{NC\text{-}UEi}$ vs. $SNR_{UEi\text{-}BS}$ performances obtained for connection graphs given by (23).

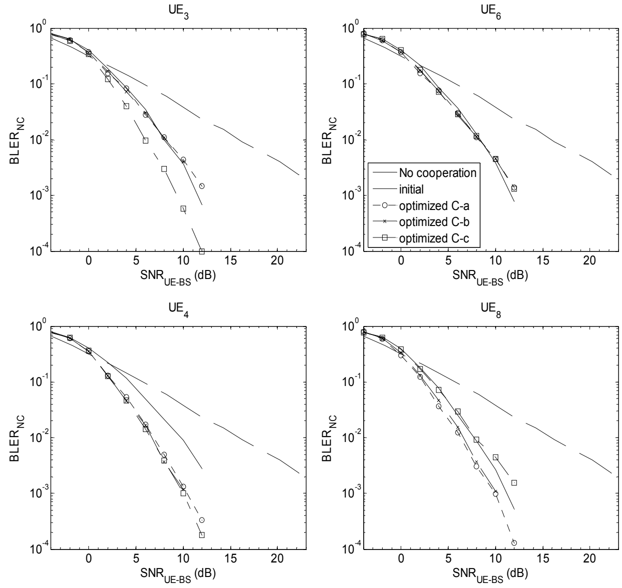

Fig. 12. $BLER_{NC\text{-}UEi}$ vs. $SNR_{UEi\text{-}BS}$ performances obtained for connection graphs given by (24).

The cooperative and non-cooperative transmissions obey the power constraint given by (4), all nodes have the same transmission power and all UE_i-BS and RN_j-BS links have the same SNR. The BLER performances provided by the direct UE_i-BS links and those provided by the NC algorithm operating on the non-optimized graphs are presented as references.

The results presented in Fig. 11 and Fig. 12 led to the following conclusions:

1. The improvement of the BLER performances ensured for each UE_i increases with the decrease of the direct link's $BLER_{UEi\text{-}BS}$ due to the diversity gain brought by the proposed cooperation protocol. This improvement varies for each UE_i depending on the structure of the connection/cooperation graph.

2. The connection/cooperation graph's optimization performed for a great value of the direct links BLER (i.e. $BLER_{th} = 0.1$ and small $SNR_{UEi\text{-}BS}$) remains valid, in most of the cases and for all three graph optimization criteria, at smaller values of $BLER_{th}$ (i.e. larger $SNR_{UEi\text{-}BS}$). Therefore, the optimization process could be performed only for one $BLER_{th}$ value. Still, in particular cases and for particular SNR values the $BLER_{NC\text{-}UEi}$ ensured for some users over the optimized graphs are approximately equal to or greater than the ones provided by the non-optimized graphs – see Fig. 12 UE_3, UE_6 and UE_8. This means that in order to ensure that $BLER_{NC\text{-}UEi} < BLER'_{NC\text{-}UEi}$, $\forall i = 1,\ldots,N_u$, $\forall SNR_{UEi\text{-}BS}$ and $\forall SNR_{RNj\text{-}BS}$, $\forall j = 1,\ldots,N_r$, the optimization process has to be performed separately for different $SNR_{UEi\text{-}BS}$ and $SNR_{RNj\text{-}BS}$ intervals.

3. The $BLER_{NC\text{-}UEi}$ values provided by the three optimization criteria are close, and therefore, if no additional requirements are imposed, criteria *C-b* or *C-c* should be used, due to their smaller computational effort.

7. Conclusions

The proposed NC-based cooperation protocol is a relatively simple but effective method to improve the individual user's uplink transmission BLER performances in wireless networks. The protocol can be used in any network topology that includes multiple sources and intermediate nodes with data processing capabilities. Utilization of such protocol in Single-Source Multi-Relay (1SMR) topologies, like the one corresponding to the downlink transmission in a cellular network, would require data exchange (i.e. separate data communications) between the RNs or the UEs, which would significantly complicate the management of radio resources allocated to the cell.

The paper proposes also a theoretical evaluation of the XOR-based cooperation protocol. This evaluation uses a graph-based characterization of the cooperation cluster and specific graph parameters, such as the outage sets, necessary for the performance analysis, are defined. The theoretical method developed was validated by extensive computer simulations.

In addition, an algorithm intended for structuring and optimization of the cooperation cluster was developed. This algorithm is based on genetic algorithms and it is a flexible and effective one, benefiting of the GA's capability to solve complex optimization problems with contradicting requirements. The proposed graph optimization algorithm was tested on several connection graphs of medium complexity, both a significant decrease of the network outage probability and a smaller average BLER per user being obtained. As for the individual user's BLER, a decrease is obtained in most scenarios for all or for the majority of the UEs involved, depending on the initial connection graph's structure and on the optimization criterion employed. The cluster optimization algorithm also decreases significantly the number of UE_i-RN links involved, which simplifies the dynamic manage-ment of the cluster's radio resources. The performed computer simulations showed that the cooperation graph's optimization, accomplished for a set of the transmission links BLERs, holds for a rather wide range of BLER values, and therefore another optimization process would not be required as long as the BLERs of the involved links rest in a certain domain.

As a drawback of the proposed cluster structuring algorithm can be mentioned the relatively slow convergence speed. Despite that, by appropriate formulation of the optimization criterion and selection of the GA's parameters, the speed of the optimization process can be improved, making it suitable for nomadic wireless networks or for wireless cooperation clusters including pedestrian mobile users.

Acknowledgement

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