Outage Probability and Power Allocation for Two-Way DF Relay Networks with Relay Selection

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Abstract. *In this paper, we investigate the outage probability and power allocation for the two-way decode-and-forward (DF) relay networks with relay selection. Specially, we consider independent but not necessarily identical distributed Rayleigh fading channels. Firstly, we derive an exact closed form outage probability expression. To shed light on the relation between the outage probability and the power allocation factor, an upper bound for the outage probability is derived, too. We then propose a power allocation scheme in the sense of minimizing this upper bound. Monte Carlo simulations are conducted to show that the derived outage probability expression excellently matches simulation results, and our proposed power allocation scheme performs effectively.*

Keywords

Two-way relay networks, relay selection, decode-and-forward, outage probability, power allocation.

1. Introduction

Recently, cooperative relaying has emerged as a promising technology for providing high throughput and reliability, and has generated a lot of research interests [1–5]. In conventional cooperative relay networks, so-called one-way relay networks, the transmission from source to destination consists of two time-slots. Thus, it takes four time-slots for two users to exchange information, in such scenarios as voice service, video-conferencing, etc. To eliminate the pre-log factor one-half in sum-rate expression and further improve bandwidth efficiency, half-duplex two-way relay cooperation has been proposed in [6], where two sources transmit signals simultaneously to one relay or multiple relays in the first time-slot, and the relay(s) broadcasts the received signals to the two sources in the second time-slot.

Because of the promising advantage of two-way relay cooperation, kinds of relay strategies, performance analysis and power allocation schemes have been investigated [7–15]. In general, the amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF) protocols used in one-way relay networks can be applied to two-way relay networks [7]. For a two-way relay network with one relay node, Ref. [8] analyzed the outage probability of AF and DF protocols, and proposed an adaptively switching scheme between AF and DF protocol based on the decoding ability at the relay. The error performance of binary phase shift keying (BPSK) modulation was analyzed for two-way DF relay networks in [9], where the relay node performs XOR operation on its received signals. In practical scenarios such as in cellular networks, there always exists multiple users in a network and the diversity order can be increased with the help of other users. To exploit this merit, Ref. [10] proposed a distributed space-time coding scheme for two-way relay networks, which can achieve full diversity order if the number of symbols in a frame is no less than the number of relays. Since the scheme in [10] requires synchronization among all relays, which is a difficult task when the number of relays is large, the authors in [11] have proposed an opportunistic two-way relaying scheme based on joint network coding and relay selection. In addition, the author in [12] has proposed max-min and max-sum relay selection criterions for the two-way relay networks that employ DF protocol. The author in [13] has proposed a max-min relay selection scheme for two-way AF relay networks. Power allocation is another important issue for designing a two-way relay network, and has been investigated in previous literatures. For example, [14] proposed two power allocation schemes for two-way AF relay networks. One scheme aimed to maximize the average sum rate, while the other one aimed to achieve the trade-off of outage probability between two users. In addition, [15] investigated power allocation for two-way relay networks under data rate fairness constraints.

We noticed that the performance analysis in [12] was based on the assumption that the channel gains of all the channel links were identically distributed, which does not match the practical scenarios. In this paper, we will extend the work in [12] and focus on performance analysis of the max-min relay selection scheme over independent but not necessarily identical distributed Rayleigh fading channels. To be specific, we derive an exact closed form outage probability expression. However, it is a difficult task for us to find the optimal power allocation factor from this expression. We then make an approximation in high signal-to-noise ratio (SNR) and derive an upper bound for the outage probability.

Through minimizing this upper bound, we propose a power allocation scheme, which is shown to perform effectively in high SNR.

The rest of this paper is organized as follows. The system model is introduced in Section 2. The derivation of exact outage probability expression and power allocation scheme are presented in Section 3. In Section 4, Monte Carlo simulations are conducted to verify our analytical results. Finally, Section 5 contains some conclusions.

Notations: Throughout this paper, $f_X(\bullet)$, $F_X(\bullet)$ and $P(X)$ denote the probability density function (PDF), the cumulative distribution function (CDF) and the probability of a random variable X, respectively.

2. System Model

Consider a network with $N+2$ nodes, where each node is mounted with a single antenna and operates in half-duplex mode. As depicted in Fig. 1, the channel from source S_1 to relay R_j is denoted as h_j, and the channel from source S_2 to relay R_j is denoted as g_j, $j=1,2\ldots N$. We assume h_j and g_j are complex Gaussian distributed with zero mean and variance Ω_j and Φ_j, respectively. All channels are assumed to be independent but not necessarily identical distributed. We further assume that the channels are static in one frame transmission and change from one frame to another. The time-division duplex (TDD) is adopted here, which means that the channels from R_j to S_1 and S_2 are still h_j and g_j, respectively.

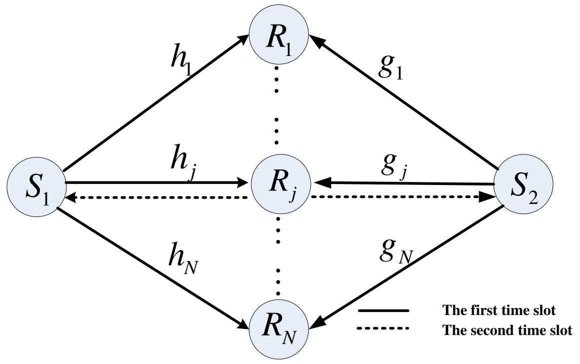

Fig. 1. The schematic of two-way relay network with relay selection.

The information exchange between S_1 and S_2 consists of two time-slots. Both sources transmit to all relays simultaneously in the first time-slot. The received signal at R_j can be written as

$$y_{j,1}=\sqrt{P_s}h_jx_1+\sqrt{P_s}g_jx_2+n_{j,1} \tag{1}$$

in which P_s denotes the transmitting power of sources, and $x_l(l=1,2)$ denotes the unit energy symbol transmitted by S_l, $n_{j,1}$ represents the additive white Gaussian noise (AWGN) with zero mean and variance N_0 at relay R_j. Without loss of generality, we assume that all nodes in the network have the same noise power N_0 and N_0 equals to one. Note that we have implicitly assumed that both S_1 and S_2 have the same transmitting power P_s, and this assumption has been adopted in [13], [14]. In the second time-slot, only the selected relay decodes its received signal, performs the XOR operation and broadcasts to both sources, where the selection criterion is as follows [12]:

$$i=\arg\max_{j=\{1,2\cdots N\}}\min\left(\left|h_j\right|^2,\left|g_j\right|^2\right) \tag{2}$$

in which i denotes the index of best relay node. We assume that the transmitting power of the selected relay node R_i in the second time-slot is P_r, which has a relation with P_s as $P_r=kP_s, k>0$, where k is called power allocation factor in this paper. When the total power P_{sum} of both sources and the relay is fixed, P_s and P_r are absolutely determined by k (seen from $P_{sum}=2P_s+P_r=(2+k)P_s=\left(2/k+1\right)P_r$), and different values of k correspond to different outage performance [13]. To improve the system performance, we aim to determine the optimal power allocation factor subject to the total power constraint in the next section.

3. Outage Probability and Power Allocation Scheme

In this section, we firstly derive the exact outage probability expression for the system described in last section. As it is cumbersome to determine the optimal power allocation factor directly from the exact outage probability expression, an upper bound is derived and the optimal power allocation factor is obtained by minimizing this upper bound.

3.1 Exact Outage Probability

Let the data rate of S_l be r_l bits per channel use (BPCU), $l=1,2$. We assume symmetric data traffic, namely both sources have the same data rate $r_1=r_2=r$ BPCU. This assumption is reasonable because the two sources exchanging information may require the same data rate, in such scenario as voice service. According to [7], [12], the achievable rate region Υ of DF protocol is the closure of the set of all points (r_1,r_2) satisfying

$$\begin{aligned}
&r_1\le\min\{\tfrac{1}{2}\log_2(1+P_r|h_i|^2),\,\tfrac{1}{2}\log_2(1+P_s|g_i|^2)\},\\
&r_2\le\min\{\tfrac{1}{2}\log_2(1+P_s|h_i|^2),\,\tfrac{1}{2}\log_2(1+P_r|g_i|^2)\},\\
&r_1+r_2\le\tfrac{1}{2}\log_2(1+P_s|h_i|^2+P_s|g_i|^2).
\end{aligned} \tag{3}$$

The problem of deriving the exact outage probability expression can be transformed to determine the probability that a given data rate lies in the achievable data rate region Υ. Hereafter, we focus on deriving the probability of that (3) holds for a desired data rate r at both sources.

$$f_{z_i}(u) = \sum_{r_i=1}^{N-1} \sum_{s_i=1}^{\binom{N-1}{r_i}} (-1)^{r_i+1} \left\{ \left[\sum_{t_i=1}^{r_i} \left(\lambda_{(t_i+s_i-1)_{N-1}} + \theta_{(t_i+s_i-1)_{N-1}} \right) \right] \exp\left[-u \sum_{t_i=1}^{r_i} \left(\lambda_{(t_i+s_i-1)_{N-1}} + \theta_{(t_i+s_i-1)_{N-1}} \right) \right] \right\} \quad (4)$$

$$p_i = \int_0^\infty f_{z_i}(z_i) P \left[\underbrace{\min\left(P_r|h_i|^2, P_s|h_i|^2, P_r|g_i|^2, P_s|g_i|^2\right) \geq 2^{2r}-1, P_s|h_i|^2 + P_s|g_i|^2 \geq 2^{4r}-1,}_{\Psi} |h_i|^2 \geq z_i, |g_i|^2 \geq z_i \right] dz_i \quad (7)$$

Supposing relay node R_i is the selected node in accord with (2), we define a set Ξ_i that consists of all the relay nodes except R_i, namely $\Xi_i = \{R_j | j = 1 \cdots N, j \neq i\}$. To simplify the notation in the following, we reorder the relay nodes in Ξ_i as $\Xi_i' = \{R_{j'} | j' = 1, 2 \cdots N-1\}$. We also define $z_i = \max_{j'=\{1,2\cdots N-1\}} \min(|h_{j'}|^2, |g_{j'}|^2)$. In the following lemma, we present the PDF of z_i.

Lemma: Let's define $\lambda_j = 1/\Omega_j$ and $\theta_j = 1/\Phi_j$, $j = 1, 2, \cdots N$, then the PDF of z_i is given by (4) (presented at the top of this page). In (4), r_i, s_i and t_i are the indices of relay nodes in Ξ_i', $\left(\begin{array}{c} N-1 \\ r_i \end{array} \right) = \frac{(N-1)!}{r_i!(N-r_i-1)!}$ and the operator $(\bullet)_{N-1}$ is defined as

$$(s_i)_{N-1} = \begin{cases} s_i & 1 \leq s_i \leq N-1, \\ s_i - N + 1 & s_i > N-1. \end{cases}$$

Proof: We have the following equations

$$\begin{aligned} & P[\min(|h_{j'}|^2, |g_{j'}|^2) \leq u] \\ & = 1 - P[|h_{j'}|^2 > u] P[|g_{j'}|^2 > u] \qquad (a) \\ & = 1 - e^{-(\lambda_{j'} + \theta_{j'})u}. \qquad (b) \end{aligned} \quad (5)$$

The step (a) in (5) holds for the fact that $h_{j'}$ and $g_{j'}$ are independent. The step (b) in (5) derives from that $|h_{j'}|^2$ and $|g_{j'}|^2$ are exponential distributed with parameters $\lambda_{j'}$ and $\theta_{j'}$, respectively. By using order statistics [16], the CDF of z_i is $F_{z_i}(u) = P(z_i \leq u) = \prod_{j'=1}^{N-1} [1 - e^{-(\lambda_{j'}+\theta_{j'})u}]$. Then, (4) is directly obtained by differentiating $F_{z_i}(u)$ with respect to u. ■

With the above lemma, it is sufficient for us to investigate the exact outage probability in the following. Let the probability that a desired data rate r lies in the achievable region Υ when R_i is selected be

$$p_i = P \left[r \in \Upsilon, i = \max_{j=\{1,2\cdots N\}} \min(|h_j|^2, |g_j|^2) \right]. \quad (6)$$

Then, the outage probability P_{out} is given by $P_{out} = 1 - \sum_{i=1}^{N} p_i$. Thus, the remaining problem is to determine p_i. With the definition of z_i and achievable region Υ in (3), p_i can be rewritten by (7)(shown at the top of this page). Since it is intractable to obtain an unified region for Ψ in (7) for arbitrary values of P_s and P_r (or equivalently arbitrary value of k), we consider the following three cases:

Case A: $0 < k \leq 2(2^{2r}-1)/(2^{4r}-1)$: For notation simplicity, let's define $m_r = (2^{2r}-1)/P_r$ and $n = (2^{4r}-1)/P_s$. In this case, we have $P_r < P_s$, $m_r \geq n/2$. Then, Ψ reduces to $\Psi = \left\{|h_i|^2 \geq m_r, |g_i|^2 \geq m_r, |h_i|^2 + |g_i|^2 \geq n\right\}$ (as illustrated in Fig. 2(a)).

Case B: $2(2^{2r}-1)/(2^{4r}-1) \leq k \leq 1$: In this case, $P_r \leq P_s$, $m_r \leq n/2$ and Ψ can by written by $\Psi = \left\{|h_i|^2 \geq m_r, |g_i|^2 \geq m_r, |h_i|^2 + |g_i|^2 \geq n\right\}$ (as illustrated in Fig. 2(b)). Note that the difference between Fig. 1(b) and Fig. 1(a) is caused by $m_r \leq n/2$ when $2(2^{2r}-1)/(2^{4r}-1) \leq k \leq 1$.

Case C: $k \geq 1$: We define $m_s = (2^{2r}-1)/P_s$ and then Ψ reduces to $\Psi = \left\{|h_i|^2 \geq m_s, |g_i|^2 \geq m_s, |h_i|^2 + |g_i|^2 \geq n\right\}$ (as illustrated in Fig. 2(c)). It should be noted that $m_s \leq n/2$ always holds when $k \geq 1$.

By separately considering the above three cases and solving the integration in (7), we can obtain the exact outage probability P_{out}, which is shown in the following theorem.

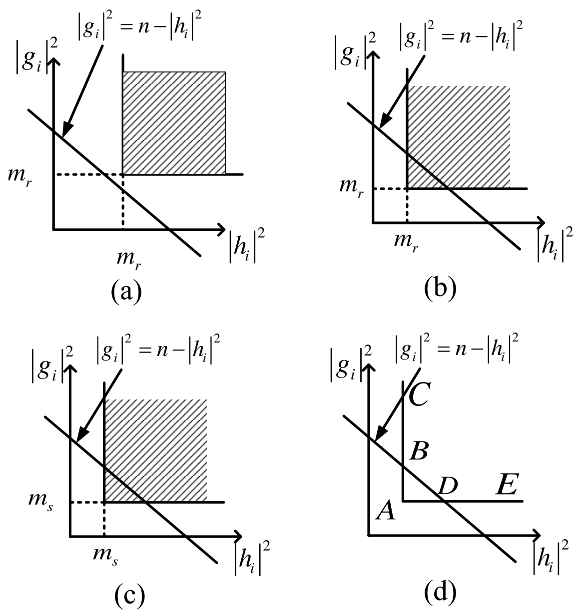

Fig. 2. Illustration of region Ψ and integration region.

$$h_i(x,y,z) = \sum_{r_i=1}^{N-1} \sum_{s_i=1}^{\binom{N-1}{r_i}} \frac{(-1)^{r_i+1}\left[\sum\limits_{t_i=1}^{r_i}(\lambda_{(t_i+s_i-1)_{N-1}}+\theta_{(t_i+s_i-1)_{N-1}})\right]}{\left[x+y+\sum\limits_{t_i=1}^{r_i}(\lambda_{(t_i+s_i-1)_{N-1}}+\theta_{(t_i+s_i-1)_{N-1}})\right]} \exp\left\{-\left[x+y+\sum_{t_i=1}^{r_i}(\lambda_{(t_i+s_i-1)_{N-1}}+\theta_{(t_i+s_i-1)_{N-1}})\right]z\right\},$$

$$g_i(x) = 1-\sum_{r_i=1}^{N-1} \sum_{s_i=1}^{\binom{N-1}{r_i}} (-1)^{r_i+1} \exp\left\{-\sum_{t_i=1}^{r_i}\left(\lambda_{(t_i+s_i-1)_{N-1}}+\theta_{(t_i+s_i-1)_{N-1}}\right)x\right\},$$

$$q_i(x,y,z) = \sum_{r_i=1}^{N-1} \sum_{s_i=1}^{\binom{N-1}{r_i}} (-1)^{r_i+1} \left\{\begin{array}{l} [(xy+1)e^{-xy}-2xe^{-xy}y]\exp\left[-\sum\limits_{t_i=1}^{r_i}(\lambda_{(t_i+s_i-1)_{N-1}}+\theta_{(t_i+s_i-1)_{N-1}})z\right] \\ -[(xy+1)e^{-xy}-xe^{-xy}y]\exp\left[-\sum\limits_{t_i=1}^{r_i}(\lambda_{(t_i+s_i-1)_{N-1}}+\theta_{(t_i+s_i-1)_{N-1}})y/2\right] \\ +\dfrac{2xe^{-xn}\left[\exp(-\sum\limits_{t_i=1}^{r_i}(\lambda_{(t_i+s_i-1)_{N-1}}+\theta_{(t_i+s_i-1)_{N-1}})y/2)-\exp(-\sum\limits_{t_i=1}^{r_i}(\lambda_{(t_i+s_i-1)_{N-1}}+\theta_{(t_i+s_i-1)_{N-1}})z)\right]}{\sum\limits_{t_i=1}^{r_i}(\lambda_{(t_i+s_i-1)_{N-1}}+\theta_{(t_i+s_i-1)_{N-1}})} \end{array}\right\}.$$

Theorem 1: The exact outage probability P_{out} for the two-way DF relay network that employs max-min relay selection scheme is

$$P_{out} = \begin{cases} 1-\sum\limits_{i=1}^{N} p_i^1 & 0<k\le 2(2^{2r}-1)/\left(2^{4r}-1\right), \\ 1-\sum\limits_{i=1}^{N} p_i^2 & 2(2^{2r}-1)/\left(2^{4r}-1\right)\le k\le 1, \\ 1-\sum\limits_{i=1}^{N} p_i^3 & k\ge 1 \end{cases} \quad (8)$$

in which p_i^1, p_i^2 and p_i^3 are expressed as follows:

$$p_i^1 = h_i(\lambda_i,\theta_i,m_r)+e^{-(\lambda_i+\theta_i)m_r}g_i(m_r),$$

when $\lambda_i \ne \theta_i$,

$$\begin{aligned} p_i^2 &= h_i(\lambda_i,\theta_i,n/2)+\tfrac{\lambda_i e^{-\theta_i n}}{\lambda_i-\theta_i}\left[h_i(\lambda_i,-\theta_i,m_r)-h_i(\lambda_i,-\theta_i,n/2)\right] \\ &-\tfrac{\theta_i e^{-\lambda_i n}}{\lambda_i-\theta_i}\left[h_i(-\lambda_i,\theta_i,m_r)-h_i(-\lambda_i,\theta_i,n/2)\right] \\ &+\tfrac{\left[\lambda_i e^{-\theta_i n}e^{-(\lambda_i-\theta_i)m_r}-\theta_i e^{-\lambda_i n}e^{-(\theta_i-\lambda_i)m_r}\right]g_i(m_r)}{\lambda_i-\theta_i}, \\ p_i^3 &= h_i(\lambda_i,\theta_i,n/2)+\tfrac{\lambda_i e^{-\theta_i n}}{\lambda_i-\theta_i}\left[h_i(\lambda_i,-\theta_i,m_s)-h_i(\lambda_i,-\theta_i,n/2)\right] \\ &-\tfrac{\theta_i e^{-\lambda_i n}}{\lambda_i-\theta_i}\left[h_i(-\lambda_i,\theta_i,m_s)-h_i(-\lambda_i,\theta_i,n/2)\right] \\ &+\tfrac{\left[\lambda_i e^{-\theta_i n}e^{-(\lambda_i-\theta_i)m_s}-\theta_i e^{-\lambda_i n}e^{-(\theta_i-\lambda_i)m_s}\right]g_i(m_s)}{\lambda_i-\theta_i}, \end{aligned} \quad (9)$$

when $\lambda_i = \theta_i$,

$$\begin{aligned} p_i^2 &= h_i(\lambda_i,\theta_i,n/2)+q_i(\lambda_i,n,m_r) \\ &+\left[(\lambda_i n+1)e^{-\lambda_i n}-2\lambda_i e^{-\lambda_i n}m_r\right]g_i(m_r), \\ p_i^3 &= h_i(\lambda_i,\theta_i,n/2)+q_i(\lambda_i,n,m_s) \\ &+\left[(\lambda_i n+1)e^{-\lambda_i n}-2\lambda_i e^{-\lambda_i n}m_s\right]g_i(m_s). \end{aligned} \quad (10)$$

The functions $h_i(x,y,z)$, $g_i(x)$ and $q_i(x,y,z)$ are given at the top of this page.

Proof: See Appendix.■

Though the exact outage probability expression is complicated, it provides us a way to compare with other relay selection schemes, such as max-sum DF scheme in [12] and max-min AF scheme in [13]. In addition, it acts as the basis for us to investigate the power allocation in the next subsection.

3.2 Power Allocation Scheme

In this subsection, we study the power allocation problem in the sense of minimizing the outage probability with the total power P_{sum} constraint. As the outage probability expression (8) is complicated, we try to get an upper bound for the outage probability in high SNR regime, and obtain the optimal power allocation factor with respect to minimizing this upper bound.

Substituting $P_r = kP_s$ and $P_{sum} = 2P_s + P_r$ into $m_r = \left(2^{2r}-1\right)/P_r$ and $m_s = \left(2^{2r}-1\right)/P_s$, we get $m_r = \left(2^{2r}-1\right)\left(1+2/k\right)/P_{sum}$ and $m_s = \left(2^{2r}-1\right)(k+2)/P_{sum}$. Thus, for a fixed power allocation factor k, m_r and m_s approach zero as the total power P_{sum} approaches infinity. From the proof of theorem 1, we know that the integration range of the last component in p_i is of small value in high SNR. So, we can reasonably ignore the last components in p_i^1, p_i^2 and p_i^3. To simplify calculation, we slightly enlarge the integration region of $P\left[x\ge z_i, y\ge z_i, x+y\ge n\right]$ in B.2 (as shown in Fig. 2(d), we consider the integration region EAC instead of EDBC. The corresponding calculation for case C follows the same way as in B.2), and then get an upper bound of the outage probability P_{out}^u, which is expressed as

$$P_{out}^u = \begin{cases} 1-\sum\limits_{i=1}^{N} h_i\left(\lambda_i,\theta_i,m_r\right) & 0<k\le 1, \\ 1-\sum\limits_{i=1}^{N} h_i\left(\lambda_i,\theta_i,m_s\right) & k\ge 1. \end{cases} \quad (11)$$

Theorem 2: The optimal power allocation factor with respect to minimizing P_{out}^u is $k=1$.

Proof: From the definition of $h_i(x,y,z)$, we get

$$\frac{\partial h_i(\lambda_i,\theta_i,z)}{\partial z} = -e^{-(\lambda_i+\theta_i)z}f_{z_i}(z)\le 0. \quad (12)$$

The inequality in (10) stems from that the PDF $f_{z_i}(z)$ is always larger or equal to zero for arbitrary z. (10) indicates that $h(\lambda_i, \theta_i, z)$ is a monotonically decreasing function with respect to z. Based on the following equations

$$m_r = \frac{2^{2r}-1}{P_{sum}}(1+\frac{2}{k}), \quad m_s = \frac{2^{2r}-1}{P_{sum}}(k+2), \tag{13}$$

we know that m_r and m_s are monotonically decreasing and increasing functions of k, respectively. Consequently, we can reasonably deduce that $1-\sum_{i=1}^{N} h_i(\lambda_i, \theta_i, m_r)$ in (11) monotonically decreases and $1-\sum_{i=1}^{N} h_i(\lambda_i, \theta_i, m_s)$ in (11) monotonically increases as k increases. From the above discussion, we see that P_{out}^{u} achieves its minimum when $k = 1$. Thus, theorem 2 is proved.■

With theorem 2, we propose to allocate equal power among all nodes (including two source nodes and one selected relay node) in the network, i.e., $P_s = P_r$. Since our proposed power allocation factor is independent of channel fading parameters, it is simple to implement in practical networks. As will be seen in the next section, this scheme is nearly optimal in high SNR.

4. Simulation Results

In this section, we perform Monte Carlo simulations to verify our analytical results. In all simulations, we assume the data rate $r = 1$ BPCU and $N = 2, 3$. The corresponding channel parameters $\lambda_j, \theta_j, j = 1, 2, 3$ are randomly generated by MATLAB software before simulation. The MATLAB output is $\lambda_1 = 0.4326, \theta_1 = 1.6656, \lambda_2 = 0.1253, \theta_2 = 0.1253$ and $\lambda_3 = 1.1465, \theta_3 = 1.1909$. In the process of simulation, all the channel parameters $\lambda_j, \theta_j, j = 1, 2, 3$ are fixed. For fair comparison, we plot the outage probability curves as a function of P_{sum}.

Fig. 3 presents simulated outage probability for $N = 2, 3$ to corroborate the analytical exact outage probability expression (8). Since $2(2^{2r}-1)/(2^{4r}-1) = 0.4$ for $r = 1$ BPCU, we choose three typical values for k, namely $k = 0.3, 0.6, 1.2$. As can be seen clearly from all the curves in this figure, analytical and simulated outage probability curves match excellently, which verifies the correctness of theorem 1. By comparing the curves for $N = 2$ with those for $N = 3$, it is observed the curves corresponding to $N = 3$ have a faster decreasing speed than those corresponding to $N = 2$ in high SNR. This observation indicates that we should employ more relays to help sources exchanging information from the viewpoint of improving transmission reliability. We also plot the simulated results and upper bound in Fig. 4. Though the bound is relatively loose, as observed from the figure, the power allocation factor derived from this bound is nearly optimal in high SNR, which is demonstrated in Fig. 5 and Fig. 6.

The effectiveness of power allocation scheme proposed in last section is illustrated in Fig. 5 and Fig. 6. The simulated outage probability with $k = 0.5, 1, 1.5$ is plotted in Fig. 5. As expected, the curve corresponding to $k = 1$ outperforms the other two curves corresponding to $k = 0.5, 1.5$ from medium to high SNR regime. For example, when the power allocation factor changes from 0.5 to 1, a SNR gain of at least 2 dB is achieved when the outage probability equals to 10^{-6} and N equals to 3. Interestingly, we find that $k = 1.5$ performs better than $k = 0.5$. This phenomenon is more obvious in Fig. 6. As shown in Fig. 6, the slope of the outage probability against the k is steeper when $k \in (0, 1]$ compared to the case of $k \in [1, \infty)$. This characteristic provides us the rule for power allocation in practice: more power should be allocated to relay node when we can not guarantee that the power allocation factor k exactly equals to one. Of course, Fig. 6 also indicates that $k = 1$ is an optimum value relative to other values in the figure, which proves the effectiveness our proposed power allocation scheme.

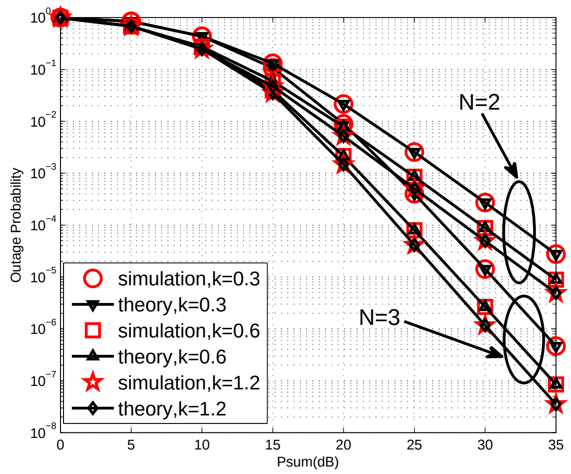

Fig. 3. Outage probability of two-way DF relay networks with max-min relay selection criteria, $N = 2, 3$: simulated results versus exact analytical results.

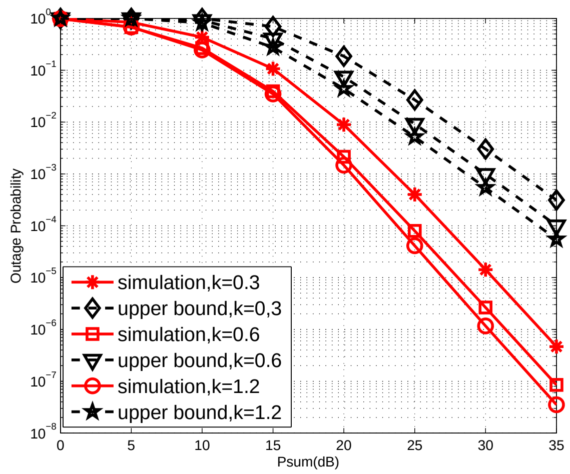

Fig. 4. Outage probability of two-way DF relay networks with max-min relay selection criteria, $N = 3$: simulated results versus upper bound.

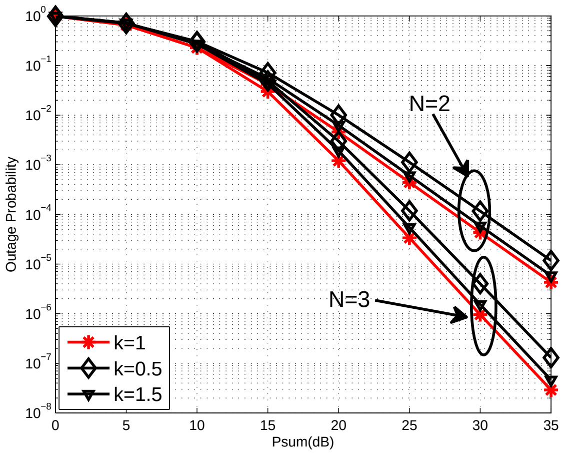

Fig. 5. Simulated outage probability using the power allocation factor $k = 0.5, 1, 1.5$ and $N = 2, 3$.

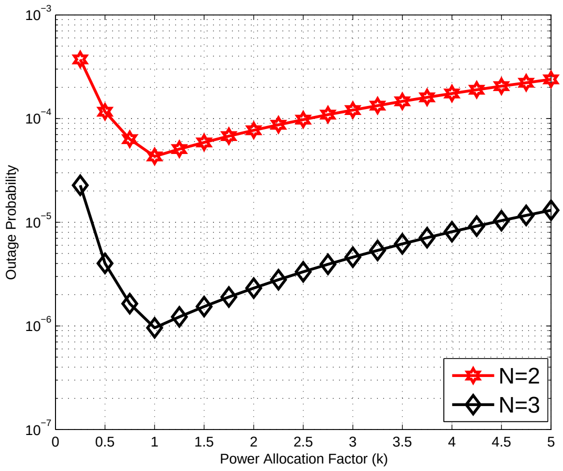

Fig. 6. Simulated outage probability with various power allocation factors, $N = 2, 3$, $P_{sum} = 30$ dB.

5. Conclusion

This paper has investigated the outage performance of two-way DF relay networks that employ max-min relay selection. We considered the general independent but not necessarily identical distributed Rayleigh fading channels. We derived the exact closed form outage probability expression, which provided us the theoretical basis for comparing with other two-way relaying schemes, and for system design in the future. Monte Carlo simulations have been conducted to verify the correctness of our derived exact outage probability expression. Both analytical and simulation results have shown the benefits of employing multiple relays for cooperatively transmission. To investigate the relation between outage probability and power allocation factor, we also provided an upper bound for outage probability, and then obtained the optimal power allocation factor in the sense of minimizing this upper bound. It has been shown from simulation results that our proposed power allocation factor value outperformed other values from medium to high SNR regime. Moreover, the proposed power allocation scheme has a low implementation complexity in practice, since it does not rely on specific channel conditions. To investigate the outage performance and optimal power allocation for the network with different data rate requirements will be our future work.

Appendix: Proof of Theorem 1

In this appendix, we present the derivation of exact outage probability P_{out} in detail. Since $P_{out} = 1 - \sum_{i=1}^{N} p_i$, we focus on deriving p_i in the following. For notation simplicity, we define $x = |h_i|^2$ and $y = |g_i|^2$ in this appendix. The probability density functions of x and y are $f_x(x) = \lambda_i e^{-\lambda_i x}$ and $f_y(y) = \theta_i e^{-\theta_i y}$, respectively. Now, we separate k into three cases to derive p_i.

Case A: $0 < k \le 2(2^{2r}-1)/(2^{4r}-1)$.

In this case, we have $m_r \ge n/2$ and p_i can be expressed as

$$p_i = \int_0^\infty f_{z_i}(z_i) P\begin{bmatrix} x \ge m_r, y \ge m_r, \\ x \ge z_i, y \ge z_i, x+y \ge n \end{bmatrix} dz_i. \quad (14)$$

Since (14) can not be calculated directly, we partition z_i into two parts.

A. 1 when $z_i \ge m_r$,

$$\begin{aligned} p_{i,a1} &= \int_{m_r}^\infty f_{z_i}(z_i) P[x \ge z_i, y \ge z_i, x+y \ge n] dz_i \\ &= \int_{m_r}^\infty f_{z_i}(z_i) e^{-(\lambda_i+\theta_i)z_i} dz_i = h_i(\lambda_i, \theta_i, m_r), \end{aligned} \quad (15)$$

A. 2 when $0 \le z_i < m_r$,

$$\begin{aligned} p_{i,a2} &= \int_0^{m_r} f_{z_i}(z_i) P[x \ge m_r, y \ge m_r, x+y \ge n] dz_i \\ &= \int_0^{m_r} f_{z_i}(z_i) e^{-(\lambda_i+\theta_i)m_r} dz_i = e^{-(\lambda_i+\theta_i)m_r} g_i(m_r). \end{aligned} \quad (16)$$

Adding (15) and (16) we can represent p_i in closed form as $p_i = p_i^1 = h_i(\lambda_i, \theta_i, m_r) + e^{-(\lambda_i+\theta_i)m_r} g_i(m_r)$.

Case B: $2(2^{2r}-1)/(2^{4r}-1) \le k \le 1$.

In this case, we have $m_r \le n/2$ and p_i can be given by

$$p_i = \int_0^\infty f_{z_i}(z_i) P\begin{bmatrix} x \ge m_r, y \ge m_r, \\ x \ge z_i, y \ge z_i, x+y \ge n \end{bmatrix} dz_i. \quad (17)$$

We partition z_i into three parts to calculate p_i.

B. 1 when $z_i \ge n/2$,

$$\begin{aligned} p_{i,b1} &= \int_{n/2}^\infty f_{z_i}(z_i) P\begin{bmatrix} x \ge z_i, y \ge z_i, \\ x+y \ge n \end{bmatrix} dz_i \\ &= \int_{n/2}^\infty f_{z_i}(z_i) e^{-(\lambda_i+\theta_i)z_i} dz_i = h_i(\lambda_i, \theta_i, n/2), \end{aligned} \quad (18)$$

B. 2 when $m_r \le z_i < n/2$,

$$
\begin{aligned}
p_{i,b2} &= \int_{m_r}^{n/2} f_{z_i}(z_i) P[x \geq z_i, y \geq z_i, x+y \geq n] dz_i \\
&= \int_{m_r}^{n/2} f_{z_i}(z_i) \frac{\lambda_i e^{-\theta_i n} e^{-(\lambda_i-\theta_i)z_i} - \theta_i e^{-\lambda_i n} e^{-(\theta_i-\lambda_i)z_i}}{\lambda_i-\theta_i} dz_i \\
&= \frac{\lambda_i e^{-\theta_i n}}{\lambda_i-\theta_i} \left[h_i(\lambda_i,-\theta_i,m_r) - h_i(\lambda_i,-\theta_i,n/2)\right] \\
&\quad - \frac{\theta_i e^{-\lambda_i n}}{\lambda_i-\theta_i} \left[h_i(-\lambda_i,\theta_i,m_r) - h_i(-\lambda_i,\theta_i,n/2)\right].
\end{aligned} \tag{19}
$$

Note that we have implicitly assumed $\lambda_i \neq \theta_i$. When $\lambda_i = \theta_i$, $p_{i,b2}$can be expressed as

$$
\begin{aligned}
p_{i,b2} &= \int_{m_r}^{n/2} f_{z_i}(z_i) \left[(\lambda_i n+1)e^{-\lambda_i n} - 2\lambda_i e^{-\lambda_i n} z_i\right] dz_i \\
&= q_i(\lambda_i, n, m_r).
\end{aligned} \tag{20}
$$

B. 3 when $0 \leq z_i < m_r$,

$$
\begin{aligned}
p_{i,b3} &= \int_{0}^{m_r} f_{z_i}(z_i) P[x \geq m_r, y \geq m_r, x+y \geq n] dz_i \\
&= \int_{0}^{m_r} f_{z_i}(z_i) \frac{\lambda_i e^{-\theta_i n} e^{-(\lambda_i-\theta_i)m_r} - \theta_i e^{-\lambda_i n} e^{-(\theta_i-\lambda_i)m_r}}{\lambda_i-\theta_i} dz_i \\
&= \frac{\lambda_i e^{-\theta_i n} e^{-(\lambda_i-\theta_i)m_r} - \theta_i e^{-\lambda_i n} e^{-(\theta_i-\lambda_i)m_r}}{\lambda_i-\theta_i} g_i(m_r).
\end{aligned} \tag{21}
$$

(21) holds true for $\lambda_i \neq \theta_i$. When $\lambda_i = \theta_i$, $p_{i,b3}$ is

$$
p_{i,b3} = \left[(\lambda_i n+1)e^{-\lambda_i n} - 2\lambda_i e^{-\lambda_i n} m_r\right] \times g_i(m_r). \tag{22}
$$

From above derivation, we get $p_i = p_i^2$ by adding (18), (19) and (21), when $\lambda_i \neq \theta_i$. When $\lambda_i = \theta_i$, p_i^2 is obtained by adding (18), (20) and (22).

Case C. $k \geq 1$.

The derivation of p_i in this case can be accomplished by following the procedure in case B with m_r replaced by m_s. (The similarity of case B and Case C can be observed from Fig. 2(b) and Fig. 2(c)).

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