# The Spectrum Sharing in Cognitive Radio Networks Based on Competitive Price Game

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Abstract. The competitive price game model is used to analyze the spectrum sharing in the cognitive radio networks. This paper focuses on the improved spectrum sharing problem with constraints of available spectrum resource from primary users. The constrained spectrum sharing model based on competitive price game is established. In order to achieve the spectrum sharing strategy that satisfies the constraints from primary users, the Rockafeller multiplier method is applied to deal with the constraints of available licensed spectrum resource, and the improved profit function is achieved, which can be used to measure the impact of shared spectrum price strategies on the system profit. However, in the competitive price game based spectrum sharing problem of practical cognitive radio network, primary users have to determine the shared spectrum price without the acknowledgement of the other primary user's price strategies simultaneously. Thus a fast gradient iterative calculation method of equilibrium price is proposed, only with acknowledgement of the shared spectrum price strategies during last cycle. Through the adaptive iteration at the direction with largest gradient of improved profit function, the equilibrium price strategies can be achieved rapidly. It can also avoid the predefinition of the adjustment factor according to the parameters of communication system in conventional linear iteration method. Simulation results show that the proposed competitive price game based spectrum sharing model can be applied in the cognitive radio networks with constraints of available licensed spectrum resource, and it has better convergence performance.

# Keywords

Cognitive radio, spectrum sharing, game theory, competitive price model.

## 1. Introduction

Cognitive radio is viewed as an effective approach for improving the utilization of the radio spectrum [1], [2]. The cognitive transceivers have flexible spectrum sensing ability, and can adjust transmission parameters adaptively according to the ambient environment. The spare spectrum

of licensed users (primary user) can be accessed by the cognitive users (secondary user) dynamically without causing harmful interference. As the behaviors of the primary users and secondary users interact with each other, game theory, which is viewed as an effective tool for the analysis of interactive decision making, is applied in the spectrum sharing problem of cognitive radio networks [3], [4]. Duist Niyato introduced the competitive price model to the spectrum sharing problem in cognitive radio networks, and discussed the Nash equilibrium price strategies that maximize the profits of primary users. As the price strategies of other primary users are usually not available simultaneously, the iterative equilibrium price calculation was further analyzed [5], [6]. The spectrum sharing problem with price strategies offered simultaneously and sequentially is also discussed in [7]. Xuezhi Tan discussed the selection of adjustment factor in the calculation of the equilibrium price [8]. The calculation of the equilibrium shared spectrum price is analyzed in [9]. But only the spectrum sharing problem with the assumption that the spectrum demand of the cognitive users could be satisfied was discussed in the research above. In order to guarantee the communication quality of the primary users in the practical application, the spectrum sharing problem with the constraints of maximum available spectrum resource is more universal. In this paper, we develop an improved spectrum sharing model based on competitive price game with the consideration of limited spectrum resources from primary users, and discuss the iterative calculation of the equilibrium shared spectrum price. We proposed a fast gradient iterative method based equilibrium price calculation scheme, which can avoid the predefinition of the adjustment factor, and achieve the equilibrium price iteratively with better convergence performance.

The rest of this paper is organized as follows. In section 2, we describe the system model of spectrum sharing in cognitive radio networks. In section 3, the spectrum sharing problem based on competitive price game is analyzed, with the consideration of constraints of available spectrum resource from the primary users. In section 4, the calculation of the equilibrium price without acknowledgement of the price strategies from other primary users is further analyzed. Section 5 presents the simulation results, and section 6 concludes the paper.

# 2. System Model

Considering a wireless system with multiple primary users, the communication of secondary users is served by a secondary service for simplicity. The spectrum sharing problem between the multiple primary users and secondary users can be equivalent with the spectrum sharing problem between multiple primary users and single secondary user service. Assuming N primary users within the cognitive radio network, primary user *i* tends to share portion of its spare spectrum with secondary user at price  $p_i$ . The communication requirements of secondary user should be satisfied with assurance of primary users' communication qualities, and certain economic revenue is obtained by the primary users through spectrum sharing. The demanded spectrum size of secondary user here is determined by its transmission efficiency and the shared spectrum price that provided by primary users. The transmission efficiency of secondary user is related to the modulation and channel qualities, which can be expressed as follows [10]:

$$k = \log_2(1 + K\gamma) \tag{1}$$

where  $\gamma$  is the signal to noise ratio of the secondary receiver,  $K = 1.5/[\ln(0.2/BER^{tar})]$ ,  $BER^{tar}$  is the required bit error performance of the secondary user.

The primary users tend to maximize their profits competitively by controlling price strategies of the shared spectrum, which is defined as Bertrand price model in economics, and it is used to analyze the spectrum sharing problem in cognitive radio networks. The primary users provide the price strategies of shared spectrum, and the demanded spectrum size of the secondary user is determined from its utility function, which is relevant to the price strategies provided by primary users. The spectrum sharing profits of primary user depend on the economic revenue and the cost due to spectrum sharing. Here, the cost of spectrum sharing is defined as the degradation of the quality of service (QoS). The primary users constantly adjust the price strategies of shared spectrum to achieve the maxima of their own profits.

# 3. Constrained Spectrum Sharing Model Based on Competitive Price Game

The demanded spectrum size of secondary user can be calculated through the quadratic utility function that is described as follows [5]:

$$U(\boldsymbol{b}) = \sum_{i=1}^{N} b_i k_i^{(s)} - \frac{1}{2} \left( \sum_{i=1}^{N} b_i^2 + 2\nu \sum_{j \neq i} b_j b_j \right) - \sum_{i=1}^{N} p_i b_i \quad (2)$$

where **b** is the set of the shared spectrum size from different primary users,  $b = \{b_1, b_2, ..., b_N\}$ ;  $p_i$  is the price strategy of shared spectrum from primary user *i*,  $p = \{p_1, p_2, ..., p_N\}$ ;  $k_i^{(s)}$  is the spectrum efficiency of secondary user that can be achieved by occupying the spectrum resource from primary user *i*; *v* is the spectrum substitutability factor, and  $v \in [0,1]$ . When v = 0, the spectrum resource of different primary users can't be substituted, while v = 1, the spectrum resource of different primary users can be substituted freely. The demanded spectrum size by secondary users can be get by

$$\frac{\partial U(\boldsymbol{b})}{\partial b_i} = 0.$$
 (3)

Thus we can get

$$k_i^{(s)} - b_i - v \sum_{j \neq i} b_j - p_i = 0.$$
 (4)

The size of shared spectrum can be rewritten as the linear equations according to (4)

$$\begin{cases} b_1 + vb_2 + vb_3 + \dots + vb_N = k_1^{(s)} - p_1 \\ vb_1 + b_2 + vb_3 + \dots + vb_N = k_2^{(s)} - p_2 \\ \vdots \\ vb_1 + vb_2 + vb_3 + \dots + b_N = k_N^{(s)} - p_N \end{cases}$$
(5)

The demanded spectrum size of the secondary user can be expressed as  $\boldsymbol{D} = [b_1 \ b_2 \dots b_N]^T$ ,  $\boldsymbol{F} = [k_1^{(s)} - p_1 \ k_2^{(s)} - p_2 \dots k_N^{(s)} - p_N]^T$ , and  $\boldsymbol{A}$  is the  $N \times N$  coefficients matrix. The size of demanded spectrum can be obtained by

$$\boldsymbol{D} = \boldsymbol{A}^{-1} \boldsymbol{F} \tag{6}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 1 & v & v & \dots & v \\ v & 1 & v & \dots & v \\ v & v & 1 & \dots & v \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ v & v & v & \dots & 1 \end{bmatrix}$$

The elements in the inverse of A matrix can be achieved by elementary transform.

$$a'_{ij} = \begin{cases} \frac{1 + (N-2)v}{(1-v)(1+(N-1)v)} & i = j\\ \frac{v}{(1-v)(1+(N-1)v)} & i \neq j \end{cases}$$
(7)

With the acknowledgement of shared spectrum price strategies p, the size of demanded spectrum can be expressed as follows:

$$D_{i}(\boldsymbol{p}) = \frac{(k_{i}^{(s)} - p_{i})(\nu(N-2) + 1) - \nu \sum_{j \neq i} (k_{j}^{(s)} - p_{j})}{(1 - \nu)(\nu(N-1) + 1)} .$$
(8)

The cost of spectrum sharing in this paper is defined by the QoS degradation of primary users. The revenue function  $R_i$  and cost function  $C_i$  for the primary user *i* that provides certain size of shared spectrum are defined as follows respectively[5].

$$R_i = c_1 M_i \,, \tag{9}$$

$$C_{i}(b_{i}) = c_{2}M_{i}(B_{i}^{req} - k_{i}^{(p)}\frac{W_{i} - b_{i}}{M_{i}})$$
(10)

where  $c_1$ ,  $c_2$  are constants that denote the weights between revenue function and cost function,  $B_i^{\text{req}}$  is the bandwidth requirement of primary user,  $W_i$  is the total size of available spectrum,  $M_i$  is the number of primary connections, and  $k_i^{(p)}$  is the transmission efficiency of primary users. Thus, the profit function of primary user *i* can be expressed as

$$P_{i}(\boldsymbol{p}) = b_{i}p_{i} + R_{i} - C_{i}(b_{i}).$$
(11)

In Bertrand price model, the players of the game are the primary users, the strategies of the players are prices of shared spectrum, i.e.  $\{p_i\}$ , and the utility function is the achieved profit, i.e.  $\{P_i\}$ . If the spectrum demand of cognitive user can be satisfied by the primary users, i.e. the available spectrum resource of the primary users are larger than the spectrum demand  $D_i(p)$  of the secondary user, the profit function of primary users *i* is given by [5]

$$P_{i}(\boldsymbol{p}) = p_{i}D_{i}(\boldsymbol{p}) + c_{1}M_{i} - c_{2}M_{i}\left(B_{i}^{req} - k_{i}^{(p)}\frac{W_{i} - D_{i}(\boldsymbol{p})}{M_{i}}\right)^{2}.$$
 (12)

However, in practical spectrum sharing model for cognitive radio networks, the spectrum resources of the primary users are usually restricted. The spectrum demand of the secondary users will not be definitely satisfied by the primary users. Thus, it needs to consider the constraints of the shared spectrum size.  $B_i^{max}$  is the maximum shared spectrum size of primary user *i*, and the competitive price game based spectrum sharing model with constraints of shared spectrum for cognitive radio networks can be formulated as (13), i.e. determine the price strategy  $p^*$  satisfies

$$p^{*} = \arg \max_{p} \left( \sum_{i=1}^{N} P_{i}(p) \right)$$
  
s.t.  $D_{i}(p) > 0, i = 1, 2, ..., N$   
 $D_{i}(p) < B_{i}^{\max}, i = 1, 2, ..., N$  (13)

where  $p^*$  is also defined as the Nash equilibrium price of the competitive price game. In order to achieve the  $p^*$  that satisfies (13), the relaxation variables  $y_{1i}^2$ ,  $y_{2i}^2$  are applied. There must exist specific variable  $y_{ji}^2$  equivalent with  $D_i(p)$ . Thus the improved profit function can be obtained

$$\phi(\mathbf{p}) = \sum_{i=1}^{N} P_i(\mathbf{p}) - \frac{\sigma}{2} \sum_{i=1}^{N} \left( D_i(\mathbf{p}) - y_{l_i}^2 \right)^2 + \sum_{i=1}^{N} \omega_{l_i} \left( D_i(\mathbf{p}) - y_{l_i}^2 \right) \\ - \frac{\sigma}{2} \sum_{i=1}^{N} \left( B_1^{\max} - D_i(\mathbf{p}) - y_{2_i}^2 \right)^2 + \sum_{i=1}^{N} \omega_{2_i} \left( B_1^{\max} - D_i(\mathbf{p}) - y_{2_i}^2 \right)$$
(14)

where  $\sigma$ ,  $\omega_{1i}$ ,  $\omega_{2i}$  are constant variables, with further simplification of (14), we can obtain

$$\phi(\mathbf{p}) = \sum_{i=1}^{N} P_i(\mathbf{p}) - \frac{\sigma}{2} \sum_{i=1}^{N} \left( \left( D_i(\mathbf{p}) - y_{1i}^2 - \frac{\omega_{1i}}{\sigma} \right)^2 - \frac{\omega_{2i}^2}{\sigma^2} \right) \quad (15)$$
$$- \frac{\sigma}{2} \sum_{i=1}^{N} \left( \left( B_1^{\max} - D_i(\mathbf{p}) - y_{2i}^2 - \frac{\omega_{2i}}{\sigma} \right)^2 - \frac{\omega_{2i}^2}{\sigma^2} \right)$$

The value of  $y_{1i}^2$ ,  $y_{2i}^2$  is determined with the objective of maximizing the improved profit function, which can be given by

$$y_{1i}^2 = \max\left(0, D_i(\boldsymbol{p}) - \frac{\omega_{1i}}{\sigma}\right), y_{2i}^2 = \max\left(0, B_i^{\max} - D_i(\boldsymbol{p}) - \frac{\omega_{2i}}{\sigma}\right).$$
(16)

Substitute (16) into (15), we can get

$$\phi(\boldsymbol{p}) = \sum_{i=1}^{N} P_i(\boldsymbol{p}) - \frac{\sigma}{2} \sum_{i=1}^{N} \left( \max\left(0, \frac{\omega_{1i}}{\sigma} - D_i(\boldsymbol{p})\right)^2 - \frac{\omega_{1i}^2}{\sigma^2} \right) (17) - \frac{\sigma}{2} \sum_{i=1}^{N} \left( \max\left(0, \frac{\omega_{2i}}{\sigma} - B_i^{\max} + D_i(\boldsymbol{p})\right)^2 - \frac{\omega_{2i}^2}{\sigma^2} \right)$$

The above function can be seen as a stage function, and at the stage points, the left limit equals the right limit, thus function (17) is derivative. In order to achieve the equilibrium price strategy  $p^*$ , we can have

$$\frac{\partial \phi(\mathbf{p})}{\partial p_i} = 0 \quad i = 1, \dots, N \tag{18}$$

where

$$\frac{\partial \phi(\boldsymbol{p})}{\partial p_i} = \sum_{i=1}^{N} \frac{\partial P_i(\boldsymbol{p})}{\partial p_i} + \frac{\sigma}{2} \sum_{i=1}^{N} \max\left(0, \frac{\omega_{1i}}{\sigma} - D_i(\boldsymbol{p})\right) \frac{\partial D_i(\boldsymbol{p})}{\partial p_i} \\ - \frac{\sigma}{2} \sum_{i=1}^{N} \max\left(0, \frac{\omega_{1i}}{\sigma} + D_i(\boldsymbol{p}) - B_i^{\max}\right) \frac{\partial D_i(\boldsymbol{p})}{\partial p_i}$$

# 4. Iterative Calculation of the Shared Spectrum Price

In practical cognitive radio networks, the price strategies of other primary users cannot be achieved simultaneously, the Nash equilibrium price strategy of the competitive price model cannot be achieved through the linear equations formed by (18). Thus, it is needed to further discuss the iterative price calculation method without acknowledgement of price strategies from other primary users simultaneously [5].

Assume the price strategies of other primary users cannot be achieved simultaneously, but the shared spectrum price strategies of primary users during last cycle is available. The Nash equilibrium price that maximizes the system profits of the primary users can be achieved iteratively.

#### 4.1 The Linear Iterative Method

The linear gradient descent algorithm is one of the effective tools to calculate the maximum and minimum value of continuous target function during the optimization problems. For the primary users, the profit function is convex with the variation of shared spectrum price. At the  $k^{\text{th}}$  iteration,  $p_i^k$  is the shared spectrum price of primary user *i*,  $P_i^k(\mathbf{p})$  is the profit of primary user *i*, and  $\varphi^k(\mathbf{p})$  is the

improved system profit function. If the available spectrum resource of the primary users is larger than the spectrum demand of the secondary user and  $\partial \varphi(\mathbf{p})/\partial p_i^k$  is positive, the value of the improved system profit function of primary users will increase iteratively with the increase of  $p_i^k$  until the maximum of the improved system profit function is obtained or the spectrum constraints of the primary users cannot be satisfied. When the available spectrum resource constraints can be satisfied and  $\partial \varphi(\mathbf{p}) / \partial p_i^k$  is negative, with the linear iteration as (19), the value of the improved system profit function will decrease until the equilibrium shared spectrum price is achieved. It is due to the cognitive user tends to demand less spectrum resource when the price of the shared spectrum is too high. Besides, when the spectrum demand of the secondary user can not be satisfied by the available spectrum resource of primary users, i.e. the cognitive user tends to apply for more spectrum resource than spectrum constraints, the value of the improved profit function will also decreases. Thus the equilibrium price of the shared spectrum that maximizes the improved system profit function and satisfies the constraints of available spectrum resource can be achieved iteratively.

$$p_i^{k+1} = p_i^k + a \frac{\partial \phi(\boldsymbol{p})}{\partial p_i^k} \tag{19}$$

where *a* is the nonnegative adjustment factor. With appropriate value of adjustment factor, the shared spectrum price that maximizes the improved system profit function can be achieved iteratively.

The value of adjustment factor is very important for the calculation of the equilibrium shared spectrum price. If the adjustment factor is too large, strong fluctuations will exist in the calculation of shared spectrum price, or even the equilibrium shared spectrum price can't be achieved. On the contrary, if the value of the adjustment factor is too small, the variation of calculated price changes slowly, and more iterations are also needed to achieve the equilibrium shared spectrum price. Thus the appropriate adjustment value is necessary for iterative method to achieve the equilibrium price of the competitive price game based spectrum sharing model.

#### 4.2 The Fast Gradient Iterative Method

Based on the above research, a fast gradient iterative method is proposed to obtain the equilibrium price of shared spectrum. It does not need the predefinition of the adjustment factor according to the transmission parameters of the cognitive radio networks, and it can achieve the equilibrium price of the shared spectrum with less iterations.

The steepest gradient descend algorithm, which is firstly proposed by the French mathematician Cauchy, is one of the important optimization methods. The iterations in the steepest descent direction of the target function are applied to achieve the minimum value of the target function. This paper applies principle of the steepest gradient descend algorithm, and uses the direction with the fast variation of the improved system profit function to calculate the equilibrium price of shared spectrum iteratively. The adjustment factor of the fast gradient method is adjusted to guarantee the shared spectrum price iterations in the gradient direction of the improved system profit function. The target function here is defined as

$$f(\mathbf{p}) = \arg\max\phi(\mathbf{p}). \tag{20}$$

The directional gradient of the target function  $d_i^k$  at the price strategy  $p_k$  can be defined as

$$d_{i}^{k} = \frac{\partial \phi(\boldsymbol{p})}{\partial p_{i}}$$

$$= \sum_{i=1}^{N} \frac{\partial P_{i}(\boldsymbol{p})}{\partial p_{i}} + \frac{\sigma}{2} \sum_{i=1}^{N} \max\left(0, \frac{\omega_{\text{L}i}}{\sigma} - D_{i}(\boldsymbol{p})\right) \frac{\partial D_{i}(\boldsymbol{p})}{\partial p_{i}} \quad (21)$$

$$- \frac{\sigma}{2} \sum_{i=1}^{N} \max\left(0, \frac{\omega_{\text{L}i}}{\sigma} + D_{i}(\boldsymbol{p}) - B_{i}^{\max}\right) \frac{\partial D_{i}(\boldsymbol{p})}{\partial p_{i}}$$

With one-dimensional investigation along the direction  $d^k$ , and the value of the adjustment factor  $\lambda$  is updated to guarantee the direction with fast variation.

$$\lambda^{k} = \max_{i} \phi(\boldsymbol{p}^{k} + \lambda \boldsymbol{d}^{k}). \qquad (22)$$

Thus

$$\sum_{i=1}^{N} \frac{\partial P_i(\boldsymbol{p} + \lambda \boldsymbol{d})}{\partial \lambda} + \frac{\sigma}{2} \sum_{i=1}^{N} \max\left(0, \frac{\omega_{1i}}{\sigma} - D_i(\boldsymbol{p} + \lambda \boldsymbol{d})\right) \frac{\partial D_i(\boldsymbol{p} + \lambda \boldsymbol{d})}{\partial \lambda}$$
$$- \frac{\sigma}{2} \sum_{i=1}^{N} \max\left(0, \frac{\omega_{1i}}{\sigma} + D_i(\boldsymbol{p} + \lambda \boldsymbol{d}) - B_i^{\max}\right) \frac{\partial D_i(\boldsymbol{p} + \lambda \boldsymbol{d})}{\partial \lambda} = 0$$
(23)

where

$$\frac{\partial P_i(\boldsymbol{p} + \lambda \boldsymbol{d})}{\partial \lambda} = d_i D_i(\boldsymbol{p} + \lambda \boldsymbol{d}) + \left(p_i + \lambda d_i\right) \frac{\partial D_i(\boldsymbol{p} + \lambda \boldsymbol{d})}{\partial \lambda}$$
$$- 2c_2 k_i^{(p)} \left(B_i^{req} - k_i^{(p)} \frac{W_i - D_i(\boldsymbol{p} + \lambda \boldsymbol{d})}{M_i}\right) \frac{\partial D_i(\boldsymbol{p} + \lambda \boldsymbol{d})}{\partial \lambda}$$
$$\frac{\partial D_i(\boldsymbol{p} + \lambda \boldsymbol{d})}{\partial \lambda} = \frac{v \sum_{j \neq i} d_j - (v(N-2)+1)d_i}{(1+v)((N-1)v+1)}$$

The price strategy of shared spectrum is calculated according to (24), until the variation of the price strategy can satisfy the precision requirement.

$$p_i^{k+1} = p_i^k + \lambda^k d_i^k \,. \tag{24}$$

# 5. Simulation Results

We consider the cognitive radio networks with two primary users and one secondary service, the total spectrum bandwidth of each primary user is 20 MHz, and the primary connections are set as  $M_1 = 10$  and  $M_2 = 14$  respectively. The required communication bandwidth of primary users  $B_i^{\text{req}}$  is 2 MHz. The weights of the revenue and cost function  $c_1 = c_2 = 2$ . The target BER performance is  $10^{-4}$ , and the SNR is 20 dB, the spectrum substitutability factor v = 0.4. The initial value of the shared spectrum price is set to be 0 in the iterative method to calculate the Nash equilibrium price.

Fig. 1 shows the system profit of primary users with variable shared spectrum price when sufficient spectrum resource can be provided by primary users. It can be concluded from Fig. 1 that the system profit is convex with the variation of shared spectrum price, the maximum system profit can be achieved at certain price strategy.

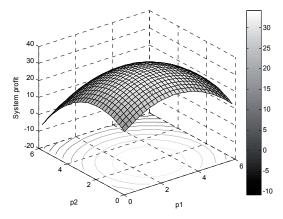


Fig. 1. The total revenue of primary users with variation of shared spectrum price.

Fig. 2 shows the simulation results of shared spectrum size under different spectrum constraints of primary users, among which the available spectrum constraint of primary users 2  $B_2^{\text{max}}$  is fixed as 1 MHz and the available spectrum constraints of primary users  $1 B_1^{\text{max}}$  is set to be varied from restricted to sufficient respectively. It can be concluded from Fig. 2 that, when the available spectrum resource from primary user 1 restricted, the secondary user tends to demand the remaining shared spectrum from primary user 2. With the increase of  $B_1^{\text{max}}$ , the shared spectrum size from primary user 1 is increased, and the shared spectrum size from primary user 2 is decreased accordingly. When the available spectrum of primary user 1 is larger than the demanded spectrum size of secondary user, the shared spectrum size from primary user 1 will not increase with the increase of  $B_1^{\text{max}}$ , and the secondary user tends to demand the reasonable spectrum size that maximizes its own utility function.

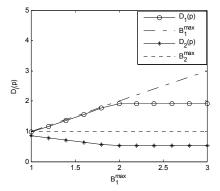
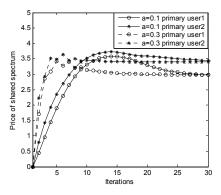
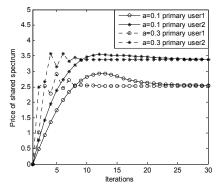


Fig. 2. The calculation of Nash equilibrium price by linear equations.

Fig. 3 shows the calculation of the Nash equilibrium price by the linear iterative method under different adjustment factors. It can be concluded from Fig. 3 that the value of the adjustment factor has significant impact on the iterative calculation of the Nash equilibrium price. When the adjustment factor is small, the price of shared spectrum varies slightly, and the convergence speed in calculation of the shared spectrum price is slow; when the adjustment factor is large, although the price of the shared spectrum varies significantly, it may also affect the convergence speed of the linear gradient method due to strong fluctuations.



(a) with restricted primary resource,  $B_1^{\text{max}}=1$  and  $B_2^{\text{max}}=1$ 



(b) with sufficient primary resource,  $B_1^{\text{max}}=2$  and  $B_2^{\text{max}}=1$ 

Fig. 3. The iterative calculation of Nash equilibrium price under different adjustment factor.

Fig. 4 shows simulation result of linear iterative method with inadequate adjustment factor. It can be concluded from Fig.4 that inadequate adjustment factor may lead to unstable shared spectrum price strategy, and the value of adjustment factor has significant impact on the convergence of the linear iterative method.

Fig. 5 shows the simulation results of equilibrium price by fast gradient iterative method proposed in this paper, it is consistent with the achieved shared spectrum price in Fig. 3. The fast gradient method also does not need the price strategies of other primary users simultaneously. It can adjust the price strategies in the direction with the fastest variations of the system profit function, and it can achieve the equilibrium price strategy that satisfies the available spectrum resource constraints from the primary users.

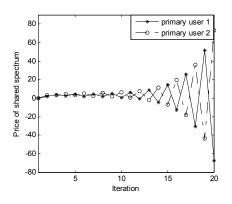
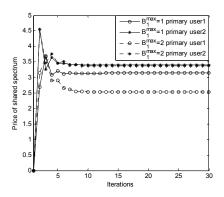


Fig. 4. The calculation of Nash equilibrium price with inadequate adjustment factor.



**Fig. 5.** The calculation of Nash equilibrium price through fast gradient iterative scheme.

Fig. 6 shows the simulation results of the convergence performance of the fast gradient method and the conventional linear gradient method. It can be concluded from Fig. 6 that the fast gradient iterative method has better convergence speed in the calculation of the equilibrium price, and it can also avoid the predefinition of the adjustment factor according to the communication parameters in the cognitive radio networks.

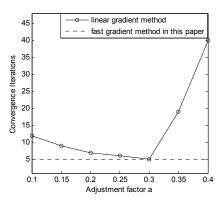


Fig. 6. The convergence speed of linear iterative scheme and fast gradient iterative scheme.

# 6. Conclusions

The competitive price game model is applied to analyze the spectrum sharing problem in cognitive radio networks. The primary users can obtain economic revenue by sharing portion of their spare spectrum, and the primary users provide price strategies of the shared spectrum competitively, and the secondary users determine the demanded spectrum size according to the price of shared spectrum and its own communication parameters. The primary users tend to select the shared spectrum price that maximizes their system profit. The spectrum sharing problem with constraints based on competitive price game model is further analyzed in this paper. The improved system profit function is formed by the Rockafeller multiplier algorithm, and the shared spectrum price strategy that maximizes the system profit and satisfies the constraints of the available spectrum resource from the primary users can be achieved. A fast gradient algorithm based iterative method is proposed to calculate the shared spectrum price. It can achieve the shared spectrum price that satisfies the available spectrum constraints without the dependence on the price strategies from other primary users simultaneously, and has better convergence performance.

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