Improved Design and Implementation of Variational Bayesian Iterative Receivers

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Abstract. *It was recently shown that detection performance can be significantly improved if the statistics of channel estimation errors are available and properly used at the receiver. Although deriving the statistics of channel estimation errors is rather straightforward for pilot-only channel estimation methods, it is not the case for semi-blind receivers such as variational Bayesian (VB) receivers. We have shown in a recent contribution that by a modified formulation of the VB formalism, one can reduce the impact of channel uncertainty on the decoder performance. In this paper, we propose different practical VB receiver implementation techniques that lead to further performance improvement. The adequacy of the proposed receiver design compared to classically-used VB receivers is demonstrated by simulations for orthogonal frequency-division multiplexing (OFDM) systems.*

Keywords

VBEM algorithm, joint iterative data detection and channel estimation, channel estimation errors, OFDM.

1. Introduction

OFDM is adopted in many standards since it offers a simple way for achieving high data-rate wireless transmission over frequency-selective fading channels. It is well known that reliable coherent data detection is not possible unless an accurate estimate of the channel is available at the receiver. If the channel changes slowly, pilot symbol assisted modulation (PSAM) [1] has been shown to be an effective solution for obtaining channel state information (CSI) at the receiver. However, obtaining an accurate estimate in highly mobile environments only through the use of pilots, would require inserting multiple training symbols per frame, which can result in a significant reduction of the spectral efficiency.

Semi-blind (data-aided) channel estimation methods can enhance system performance by exploiting unknown data symbols in addition to few pilots in the channel estimation process [2]. Recently, variational Bayesian (VB) inference [3], which is closely related to mean-field methods in statistical physics, has been proposed as an effective method for tractable receiver design. The VB method is applied in [4] for joint data detection and channel tracking of an OFDM system. A similar contribution is provided in [5] for an OFDM system working under a time-varying multipath channel. In [6], variational inference is used for joint symbol detection and phase noise estimation in OFDM system. Regardless of the deployed technique (pilot-only or data-aided), channel estimation is an *imperfect* process and the poor quality of channel estimates degrades the performance of decoding at the receiver.

In [7], assuming *pilot-only* channel estimation, Sadough *et. al.* showed that the channel estimation error distribution plays an important role in improving the receiver's performance. The general framework initially proposed in [7], was used in [7] to improve the decoding performance of bit interleaved coded modulation (BICM) and in [8] and [9] to derive an improved detector based on soft-parallel-interference cancellation for MIMO systems, for the case where the channel is estimated by means of pilot symbols and where the availability of the channel estimation error statistics was quite straightforward.

Recently, in [10] [11], the authors have addressed the case of iterative semi-blind receivers. More precisely, in [10] the authors have proposed a receiver robust to channel estimation errors for the case where the channel is estimated by the expectation-maximization (EM) algorithm. A similar contribution is provided in [11] for VB receivers where the estimation error statistics are supplied leading to a reduction of the impact of channel estimation inaccuracies at each receiver iterations.

In contrast to [10] and [11] where performance improvement was obtained through modifications in the formulation of the EM[1]/VB estimator leading to improved detection metrics, in this paper we follow a practical approach related to the implementation of VB iterative receivers. More precisely, we investigate some practical implementation issues for the improved receiver proposed in [11]. The approach presented in [11] uses the same architecture in all

[1]EM stands for expectation-maximization.

channel conditions. In this work, our aim consists in obtaining additional gains compared to [11] by combining pilot-only channel estimation with the semiblind VB estimator and then by adapting the receiver implementation to the channel dynamic. In brief, the main contributions of this paper can be summarized as follows.

- With the aim of improving detection accuracy, a new receiver architecture that combines pilot-only channel estimation with semiblind VB estimation is proposed,
- Three scenarios for receiver implementation that are appropriate for different channel conditions such as quasi-static, block fading or fast fading channels are proposed,
- The number of receiver iterations is optimized with the aim of minimizing the bit error rate (BER).

Notational conventions are as follows. $\mathbb{E}_{\mathbf{x}}[.]$ or $\langle . \rangle_{p(\mathbf{x})}$ refer to expectation with respect to the random vector $\mathbf{x}$, $\propto$ denotes equality up to a normalization factor, $\mathcal{CN}\big(\mathbf{m}, \mathbf{\Sigma}\big)$ denotes complex Gaussian vector distribution with mean $\mathbf{m}$ and covariance matrix $\mathbf{\Sigma}$; $|.|$ and $\|.\|$ denote absolute value and vector norm, respectively and finally $(.)^{\dagger}$ denote Hermitian transpose.

2. System and Channel Model

The considered transmitter architecture is depicted in Fig. 1. We consider a BICM with an OFDM system employing M subcarriers. We assume a Rayleigh distributed block-fading multipath channel model where each frame of size M_{frame} symbols corresponds to M_{block} independent fading blocks. Notice that in our model, $M_{\text{block}} = 1$ returns to the quasi-static channel model whereas $M_{\text{block}} = M_{\text{frame}}$ returns to the fast-fading channel model. Since the channel is assumed to be block-fading, for estimating the k-th complex channel frequency coefficient H_k, we receive $N = M_{\text{frame}}/M_{\text{block}}$ independent observations. At the receiver, after removing the cyclic prefix, the signal corresponding to k-th subcarrier in a given fading block writes

$$\mathbf{y}_k = H_k \mathbf{s}_k + \mathbf{z}_k \qquad \text{for } k = 1, \ldots, M \tag{1}$$

where the $(1 \times N)$ vector $\mathbf{y}_k = [y_{1,k}, \ldots, y_{N,k}]$, the entries of the noise vector $\mathbf{z}_k$ are assumed to be zero-mean circularly symmetric complex Gaussian (ZMCSCG) with distribution $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \sigma_z^2 \mathbf{I}_M)$, and the definition of $\mathbf{s}_k$ and $\mathbf{z}_k$ follow that of $\mathbf{y}_k$.

3. Pilot-based Channel Estimation

Consider the estimation of the k-th channel frequency coefficient H_k with N pilot symbols $\tilde{s}_i$ gathered in the row vector $\tilde{\mathbf{s}}_k = [\tilde{s}_0, \ldots, \tilde{s}_{N-1}]$, for $k = 1, ..., M$.

According to the observation model (1), during a given channel training interval, we receive

$$\tilde{\mathbf{y}}_k = H_k \tilde{\mathbf{s}}_k + \tilde{\mathbf{z}}_k \qquad \text{for } k = 1, ..., M \tag{2}$$

where the entries of the noise vector $\tilde{\mathbf{z}}_k$ have the same distribution as those of $\mathbf{z}$ in (1). Moreover, the definition of $\tilde{\mathbf{y}}_k$ and $\tilde{\mathbf{z}}_k$ follow that of $\tilde{\mathbf{s}}_k$.

The average power E_T of the k-th training vector $\tilde{\mathbf{s}}_k$ is

$$E_T \triangleq \frac{1}{N} \|\tilde{\mathbf{s}}_k\|^2. \tag{3}$$

Here, we assume equi-powered training vectors for all subcarriers. The least-squares estimate of H_k is obtained by minimizing $\|\tilde{\mathbf{y}}_k - H_k \tilde{\mathbf{s}}_k\|^2$ with respect to H_k which coincides here with the ML estimate. This yields

$$\begin{aligned} \widehat{H}_k^{\text{ML}} &= \tilde{\mathbf{y}}_k \tilde{\mathbf{s}}_k^{\dagger} \left(\tilde{\mathbf{s}}_k \tilde{\mathbf{s}}_k^{\dagger}\right)^{-1} \\ &= H_k + \varepsilon_k \qquad \text{for } k = 1, ..., M \end{aligned} \tag{4}$$

where $\varepsilon_k = \tilde{\mathbf{z}}_k \tilde{\mathbf{s}}_k^{\dagger} \left(\tilde{\mathbf{s}}_k \tilde{\mathbf{s}}_k^{\dagger}\right)^{-1}$ is the channel estimation error term. Thus, from (4), the conditional pdf of $\widehat{H}_k^{\text{ML}}$ given H_k reads

$$p(\widehat{H}_k^{\text{ML}} | H_k) = \mathcal{CN}\big(H_k, \Sigma_\varepsilon\big) \tag{5}$$

where

$$\Sigma_\varepsilon = \mathbb{E}\big[\varepsilon \varepsilon^{\dagger}\big] = \sigma_\varepsilon^2 \quad \text{ where } \sigma_\varepsilon^2 = \frac{\sigma_z^2}{N E_T} \cdot \tag{6}$$

Consider an uncorrelated i.i.d. Rayleigh channel with a *prior* distribution $H_k \sim \mathcal{CN}(0, \sigma_h^2)$. By using the latter pdf and the pdf of $(\widehat{H}_k^{\text{ML}} | H_k)$ from (5), we can derive the *posterior* distribution of the perfect channel, conditioned on its ML estimate as follows (see the appendix of [7] for details):

$$p(H_k | \widehat{H}_k^{\text{ML}}) = \mathcal{CN}(\delta \widehat{H}_k^{\text{ML}}, \, \delta \sigma_\varepsilon^2) \tag{7}$$

where

$$\delta = \frac{\sigma_h^2}{(\sigma_h^2 + \sigma_\varepsilon^2)}. \tag{8}$$

After simple and straightforward manipulations, we can write (7) in an equivalent form as

$$p(H_k | \widehat{H}_k^{\text{ML}}) = \mathcal{CN}(\mu, \beta) \tag{9}$$

where

$$\beta = \frac{\sigma_z^2 \sigma_h^2}{\sigma_z^2 + \sigma_h^2 \|\tilde{\mathbf{s}}_k\|^2}, \tag{10}$$

$$\mu = \beta \left[\frac{\tilde{\mathbf{y}}_k \tilde{\mathbf{s}}_k^{\dagger}}{\sigma_z^2} \right]. \tag{11}$$

The reason behind the above equivalent writing will be clarified in the next section. It is worth mentioning that the availability of the estimation error distribution constitutes an interesting feature of pilot assisted channel estimation that lets us derive the posterior distribution (9). This distribution will be exploited in the sequel for deriving an improved VB detector under imperfect channel estimation. For the sake of notational simplicity, we will not specify hereafter the subscript $_k$ in (1).

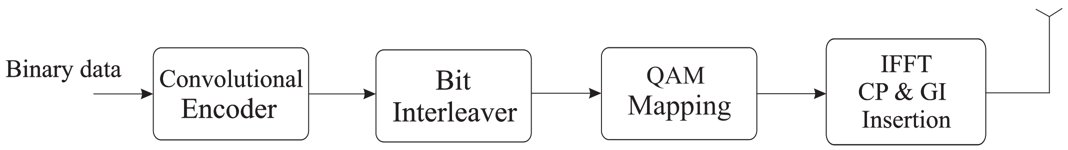

Fig. 1. Transmitter architecture for the considered BICM OFDM system.

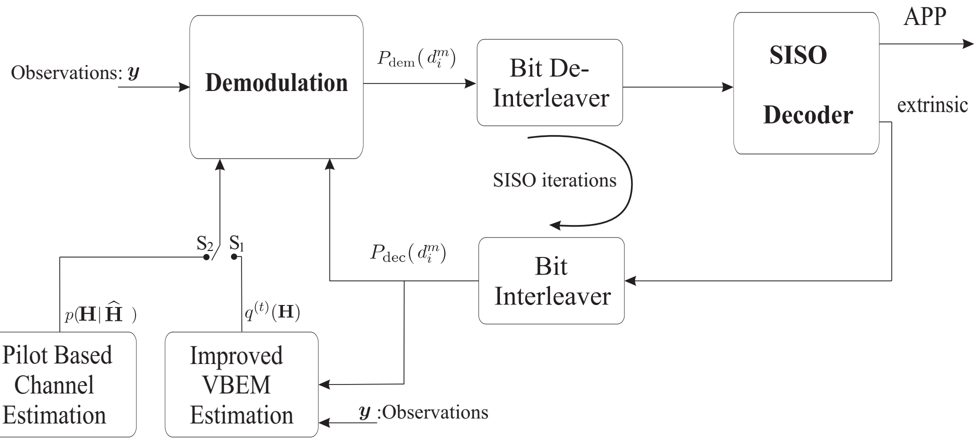

Fig. 2. Receiver architecture for the BICM OFDM system based on proposed improved VBEM algorithm.

4. Improved Iterative Receiver Design

4.1 Improved VB Receiver Formulation

Let us first explain the main part of our data-aided receiver which is based on VB inference. Starting from (1), the optimal estimate of the symbol vector **s** in the maximum *a posteriori* (MAP) sense is given by

$$\hat{\mathbf{s}}^{\mathrm{MAP}} = \arg\max_{\mathbf{s}} \; p(\mathbf{s}|\mathbf{y}). \tag{12}$$

The objective function in (12) can be written as

$$p(\mathbf{s}|\mathbf{y}) = \int p(\mathbf{s},H|\mathbf{y})\,dH = \int p(\mathbf{s}|H,\mathbf{y})\,p(H|\mathbf{y})\,dH \tag{13}$$

where H is regarded as a nuisance parameter. This integration is likely to be intractable since it involves integrals over complicated expressions. Note that in pilot-only channel estimation where the channel is estimated prior to data detection and is equal to $\widehat{H}$, one can assume that $p(H|\mathbf{y}) = \delta(H-\widehat{H})$ leading to $p(\mathbf{s}|\mathbf{y}) = p(\mathbf{s}|\mathbf{y},\widehat{H})$. Here, we assume that the channel is not known prior to data detection and thus the optimal solution is infeasible to obtain. The central idea of VB approximation [3] is to approximate the exact but intractable joint distribution into a product of marginal probabilities. Referring to our model (1), the VB method tries to find a distribution denoted by $q(\mathbf{s},H) = q(\mathbf{s})q(H)$ which approximates the exact posterior $p(\mathbf{s},H|\mathbf{y})$.

In [11], we derived a modified formulation and thus we derived modified receiver metrics as provided below

$$\mathrm{VBE\ (VBE-step):}$$

$$q^{(t)}(s_i) \propto \frac{p(s_i)}{\pi\left(\sigma_z^2 + \beta^{(t-1)}\,|s_i|^2\right)} \exp\left\{-\frac{\left|y_i - \mu^{(t-1)} s_i\right|^2}{\sigma_z^2 + \beta^{(t-1)}\,|s_i|^2}\right\}, \tag{14}$$

$$\mathrm{VBM\ (VBM-Step):}$$

$$q^{(t)}(H) = \mathcal{CN}\left(\mu^{(t)}, \beta^{(t)}\right) \tag{15}$$

where

$$\beta^{(t)} = \frac{\sigma_z^2\,\beta^{(t-1)}}{\sigma_z^2 + \beta^{(t-1)}\,\langle\|\mathbf{s}\|^2\rangle_{q^{(t)}(\mathbf{s})}}, \tag{16}$$

$$\mu^{(t)} = \beta^{(t)}\left[\frac{\mathbf{y}\langle\mathbf{s}^\dagger\rangle_{q^{(t)}(\mathbf{s})}}{\sigma_z^2} + \frac{\mu^{(t-1)}}{\beta^{(t-1)}}\right]. \tag{17}$$

For convenience, we define the parameter $H^{(t-1)} \triangleq \mu^{(t-1)}$, provided at the $(t-1)$-th iteration of the VBEM algorithm. Obviously, the above iterative VBEM algorithm requires initialization. We assume that the initialization is performed by a pilot-based channel estimation block which can provide the initial distribution $p(H|\widehat{H})$ from (9); i.e., $q^{(0)}(H) = p(H|\widehat{H})$ (see Fig. 2).

4.2 Proposed Receiver Architecture

The block diagram of the proposed iterative BICM receiver is depicted in Fig. 2. Our aim is to explain the connection between the iterative VBEM algorithm and the BICM iterative receiver. At the receiver, we perform MAP symbol detection and channel decoding in an iterative manner as proposed for instance in [13]. As shown in Fig. 2, the receiver consists in the combination of two main sub-blocks that exchange soft information with each other. The first sub-block, referred to as soft demodulator (also called demapper), produces soft information in the form of extrinsic probabilities from the input symbols and sends it to the second sub-block which is a soft-input soft-output (SISO) decoder. Each sub-block can take advantage of the quantities provided by the other sub-block as an *a priori* information. Here, SISO decoding is performed using the well known forward-backward algorithm [14]. For an in depth mathematical formulation regarding the functionality of the improved VBEM estimator block and the soft demodulator block, the reader is urged to see [7] and [11].

As shown in Fig. 2, a pilot-based channel estimator is used jointly with an elaborated VBEM estimator inside the receiver architecture. Note that this structure is the receiver implementation structure proposed in this paper, since classical receiver implementation methods suggest to use solely either a pilot-only channel estimator as in [7] or a VBEM estimator as in [11]. We will see in subsequent sections that this proposed architecture leads to an improvement of the system performance compared to classical VBEM receiver implementations. We will also see that the manner the channel estimator switches between the aforementioned two methods constitutes different scenarios that will be adapted to channel conditions.

4.3 Proposed Receiver Implementation Issues

As explained above, in the recent contribution [11], we have proposed an improved receiver based on VB inference that is characterized by equations (14) and (15). The interactions between the VBEM algorithm and the SISO decoder at the receiver was explained in the previous Subsection. In what follows, we aim at investigating the impact of practical receiver implementation techniques. More precisely, since both VBEM and SISO decoding are iterative algorithms, different implementation scheme can be employed and each implementation scenario affects differently the system performance, as explained in what follows (see also [15]). Let us denote by Q and P the number of iterations used for VBEM and SISO decoding, respectively. The simplest implementation scenario consists in performing one pass of VBEM algorithm ($Q = 1$) in each pass of SISO decoding (this scenario is denoted Scenario I). The second implementation scenario consists in performing Q iterations ($Q > 1$) for the VBEM algorithm inside each iteration of the SISO decoder (this scenario is denoted Scenario II). In Fig.2, we can see a switch which selects the VBEM estimator or the pilot-based technique for providing the demodulator with the information about the channel. By slightly modifying Scenario II, we propose a third implementation scheme (denoted Scenario III) which selects the pilot-based estimation method during a number of R SISO iterations ($R \leq P$) before switching to the VBEM estimator for following the strategy of Scenario II. The motivation behind this latter scenario is that, usually, during initial iterations of the VBEM algorithm, the quantities injected to the SISO decoder leads to unreliable probabilities at the output of the SISO decoder. For this reason, the receiver uses a conventional and simple pilot-based method during some SISO iterations (that we have to choose) before switching to the more elaborated VBEM estimator. Note that setting $R = 1$, in Scenario III it equivalent to Scenario II. Numerical results provided in the sequel aim at comparing the efficiency of each scenario and finding the necessary number of iterations.

5. Simulation Results and Discussion

Here, we provide simulation results to compare the performance provided by the proposed improved VBEM detector with conventional VB detection implemented through different scenarios explained above. We consider BICM combined with OFDM with the transmitter architecture depicted in Fig. 1, where different parameters used throughout simulations are as follows. One OFDM symbol is composed of $M = 40$ subcarriers. For channel coding, we consider the rate $1/3$ recursive systematic convolutional (RSC) code of constraint length 3 defined in octal form by $[5,7,7]_8$. The interleaver is pseudo-random and operates over the entire frame that contains a total number of 16 OFDM symbols. Data symbols belong to 16-QAM constellation with set-partition (SP) labeling. Performance evaluation is performed over the block-fading channel with parameters $M_{\text{block}} = 5$ with $N = 3$ and $M_{\text{block}} = 1$ with $N = 15$. Channel coefficients corresponding to different OFDM subcarriers are assumed uncorrelated and distributed according to the Rayleigh distribution. One OFDM pilot symbol is dedicated for initializing the channel in each fading block. Moreover, we perform a total number of 8 SISO decoding iterations (i.e., $P = 8$).

Remember that the receiver architecture is depicted in Fig. 2. Let us first analyze in Fig. 3, the BER performance obtained with conventional and improved VBEM receivers, implemented according to the implementation Scenario II. This scenario is adopted in our initial contribution [11]. Although we observe from Fig. 3 the superiority of the improved VBEM receiver implemented by Scenario II, here in what follows, we aim at finding the appropriate way for getting additional gains for the improved VBEM receiver, compared to those provided in Fig. 3.

We now aim at finding the optimal value for parameter R, i.e., the number of times the switch in Fig. 2 remains in the S_2 position. To this end, we have depicted in Fig. 4 the BER versus the parameter R for the improved VB receiver for a block-fading channel with parameter $N = 3$. We can see that the parameter R leading to the lowest BER is equal to 5. A similar plot is provided in Fig. 5, for the case where the block-fading parameter is equal to $N = 15$. We observe that in this channel condition, the optimal value for R is equal to 1, i.e., in this case it is of advantage to implement the receiver according to Scenario II. Consequently, in the following simulation results, we set $R = 5$ and $R = 1$ for Scenario III when $N = 3$ and $N = 15$, respectively.

Figure 6 compares the BER performance versus E_b/N_0 in dB obtained by using the improved VBEM algorithm for the case of a block-fading channel with parameter $N = 3$ for Scenarios I, II and III of improved VBEM algorithm. It can be seen from Fig. 6 that Scenario III (with the optimal settings derived above) provides the lowest BER performance. More precisely, we achieve a gain of about 0.75 dB at a BER of 10^{-4} with respect to Scenario I. Compared to Scenario II, we observe that the gain is even larger. Other similar plots are provided in Fig. 7 for the case where the block-fading parameter is equal to $N = 15$. In this case, we observe that at high E_b/N_0 values, different implementation scenarios perform very closely. This is due to the fact that in this case, a large number of observations ($N = 15$) are available at the receiver for estimating each subcarrier parameter and thus a fast convergence of the VBEM algorithm is achieved in all proposed implementation scenarios.

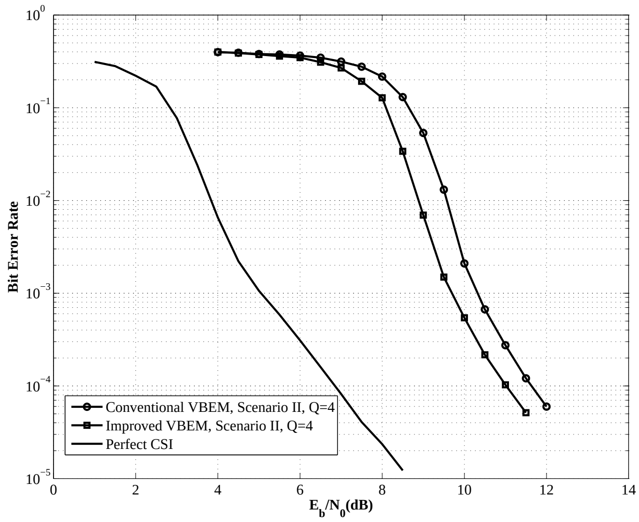

Fig. 3. BER performance of improved and conventional detector over the block-fading channel with $N = 3$. Receiver implementation is done according to Scenario II.

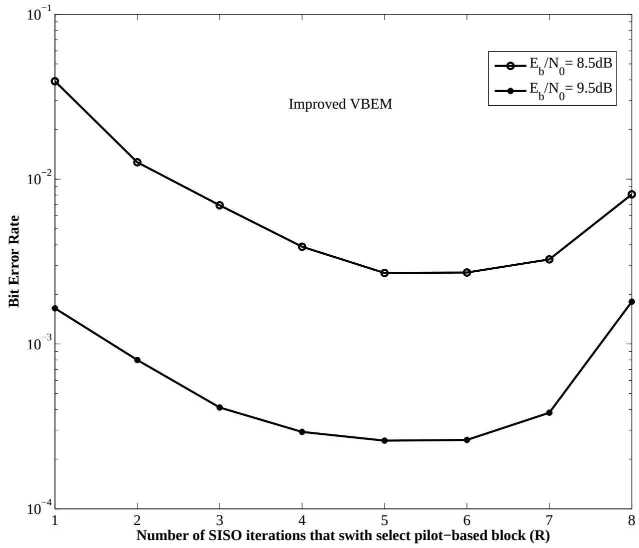

Fig. 4. BER versus parameter R for the improved VBEM algorithm implemented according to Scenario III over the block-fading channel with $N = 3$. The optimal value for R in this case is equal to 5.

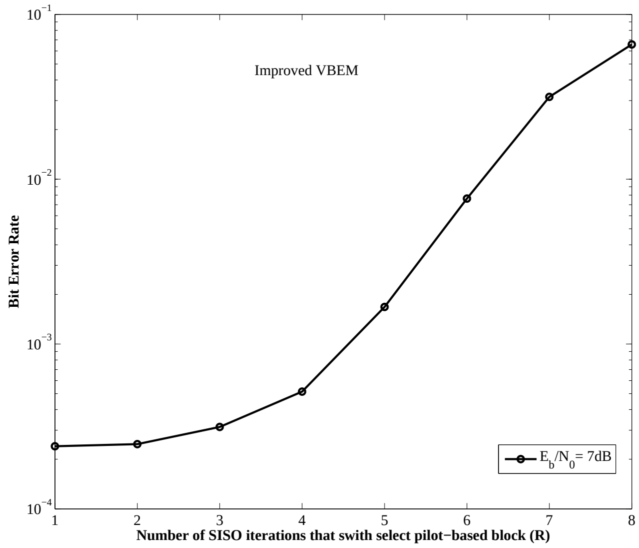

Fig. 5. BER versus parameter R for the improved VBEM algorithm implemented according to Scenario III over the block-fading channel with $N = 15$. The optimal value for R in this case is equal to 1, i.e., it is advantageous to use Scenario II.

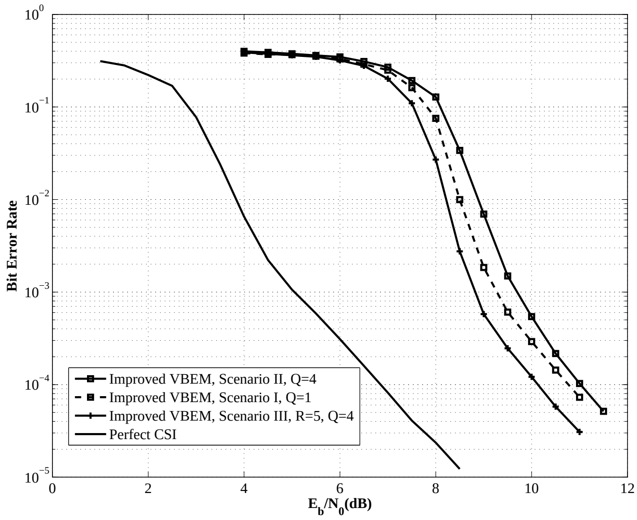

Fig. 6. BER performance obtained by using the improved VBEM receiver over the block-fading channel with $N = 3$, implemented according to Scenarios I, II and III.

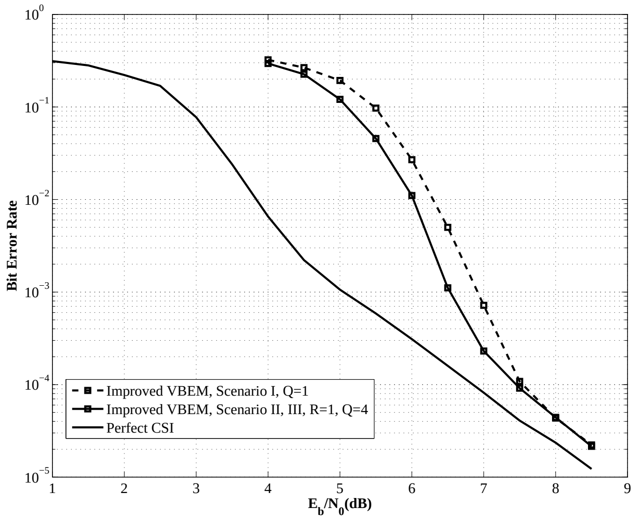

Fig. 7. BER performance obtained by using the improved VBEM receiver over the block-fading channel with $N = 15$, implemented according to Scenarios I, II and III.

6. Conclusion

We addressed the problem of receiver design based on VB inference for the practical case of imperfect channel estimation. Recently, an improved VB receiver has been proposed by the authors that provides increased robustness to channel estimation errors compared to conventional VB receivers. In this work, we completed our previous work and aimed at obtaining additional gains through an appropriate implementation of the receiver. Different scenarios for implementing practically the iterative blocks of VBEM algorithm and BICM detection were investigated. It was seen that the receiver implementation method plays an important role in improving the overall detection performance. We derived appropriate design parameters for our proposed implementation scenarios and saw that the value of these optimal parameters depends to the block-fading channel parameter N. Our numerical results provided for different block-fading channel conditions confirmed the adequacy of the proposed improved VBEM detector implementation in reducing the impact of channel estimation errors on BER performance. Although in this paper we considered the widely-used OFDM signal model, the proposed receiver design methodology holds for any transmission scenario.

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