Planar Resonators for Metamaterials

M. BLÁHA, J. MACHÁ
Č
Faculty of Electrical Engineering, Czech Technical University in Prague, Technická 2, 166 27 Prague 6, Czech Republic

mi.bla@seznam.cz, machac@fel.cvut.cz

Abstract. This paper presents the results of an investigation into a combination of electric and magnetic planar resonators in order to design the building element of a volumetric metamaterial showing simultaneously negative electric and magnetic polarizabilities under irradiation by an electromagnetic wave. Two combinations of particular planar resonators are taken into consideration. These planar resonators are an electric dipole, a split ring resonator and a double H-shape resonator. The response of the single resonant particle composed of a resonator with an electric response and a resonator with a magnetic response is strongly anisotropic. Proper spatial arrangement of these particles can make the response isotropic. This is obtained by proper placement of six planar resonators on the surface of a cube that now represents a metamaterial unit cell. The cells are distributed in space with 3D periodicity.

Keywords
Metamaterial, negative electric polarizability, negative magnetic polarizability, isotropic response, planar resonator.

1. Introduction
Volumetric metamaterials (MTM) with simultaneously negative permittivity and negative permeability, known as double negative (DNG) MTMs, are composed of unit cells. These cells have to be much smaller than the wavelength of the irradiating wave. The cell consists of a suitably shaped element or a set of these elements. The response of these elements is anisotropic. MTM with an isotropic response can be designed by suitable composition of the cells. These can be spatially arranged in a regular 3D periodic system with suitable spatial symmetry of the crystallographic groups [1]. A tetrahedral symmetrical system was used. The isotropy of the cell is proved by showing that its response does not depend on the direction and the polarization of the incident wave.

The resonant particles composing a volumetric metamaterials are generally measured in waveguides, and their equivalent characterization is determined from the complex transmission parameters [11]. Another way to make the measurement is to use an open parallel waveguide [12], [13]. More complex structure using split-post dielectric resonators was used for measuring the complex permittivity and the complex permeability of planar, uniaxially anisotropic resonant elements at microwave frequencies in [14]. This work is based on simple measurements of particles in an open parallel plate waveguide [13]; cubic cells with an isotropic response were measured in a standard R32 waveguide. The particle polarizabilities were calculated from the scattering parameters by the method proposed in [15].
2. Design of Planar Resonators

The aim of our work was to design resonant particles showing simultaneously negative electric and negative magnetic polarizabilities induced by an exciting electromagnetic wave, i.e., a DNG response. Resonators of planar structure are taken into consideration, as they are simple to manufactured using a PCB technology. The planar particle is investigated as being inserted into a waveguide according to Fig. 1.

The electric and magnetic dipole moments of a general bianisotropic element are induced by both electric field \( \mathbf{E} \) and magnetic field \( \mathbf{H} \), so there is

\[
p = \alpha^{ee} \mathbf{E} + \alpha^{em} \mathbf{H},
\]

(1)

\[
m = \alpha^{me} \mathbf{E} + \alpha^{mm} \mathbf{H},
\]

(2)

where \( \alpha^{rr} \) with \( r,s = m,e \), is the tensor of the respective polarizability. In the case of the investigated planar resonators located in a waveguide according to Fig. 1 and neglecting the cross-coupling terms, we have the scalar equation for the resonant element with a magnetic response

\[
m_x = \alpha_{xx}^{mm} H_y,
\]

(3)

and for the resonant element with an electric response

\[
p_y = \alpha_{yy}^{ee} E_x.
\]

(4)

The coordinate system is defined in Fig. 1. The electric \( \alpha_{xx}^{ee} \) and magnetic \( \alpha_{xx}^{mm} \) polarizabilities of the particle are determined from the calculated or measured scattering parameters, according to the procedure presented in [15]. The reduced electric polarizability \( \alpha_{yy}^{ee} = \alpha_{yy}^{ee} / \varepsilon_0 \) is plotted in the following plots (without notification) in order to be directly comparable with the magnetic polarizability value, \( \varepsilon_0 \) being vacuum permittivity. Using a simple homogenization procedure [16], applied for elements distributed periodically in space, the effective relative permittivity and permeability or, more precisely, the corresponding elements of tensors of these quantities, are

\[
\varepsilon_{yy}^{\text{eff}} = 1 + \frac{\alpha_{yy}^{ee}}{V \varepsilon_0 - \alpha_{yy}^{ee} / 3},
\]

(5)

\[
\mu_{xx}^{\text{eff}} = 1 + \frac{\alpha_{xx}^{mm}}{V \mu_0 - \alpha_{xx}^{mm} / 3},
\]

(6)

where \( \mu_0 \) is vacuum permeability. The values of these effective parameters depend on the cell volume \( V \), over which they are averaged, and therefore on the density of the particles distributed in space. There is no sense in using permittivity and permeability, which are statistical values, in the case when only one particle is being investigated. In the text, the electric and magnetic polarizabilities defined by (3) and (4) are used to define the single particle behavior.

The resonant particles were analyzed by the CST Microwave Studio when located in the center of an ideal TEM waveguide with perfect electric walls on top and bottom, and perfect magnetic walls on its sides, Fig. 1. The waveguide was 20 mm in width, 10 mm in height, and the distance of the reference planes was 35 mm and was filled with air. The scattering parameters result from this analysis. For the final analysis, the results were compared with the measurements, and the waveguide was left open from the sides.

The DNG planar resonant particle is composed of two particular planar resonators located on the two sides of the dielectric substrate. One resonator is of the electric type and produces negative electric polarizability, while the second resonator responds to the magnetic field and shows negative magnetic polarizability. The main problem in resonant particle design is due to mutual coupling of the two particular resonators located on the same substrate. This coupling detunes the final resonator and causes the existence of two separate resonances. The most natural version of the planar resonator was composed of an ED and an SRR producing negative electric, respectively, magnetic polarizability. Next, the ED was investigated and a resonator showing higher sensitivity to the electric field was searched. The responses of four electric field sensitive particles were investigated and compared: a planar electric dipole terminated by a loop inductor [17], a similar version of ED presented in [18], the particle presented in [19], together with the DHR presented in [10]. The DHR showed the highest sensitivity, offering an electric polarizability value up to \( \alpha_{yy}^{ee} = -10^{-6} \text{ m}^3 \).

The DNG resonant particle composed of the ED [17] and the SRR [1] was investigated as the first specimen of building blocks for DNG MTM. The layouts of these planar resonators are drawn in Fig. 2. A ceramic substrate...
with $\varepsilon_r = 20$ was used due to very low losses, as its loss factor is $10^{-4}$. This substrate has the shape of a disk 7 mm in radius and 2 mm in thickness.

The two building elements - SRR and ED - were first studied separately and tuned to the same resonant frequency 2.9 GHz when located on a disk substrate 0.4 mm in thickness. These two resonators were located at the TEM waveguide in parallel, back-to-back. Here the coupling is negligible when their mutual distance is greater than approx. 1 mm and the resonant frequency remains 2.9 GHz. Reducing the substrate distance, e.g., to 0.5 mm, we get magnetic and electric resonant frequencies of 2.84 GHz and 2.915 GHz, respectively. This arrangement of course does not meet the objective of the design, as we do not have a single solid particle. The planar resonators have to be located on a single substrate, see Figs. 2 c, d. In this case, the coupling is unfortunately stronger. To reduce the influence of this effect, the substrate thickness was increased to 2 mm in the final design.

The ED was designed with the following parameters: dimensions of dipole arms 2.95 mm in length, 0.9 mm in width, radius of inductive strip is 3.1 mm and its width is 0.2 mm, length of horizontal stubs is 2.6 mm. SRR had the following geometrical dimensions: outer radius 2.3 mm, strip width 0.5 mm, slot width 0.2 mm. Fig. 3 shows the scattering parameters of the TEM waveguide with the particle located in its center, calculated by the CST Microwave Studio.

![Fig. 3. Calculated transmission characteristics of the DNG resonant particle defined in the text.](image)

The real and imaginary parts of the corresponding electric and magnetic polarizabilities determined from the scattering parameters shown in Fig. 3 according to [15] are plotted in Fig. 4.

![Fig. 4. Electric and magnetic polarizabilities of the resonant particle calculated from the scattering parameters in Fig. 3.](image)

Based on the above discussion, the second version of the planar DNG resonant particle was composed of double H-shaped resonators [10]. The layout of this resonator is drawn in Fig. 5a. This resonator shows a number of resonances when suitably located in the space to be coupled both to an electric field and to a magnetic field [10]. The first resonance is electric. It responds to the electric field and negative electric polarizability can be induced. The second resonance is magnetic, i.e., the resonator responds to the magnetic field and negative magnetic polarizability can be induced. Consequently, the two DHRs can be used to combine the resonant particle that induces negative electric polarizability and negative magnetic polarizability at the same time. A suitable electric response is assured by a structure working at the first resonance, and a suitable magnetic response is induced by a structure working at the second resonance. To design a solid element, these two resonators are placed on the two surfaces of a common dielectric substrate. First simulations of DHRs were performed by the CST Microwave Studio in the ideal TEM waveguide with vertical limitation by PEC walls and horizontal limitations by PMC walls. However, this waveguide is not simple to fabricate. The open TEM parallel plate waveguide was therefore designed and fabricated with the dimensions presented in the following text [13].

![Fig. 5. Layout of the DHR [10] (a), electric current distribution at the first resonance (b), and at the second resonance (c), calculated by the CST Microwave Studio.](image)

The particle is located along the longitudinal axis of the waveguide, as shown in Fig. 1, in order to ensure that the electric field is parallel to the DHR central strip, and at the same time to ensure that the magnetic field is perpendicular to the surface of the substrate. A Duroid substrate with permittivity equal to 10 and thickness of 0.625 mm was used. The chip size was 8 mm x 8 mm. At resonant
frequency 3.1 GHz, the resonator is therefore 12 times smaller than the free space wavelength. The current distributions along the strips of DHR at both resonant frequencies are plotted in Fig. 5b and 5c. At the first resonance, the current is distributed along the strips as if along a folded lambda-half dipole, so the structure is sensitive to an electric field. At the second resonance, we have a current loop excited by a magnetic field, and the resonator behaves as an SRR.

The DHRs were first designed alone to offer the first and second resonances around 3 GHz, and were then located on a common substrate. Due to the mutual coupling, their response was now detuned, exhibiting two resonances. Using a manual optimization process performed in the CST Microwave Studio, the resonant particle was tuned to get a DNG response near to frequency 3.1 GHz. The next problem was that the electric resonance was now much narrower than the magnetic resonance, giving a DNG bandwidth of only a few tens of MHz when the same widths of the strips and slots of the two DHRs equal to 0.3 mm were used. This drawback was removed by widening the slots and strips of the resonator with an electric response to 1 mm. The final resonators are shown in Fig. 6a - resonator with an electric response, and in Fig. 6b - resonator with a magnetic response.

The calculated transmission characteristics of the finally tuned planar resonator located at the center of the TEM parallel plate waveguide are plotted in Fig. 7, and the real and imaginary parts of the electric and magnetic polarizabilities are plotted in Fig. 8. The polarizabilities were calculated from the data in Fig. 7 by the method presented in [15]. The resonator shows the DNG response in a frequency band of about 100 MHz.

### 3. Experiment

The behavior of the planar resonators located in the TEM waveguide was tested by measuring the scattering parameters. The waveguide cross section has to be as small as possible to get some reasonable and measurable response. A waveguide with a TEM wave was designed for this purpose [13]. The line is fed through standard coaxial-rectangular waveguide transitions. An R32 waveguide was used. The waveguide passes the propagating wave via the TEM taper to the parallel plate waveguide with transversal dimensions 20 times 10 mm. The central re-assembled part of the waveguide is used for calibration, and the measured particles are put in its center. The length of this waveguide part is 35 mm and corresponds to the distance of the reference planes. The fabricated TEM waveguide is shown in Fig. 9.
The designed resonant planar particles composed of the ED and the SRR were fabricated using a standard lithographic technique using silver layouts on disk-shaped E-20 ceramic substrates [20]. The parameters of this material are defined above. The planar resonators composed of the two DHRs located on the Duroid substrate defined above were fabricated by a standard technology for PCB. The rear and front sides of the resonators are shown in Fig. 6a, b.

The measured transmission characteristics of the selected particle composed of the ED and the SRR tuned to 2.9 GHz are plotted in Fig. 10. The real parts of the polarizabilities corresponding to the S-parameters shown in Fig. 10 are plotted in Fig. 11. This resonant particle shows a DNG response in the band between 2.87 and 2.95 GHz, i.e., in the band 80 MHz in width.

The measured scattering parameters of the selected fabricated planar resonator are plotted in Fig. 12. The electric and magnetic polarizabilities calculated from the measured S parameters shown in Fig. 12 by the method presented in [15] are plotted in Fig. 13. It is evident from this data that the planar resonant particle works as an element with a DNG response in the frequency interval from 3.075 to 3.33 GHz, i.e., in the band of 255 MHz. This frequency band is nearly the same for all fabricated resonators (90 specimens have been fabricated). However, due to the imperfect reproducibility of the fabrication process, the resonant frequencies are scattered from 2.955 to 3.163 GHz, and two separate resonances are shown in about one third of the fabricated particles, which consequently cannot be used in a DNG MTM.

The measured characteristics compared with the data calculated by the CST Microwave Studio do not show exactly equal values, but the character of all dependencies is similar.

The measured scattering parameters of the selected planar particle composed of the two DHRs are plotted in

Fig. 10. Measured transmission characteristics of the DNG resonant particle tuned to 2.9 GHz.

Fig. 11. Real parts of the electric and magnetic polarizabilities of the resonant particle showing DNG behavior.

Fig. 12. Measured scattering parameters of the selected fabricated planar resonator.

Fig. 13. Real and imaginary parts of the electric and magnetic polarizabilities calculated from the measured S parameters, plotted in Fig. 12.
4. Metamaterial Cell with an Isotropic Response

A cell of DNG metamaterial with an isotropic response was fabricated using six planar particles composed of the two DHRs, which are more sensitive to an electric field than particles composed of EDs and SRRs, compare Figs 11 and 13. The particles were selected to have equal polarizability values tuned to the same frequencies. The real parts of the electric polarizabilities of six selected particles are shown in Fig. 14. These six particles are assembled to form a cube, as described in [1], to obtain a metamaterial cell with an isotropic response.

![Fig. 14. Real parts of the electric polarizability of six particles selected to compose the DNG cell.](image)

The planar resonators are located on the surface of a cube made of polystyrene. To obtain an isotropic response, their positions are determined by a suitable crystallographic symmetry, as defined in [1]. Three cubes were built with face size equal to 10, 15, and 20 mm. Photographs of these cubic cells are presented in Fig. 15.

![Fig. 15. Cubic cells of DNG metamaterial, and their positions on the face, on the edge, and on the peak in the waveguide used to prove the isotropic response.](image)

The cubic cell was measured inserted in the center of the metallic waveguide of rectangular cross section R32. The distance of the reference planes was 37.2 mm. The cube was located in three main positions, defined in Fig. 15, on the face, on the edge, and on the peak, and their subsequent turns by 45 and 90 deg. So finally there are nine sets of measurements for each cube. The scattering parameters of the waveguide with the cubic cell with the face 20 mm in length in the defined basic positions are plotted in Fig. 16. There are only small differences between the values taken for particular cube positions. The same is shown in Fig. 17, which shows the real parts of the polarizabilities of the cube.

![Fig. 16. Measured S parameters of the cube at its basic positions.](image)

![Fig. 17. Real parts of the electric and magnetic polarizabilities calculated by [15] from the data in Fig. 16.](image)

It is evident from the data shown in Figs 16 and 17 that the unit cell in cubic form behaves as an MTM cell with an isotropic response, showing DNG behavior in a frequency band of about 70 MHz. The response does not depend on the direction of the irradiating wave propagation. The polarizabilities obtained from measurements of cubes 10 mm and 15 mm in dimensions are similar to those in Fig. 10, but the spread of their values is wider due to more intensive coupling between the planar particles, which are located closer to each other here.

5. Conclusions

This paper presents resonant particles with double negative behavior suitable for the design of volumetric metamaterials. These particles are composed from an SRR and from an electric dipole located on a single disk dielectric substrate, and from two double H-shaped resonators located on the two surfaces of a dielectric substrate. The particles were fabricated and the results of a numerical simulation were verified experimentally. The negative
electric polarizability is excited by the electric dipole and in the second case by the double H-shaped resonator working at its first resonance. The negative magnetic polarizability is due to the SRR and, in the case of the second resonator, by the double H-shaped resonator working at its second resonance. The resonant particles were fabricated, and the numerical simulation results were verified experimentally. The final simulation and the measurements were performed in the parallel plate waveguide with a TEM wave. The resonator composed of the ED and the SRR shows DNG behavior in the 80 MHz band around frequency 2.9 GHz. The frequency width of the band with a negative refractive index is about 255 MHz in the case of the planar resonator composed of DHRs. At resonant frequency 3.1 GHz, the resonator size is 12 times smaller than the corresponding free space wavelength.

The metamaterial showing an isotropic response was finally prepared by arranging the presented particles composed of double H-shaped resonators in space with suitable symmetry of the unit cell in the shape of a cube. These cells are immersed into the hosting material – air – in the 3D cubic periodic system. Double negative behavior appears in the 70 MHz band.

The experimental data verifies the simulation, the design of both planar resonators and the unit cell of the DNG metamaterial with an isotropic response to the irradiation wave.

Finally, we should draw attention to a significant effect that influences the design and measurements of resonant particles of the kind presented here. These particles are designed by simulations in the CST Microwave Studio. The particle has to be located in some waveguiding structure. The finite cross-sectional dimensions of this structure influence the resonant conditions of a particle. In fact, due to the mirroring effect caused by the waveguide symmetry, a single particle in the waveguide behaves as an infinite 2D row of particles with transversal periodicity defined by the waveguide dimensions. This results in some dependence of the particle design and the measurement on the waveguiding system that is used.

Acknowledgements

This work has been supported by the Grant Agency of the Czech Republic under project No. 102/09/0314, and by the Czech Technical University in Prague under project No. SGS10/271/OHK3/3T/13.

References


[19] PADILLA, W. J., ARONSSON, M. T., HIGSTRETE, C., LEE,


**About Authors** ...

**Michal BLÁHA** was born in Prague, Czech Republic, in 1981. He received his Ing. degree in Radio Engineering from the Czech Technical University in Prague, Czech Republic, in 2008. He is currently a PhD student at the Dept. of Electromagnetic Field, Czech Technical University in Prague. He works on metamaterials.

**Jan MACHÁČ** received his MSc and Dr.Sc. degrees from the Czech Technical University in Prague (CTU FEE), in 1977 and 1996, respectively, and his CSc. degree (PhD equivalent) from the Czechoslovak Academy of Sciences, Prague, in 1982. He was appointed a Professor at CTU FEE in 2009. He is the author or coauthor of more than 200 scientific papers in journals and conference proceedings. His main research interests are modeling waves on the planar transmission lines used in microwave and millimeter-wave techniques, leaky waves, leaky wave and planar antennas, metamaterials, propagation of electromagnetic waves in periodic structures, planar microwave filters, and ultrawideband fields.