Robust Resource Allocation for OFDM-based Cognitive Radio in the Presence of Primary User Emulation Attack

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Abstract. *Cognitive radio (CR) is a promising solution to improve the spectrum efficiency in which some unlicensed users are allowed to exploit frequency bands which are not used by licensed network. However, CR technology imposes some threats to the network. One of these threats is primary user emulation attack where some malicious users try to send fake signals similar to the primary user (PU) and prevent secondary users from accessing vacant bands. Moreover, the presence of a primary user emulation attacker (PUEA) leads to additional interference to the CR and consequently, the efficiency of conventional power loading algorithms will be degraded. In this paper, we propose a power allocation scheme in an orthogonal frequency-division multiplexing (OFDM) based CR in the presence of PUEA. Power allocation is performed with the aim of maximizing the downlink transmission capacity achieved by the cognitive user, while keeping the interference level at the PU below a predefined threshold. Simulation results confirm the efficiency of our proposed power loading scheme, compared to classical loading algorithms that do not consider the activity of malicious users in the radio environment.*

Keywords

Cognitive radio, primary user emulation attacker, spectrum sensing, orthogonal frequency-division multiplexing, power allocation.

1. Introduction

Cognitive radio (CR) has been introduced to the communication society as a promising technology to alleviate the spectrum shortage problem by enabling unlicensed users to co-exist with licensed or primary users (PU) [1]. CR systems are equipped with spectrum sensing to detect vacant bands [2], [3] and to adapt their transmitter parameters according to the environment [1]. After finding vacant bands in an opportunistic manner, CR devices transmit data in frequency bands where the PU is not transmitting [3].

Orthogonal frequency-division multiplexing (OFDM) is widely-used for CR transmission, due to its flexibility in dynamically allocating vacant spectrum resources. Since finding OFDM vacant subbands in CR is a dynamic process based on the information gathered from environment, the network is susceptible to some threats. In one of these threats, some *malicious* users send signals identical to that of primary transmitter, leading secondary users (SU)s to vacate the spectrum [5], [6], [7]. This threat is commonly referred to as primary user emulation attack which benefits from CR responsibility to vacate bands used by PU signaling. It is challenging for CR network to utilize available spectrum resources in an efficient manner [8]. However, CR and primary systems are usually operating in close and side-by-side frequency bands, and this causes mutual interference on each other, due to the non-orthogonality of transmitted signals. In addition, the presence of a primary user emulation attacker (PUEA) in the environment, not only increases the interference resources introduced to the network, but also leads to an improper power allocation in the CR network, due to sending fake signals and subsequently, the spectral efficiency of available resources will be degraded.

Although, there are lots of efforts to reduce CR and PU mutual interference, to the best of our knowledge, the disruptive effect of a PUEA on cognitive network resource allocation has not been considered in the literature so far. Conventional power loading algorithms proposed for OFDM-based CR networks [9], [10], assume a secure environment and allocate the power based on *naive* spectrum sensing. However, in the presence of a *malicious* user, an excessive interference is imposed to the CR transmission which directly affects power allocation process in the CR network. Moreover fake signals transmitted by PUEA make CR user not use available vacant subbands and then causes degradation in CR transmission rate.

Some methods are proposed in the literature to reduce the effect of PUEA fake signals in spectrum sensing. For instance, a location based scheme which performs received signal strength (RSS) measurements is proposed in [11]. A new RSS-based defense strategy against the primary user emulation attack in CR wireless networks, using belief propagation of location information is proposed in [12]. A cooperative spectrum sensing that considers PUEA is proposed in [6], in which the sensing information of different SUs is combined at a fusion center with the objective of maximizing the probability of available channels under the constraint of a target false alarm probability. In [7], the authors investigate countermeasures to combat PUEA by using estimation techniques and learning methods to obtain the key information of the environment.

Despite aforementioned solutions to combat fake signals, in practice, there are some inherent errors to distinguish between PU and PUEA signals. In this paper, considering these errors and extra interference introduced to the CR by fake PUEA's signals, we propose a robust resource allocation for OFDM-based CR network based on the calculated interference imposed to CR user by PUEA and PU. Resource allocation in our proposed method aims at maximizing CR data rate. We will show that for a predefined interference level prescribed by the PU and a predefined power constraint at the CR, resource allocation in the CR network user will be improved in such a way that the total transmission rate of the CR network under a *malicious* user is increased, compared to conventional power loading methods.

The rest of this paper is organized as follows. The system model is presented in Section 2. Section 3 provides a probabilistic framework for conventional spectrum sensing methods under PUEA. Section 4 characterizes the possible interferences imposed to our considered scenario. Section 5 presents the proposed power loading algorithm in the presence of PUEA. Simulation results and discussions about the performance of the proposed power loading algorithm are presented in Section 6 and finally, Section 7 concludes the paper.

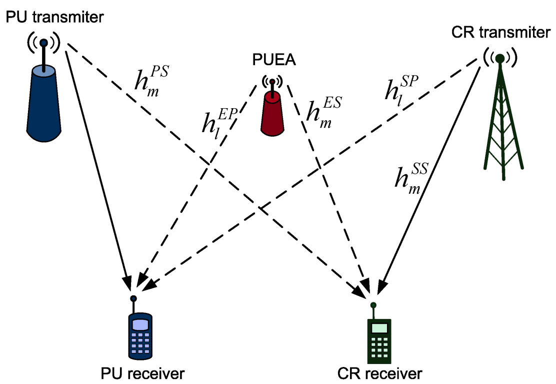

Fig. 1. System model of the considered network.

2. System Model and Main Assumptions

As shown in Fig. 1, we consider a downlink transmission scenario composed of a primary and a secondary transceiver. In our scenario, a *malicious* user (PUEA) is present in the environment that tries to send fake signals with characteristics similar to the primary signal with the aim of deceiving the CR user. We assume that CR employs an OFDM scheme with N subcarriers. As shown in Fig. 2, the total available bandwidth licensed to the PU is equal to B which is divided into N subcarriers, each of width δf. The CR user performs spectrum sensing and tries to detect vacant subbands with an opportunistic access to each of the primary subcarriers. We assume CR spectrum sensing can perfectly recognize vacant subbands and differentiate them from occupied subbands. Also, we assume that the CR network is aware of the presence of a PUEA in the radio environment and applies a given spectrum sensing method (for instance one of the methods discussed in Section 1) to distinguish between the PU and the PUEA signals in occupied subbands. However, in practice, there are errors in distinguishing between primary and attacker signals and hence signals in some subbands may be considered as PU/PUEA signal incorrectly, while they are actually received from PUEA/PU. Obviously, the occurrence of errors is inversely related to the efficiency of the applied spectrum sensing method used for separating the primary and attacker signal. So, CR network resource allocation in vacant subbands would be performed by improper knowledge of the PU activity in subbands.

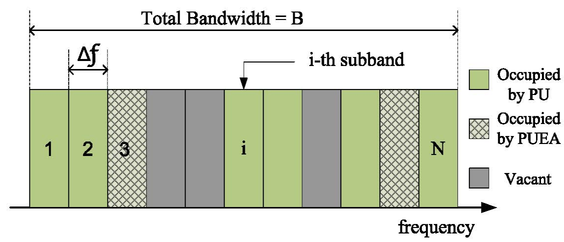

Fig. 2. Total available bandwidth.

Let h_l^{SS}, h_l^{SP}, h_l^{PS} be the fading channel gain between the SU transmitter and SU receiver, the SU transmitter and PU receiver, and the PU transmitter and SU receiver in the subcarrier l, $l \in \{1,...,N\}$, respectively. Also, we define h_l^{ES} and h_l^{EP} as fading channel gain between the PUEA and the SU receiver and the PUEA and the PU receiver, respectively. These instantaneous fading gains are assumed to follow a Rayleigh distribution and known to the CR network.

3. Spectrum Sensing in the Presence of PUEA

In this section, we derive a probabilistic framework which characterizes the accuracy and efficiency of the spectrum sensing process in the presence of the PUEA. This framework will be used in the interference analysis and proposed power allocation scheme in subsequent sections. As mentioned, frequency bandwidth licensed to the PU is equal to B, which we divide into N subbands. Subbands are indexed from 1 to N. All of these subbands are collected in a set referred to as L, where $L = \{1,...,N\}$.

We assume that spectrum sensing process is performed and vacant subbands are exactly recognized. Note that remaining subbands which are occupied, may be used by either the PU signal or the PUEA signal. Hence, after distinguishing between primary and attacker signals by one of the available methods proposed in the literature, the set L will be divided into three parts; the set of vacant subbands denoted by v, the set of subbands occupied by the PU denoted by O^p and the set of subbands occupied by the PUEA denoted by O^e. We can then write:

$$L = V \cup O^p \cup O^e. \tag{1}$$

Let us denote p_l to indicate that the PU is sending signals over subcarrier l, and e_l to indicate that the PUEA is sending fake signals over subcarrier l. As mentioned, there would be errors in assignment of the O^p and O^e sets in spectrum sensing, so, we define $\hat{p}_l$ and $\hat{e}_l$ as an estimate of p_l and e_l, estimated by CR spectrum sensing. Due to aforementioned errors, information about the presence of the PU signal in every subband is not available exactly, so the CR user may transmit its data through vacant subbands without perfect knowledge of imposed interference to the PU. In this regard, we introduce a new definition for probability of detection and probability of false alarm in spectrum sensing process due to the presence of a PUEA. We define the detection probability of PU signal and PUEA signal as:

$$P_d^p = \Pr(\hat{p}_l | p_l) \tag{2}$$

and

$$P_d^e = \Pr(\hat{e}_l | e_l), \tag{3}$$

respectively. In addition, we define the false alarm probability of PU and PUEA as:

$$P_f^p = \Pr(\hat{p}_l | e_l) \tag{4}$$

and

$$P_f^e = \Pr(\hat{e}_l | p_l), \tag{5}$$

respectively. We can now define the probability $\Pr(p_l|\hat{p}_l)$, which indicates the accuracy of applied spectrum sensing method for detecting the PU signal over subcarrier l, as:

$$\alpha_l^p \triangleq \Pr(p_l | \hat{p}_l) = \frac{P_d^p \Pr(p_l)}{\Pr(\hat{p}_l)} \tag{6}$$

where $\Pr(\hat{p}_l)$ is given by:

$$\begin{aligned}\Pr(\hat{p}_l) &= \Pr(\hat{p}_l|p_l)\Pr(p_l) + \Pr(\hat{p}_l|e_l)\Pr(e_l) \\ &= P_d^p \Pr(p_l) + P_f^p \Pr(e_l).\end{aligned} \tag{7}$$

In a similar way, we can define the probability $\Pr(e_l|\hat{e}_l)$, which indicates the accuracy of applied spectrum sensing method for detecting the PUEA signal over subcarrier l, as:

$$\alpha_l^e \triangleq \Pr(e_l | \hat{e}_l) = \frac{P_d^e \Pr(e_l)}{\Pr(\hat{e}_l)} \tag{8}$$

where $\Pr(\hat{e}_l)$ is given by

$$\begin{aligned}\Pr(\hat{e}_l) &= \Pr(\hat{e}_l|e_l)\Pr(e_l) + \Pr(\hat{e}_l|p_l)\Pr(p_l) \\ &= P_d^e \Pr(e_l) + P_f^e \Pr(p_l).\end{aligned} \tag{9}$$

In (7) and (9), $\Pr(p_l)$ and $\Pr(e_l)$ are probabilities of presence of the PU and the PUEA in the subcarrier l, respectively, which are assumed to be known to the CR network.

4. Interference Analysis in the Presence of the PUEA

Based on our system model, there are two main factors, imposing interference to the PU and the SU; **i)** out of band emissions and **ii)** the presence of fake signals in subbands, due to the presence of the PUEA. In what follows, we try to formulate interferences introduced to the PU and the CR in the presence of a PUEA and establish the main relations that let us take into account the errors, occurring in differentiating between PU and PUEA. Achieved results will be used to prevent the harmful interference from the CR to the PU with the aim of increasing data rates achieved by the CR network in the presence of a PUEA.

4.1 Interference Introduced by the PU Signal to the CR Signal

The power spectrum density (PSD) of the PU signal after M-fast Fourier transform (FFT), can be given by [13]:

$$E\{I_M^p(\omega)\} = \frac{1}{2\pi M}\int_{-\pi}^{\pi} \phi_p(e^{j\omega})\left(\frac{\sin[(\omega-\psi)M/2]}{\sin[(\omega-\psi)/2]}\right) d\psi \tag{10}$$

where $I_M^p(\omega)$ is periodogram of PU signal after M-FFT and $E\{.\}$ denotes expectation. ω represents the normalized frequency to the sampling frequency and $\phi_p(e^{j\omega})$ is the PSD of the PU signal.

The interference introduced by the PU subcarrier l to the CR subcarrier i, denoted J_{li}, can be expressed as [9]:

$$J_{li} = |h_l^{PS}|^2 \int_{d_{li}-\Delta f/2}^{d_{li}+\Delta f/2} E\{I_M^p(\omega)\}\, d\omega \tag{11}$$

where d_{li} is the frequency distance between subcarrier i and subcarrier l.

The average interference introduced by the set of PU subbands that are correctly detected as occupied through spectrum sensing, imposed to the CR subcarrier i, can be written as:

$$J_i^p = \sum_{l \in O^p} \alpha_l^p J_{li}, \quad i \in V. \tag{12}$$

Similarly, the average interference introduced by PU subcarriers which are incorrectly denoted as occupied by the PUEA, imposed to the CR subcarrier i, can be written as:

$$J_i^e = \sum_{l \in O^e} (1-\alpha_l^e) J_{li}, \quad i \in V. \tag{13}$$

So, the average interference introduced by PU subbands to the CR subcarrier i, is given by:

$$J_i = J_i^p + J_i^e, \quad i \in V. \tag{14}$$

4.2 Interference Introduced by the PUEA Signal to the CR Signal

As the PUEA tries to send signals with characteristics similar to that of the PU signal, the interference introduced by the PUEA subcarrier l to the CR subcarrier i, denoted K_{li}, can be written as:

$$K_{li} = |h_l^{ES}|^2 \int_{d_{li}-\Delta f/2}^{d_{li}+\Delta f/2} E\{I_N^e(w)\}\, dw$$

where $E\{I_N^e(w)\}$ is the PSD of the PUEA signal. Then, the average interference introduced by PUEA subbands which are correctly detected to be occupied by PUEA, on the CR subcarrier i, can be written as:

$$K_i^e = \sum_{l \in O^e} \alpha_l^e K_{li} \quad i \in V.$$

Similarly, the average interference introduced by the PUEA subcarrier l, which is incorrectly detected to be occupied by PU, on all CR subbands can be written as:

$$K_i^p = \sum_{l \in O^p} (1-\alpha_l^p) K_{li} \quad i \in V.$$

So, the average interference introduced by PU subbands to the CR subcarrier i is given by:

$$K_i = K_i^p + K_i^e \quad i \in V. \tag{15}$$

4.3 Interference Introduced by the CR Signal to the PU Signal

CR transmission is limited to work in a way that the interference imposed to primary signal remains below a predefined threshold. So, the CR network needs to calculate its imposed interference to the PU, constantly.

We assume that the CR transmission is performed by ideal Nyquist pulse and with the power of q_i in subcarrier i, so, the PSD of the transmitted signal in CR subcarrier i, can be written as [13]:

$$\phi_i(f) = q_i T_s \left(\frac{\sin \pi f T_s}{\pi f T_s}\right)^2$$

where T_s is the symbol duration.

Let I_{li} be the interference introduced by the CR subcarrier i to the PU subcarrier l, so we can write:

$$\begin{aligned} I_{li} &= |h_l^{SP}|^2 \int_{d_{li}-\Delta f/2}^{d_{li}+\Delta f/2} \phi_i(f)\, df \\ &= q_i T_s |h_l^{SP}|^2 \int_{d_{li}-\Delta f/2}^{d_{li}+\Delta f/2} \left(\frac{\sin \pi f T_s}{\pi f T_s}\right)^2 df \\ &= q_i \theta_{li} \end{aligned} \tag{16}$$

where $\theta_{il} = T_s |h_l^{SP}|^2 \int_{d_{li}-\Delta f/2}^{d_{li}+\Delta f/2} (\frac{\sin \pi f T_s}{\pi f T_s})^2 df$.

Now, the average interference introduced by CR to all PU subbands can be expressed as:

$$I = \sum_{i \in V} \sum_{l \in O^p} \alpha_l^p I_{li} + \sum_{i \in V} \sum_{l \in O^e} (1-\alpha_l^e) I_{li} \tag{17}$$

$$\triangleq \sum_{i \in V} q_i \bar{I}_i \tag{18}$$

where $\bar{I}_i \triangleq \sum_{l \in O^p} \alpha_l^p \theta_{li} + \sum_{l \in O^e} (1-\alpha_l^e)\theta_{li}$. Obviously, the CR network is allowed to transmit while $I < I_{th}$, where I_{th} is the total tolerable interference at the PU.

5. Proposed Power Allocation Scheme

In this section, we propose a power allocation method for each subcarrier of the OFDM-based CR with the aim of maximizing the achievable CR network rate in the presence of a PUEA, provided that the instantaneous interference introduced to the PU remains below a predefined threshold.

We define the signal to interference plus noise ratio (SINR) for CR subcarrier i as:

$$\gamma_i(q_i) \triangleq \frac{|h_i^{SS}|^2 q_i}{\sigma_i^2 + J_i + K_i} \tag{19}$$

where σ_i^2 denotes the variance of additive white Gaussian noise in CR subcarrier i. Considering an ideal coding scheme and using the Shannon capacity formula, the cognitive achievable data rate on subcarrier i is given by:

$$R_i(q_i) = \Delta f \log_2 [1 + \gamma_i(q_i)]. \tag{20}$$

Using (20), the optimization problem for CR power allocation in the presence of a PUEA can be expressed as:

$$\max_{q_i} R_i(q_i) \quad i \in V$$

subject to:

$$\begin{aligned} &\sum_{i \in V} q_i \bar{I}_i \le I_{th}, \\ &\sum_{i \in V} q_i \le Q, \\ &q_i \ge 0 \quad \forall i \in V \end{aligned} \tag{21}$$

where Q is the total maximum power constraint at the CR network. The optimization problem (21) is a standard convex problem that can be solved in a straightforward manner by using the Lagrange multiplier method [14]. We can easily show that:

$$q_i^* = \max\left\{0, \frac{1}{(\lambda+\mu\bar{I}_i)\ln 2} - \frac{\sigma_i^2 + J_i + K_i}{|h_i^{SS}|^2}\right\} \quad \forall i \in V \quad (22)$$

where $\lambda \geq 0$ and $\mu \geq 0$ are the Lagrange multipliers and q_i^* is the optimized power allocated to the CR subcarrier i, in the presence of the PUEA.

Proof: The proof is provided in the Appendix.

6. Simulation Results and Discussion

Throughout simulations, CR transmission is assumed to be performed by using ideal Nyquist pulse. The total available bandwidth (B) is assumed equal to 12 MHz. We assume that 24 OFDM subbands are available for cognitive transmission with a bandwidth equal to 0.5 MHz for each subcarrier. We also assume that spectrum sensing is performed and the sets V, O^e and O^p are defined as, V = [5, 6, 7, 8, 9, 16, 17, 18, 19, 22, 23, 24], O^p = [1, 2, 3, 10, 11, 14, 15] and O^e = [4, 12, 13, 20, 21].

The *a priori* probabilities for the presence of the PU and the PUEA in subbands are assumed to be known to the CR network. The probability of PU activity in O^p subbands and in O^e subbands are assumed to be equal to [0.7, 0.85, 0.65, 0.8, 0.9, 0.75, 0.6] and [0.55, 0.6, 0.75, 0.7, 0.8], respectively. The PUEA activity in subbands is considered to be equal to $1-\Pr(p_l)$.

It is well known that different spectrum sensing techniques available in the literature have different performance and accuracy. To analyze the importance of taking into account the presence of PUEA in spectrum sensing process and its relation to the spectrum sensing accuracy, we consider two different spectrum sensing methods with different performances. More precisely, we consider two given methods, denoted to as methods A and B, where method A has a better detection probability for a given false alarm than method B. For a target $P_f^p = 0.06$, P_d^p is assumed to be equal to [0.99, 0.98, 0.97, 0.99, 0.98, 0.96, 0.99] and [0.89, 0.88, 0.87, 0.89, 0.88, 0.86, 0.89] over subcarriers in set O^p for method A and method B, respectively. In addition, it is considered that P_d^p for O^e subcarriers are [0.98, 0.97, 0.99, 0.96, 0.98] and [0.88, 0.87, 0.89, 0.86, 0.88] for method A and method B, respectively. Moreover, the noise variance is set to 0 dBm and the fading coefficients for all channels are assumed to follow a Rayleigh distribution with an average channel power gain of 0 dBm.

Fig. 3 depicts the maximum CR network achieved data rate versus the average interference introduced to the PU under PUEA. It is displayed for the proposed scheme, applying method A and method B in spectrum sensing, compared to conventional power allocation which does not take the presence of the PUEA into account. The total transmit power budget of CR network is fixed and set to $Q = 7$ Watt.

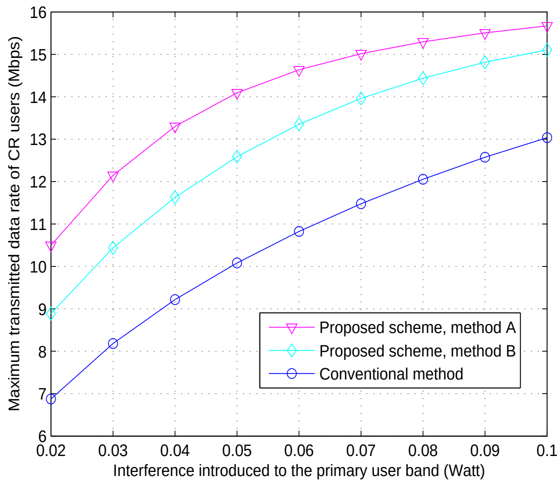

Fig. 3. Maximum CR achieved data rate versus the average interference introduced to the PU.

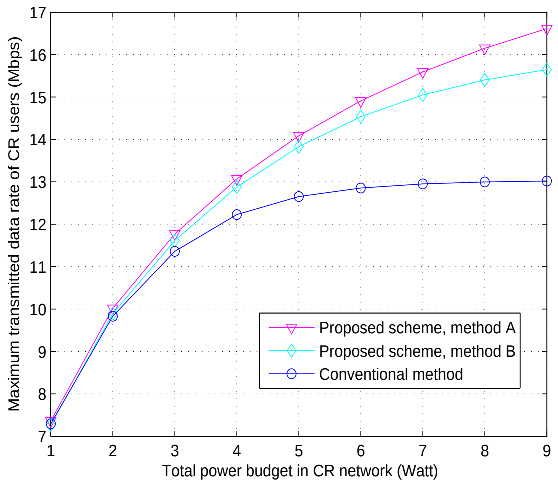

Fig. 4. Maximum CR achieved data rate versus total power budget in CR network.

This figure illustrates the efficiency of the proposed scheme which leads to higher CR data rates for a given interference level introduced to PU, compared to the conventional method. Note that the conventional power loading scheme considers all occupied subbands as PU subbands and performs power allocation in a way that the imposed interference to these subbands remains below a predefined threshold. In addition, we can see that using method A, which outperforms method B, can improve the maximum CR data rate for a given interference level introduced to the PU.

Fig. 4 illustrates the maximum CR network data rate versus total power budget in CR network for proposed scheme, applying method A and method B in spectrum sensing, compared to conventional power loading that does not take into account the presence of the PUEA in the environment. The maximum interference introduced to PU (i.e., I_{th}) is set to 0.1. We observe the superiority of our proposed method for power loading in the presence of the PUEA,

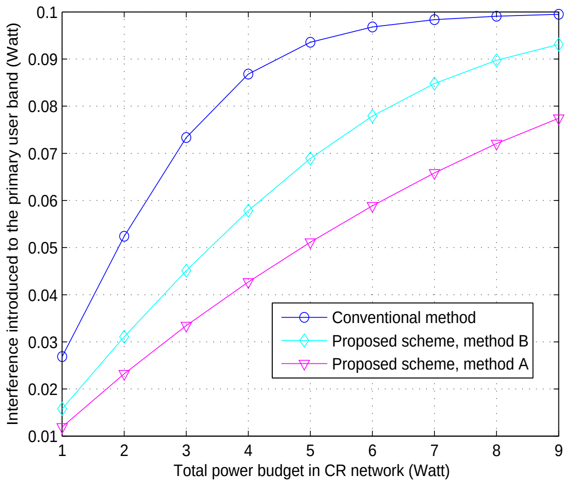

Fig. 5. The average interference introduced to the PU versus total power budget in CR network.

compared to the conventional method. In addition, for given power budget, method *A* leads to higher achievable rates for the CR network.

Fig. 5 illustrates the average interference introduced to the PU versus total power budget in CR network for proposed scheme, applying method *A* and method *B* in spectrum sensing, compared to conventional power allocation. The maximum interference introduced to PU is set this time to 0.1. It is observed that for a given CR power budget in the presence of the PUEA, our proposed power loading leads to less interference imposed on primary network and then outperforms the conventional scheme. Moreover, we observe that the efficiency of spectrum sensing method to differentiate between PU and PUEA signals, affects directly the performance of the power loading process.

7. Conclusion

Assuming the presence of a PUEA, we proposed an appropriate power allocation scheme for an OFDM-based CR system. We formulated the interference introduced to the CR and the PU network, considering the presence of an attacker and by taking into account possible errors occurring due to the presence of fake signals in the environment. Yet, the necessary QoS of the primary network is satisfied by considering the interference imposed from the CR to the PU below a tolerable threshold. Besides, we have considered the interference from the PU and PUEA to the CR network to formulate the CR downlink rate in an appropriate manner. To maximize the CR data rate under PUEA, the corresponding optimization problem was solved to find the optimal transmission power of CR in every subband. Numerical results confirmed that using the proposed method under PUEA improves the achieved data rate in the CR network compared to classical method which does not consider security aspects in dynamic spectrum access, where a malicious user is present in the radio environment. In addition, we showed that the accuracy of spectrum sensing in distinguishing between PU and PUEA signals directly affects the performance of the power loading process.

Appendix

Derivation of q_i^* in (22)

We start by converting the optimization problem in (21) to the standard form [14]. To this end, we write the problem as:

$$\min_{q_i} \left(-R_i(q_i)\right) \quad i \in V \tag{23}$$

subject to:

$$\sum_{i \in V} q_i \bar{I}_i - I_{th} \leq 0, \tag{24}$$

$$\sum_{i \in V} q_i - Q \leq 0, \tag{25}$$

$$-q_i \leq 0 \quad \forall i \in V. \tag{26}$$

The problem in (23) is a nonlinear convex problem, in other words, the objective function is nonlinear and concave with respect to q_i, so, the optimal solution can be obtained by the Karush-Kuhn-Tucker (KKT) conditions [14].

Considering Lagrange multipliers λ', μ' and ν_i' for the inequality constraints in (24), (25) and (26), respectively, we can define the Lagrangian L associated with (23), (24), (25) and (26) as:

$$L(q_i,\lambda',\mu',\nu_i') = -R_i(q_i) + \lambda'\left(\sum_{i \in V} q_i \bar{I}_i - I_{th}\right) + \mu'\left(\sum_{i \in V} q_i - Q\right) - \nu_i' q_i. \tag{27}$$

Now we express the gradient of $L(q_i,\lambda',\mu',\nu_i')$ with respect to q_i as:

$$\nabla_{q_i} L(q_i,\lambda',\mu',\nu_i') = -\frac{\Delta f}{\ln 2} \cdot \frac{1}{q_i + 1/\tilde{\gamma}_i} + \lambda' \bar{I}_i + \mu' - \nu_i' \tag{28}$$

where

$$\tilde{\gamma}_i \triangleq \frac{|h_i^{\mathrm{SS}}|^2}{\sigma_i^2 + J_i + K_i}.$$

To solve the problem, we can write the KKT conditions as:

$$\sum_{i\in V} q_i^* \bar{I}_i - I_{th} \leq 0,$$
$$\sum_{i\in V} q_i^* - Q \leq 0,$$
$$-q_i^* \leq 0 \quad \forall i \in V,$$
$$\lambda' \geq 0,$$
$$\mu' \geq 0,$$
$$\nu_i' \geq 0,$$
$$\lambda'(\sum_{i\in V} q_i^* \bar{I}_i - I_{th}) = 0,$$
$$\mu'(\sum_{i\in V} q_i^* - Q) = 0,$$
$$-\nu_i' q_i^* = 0 \quad \forall i \in V,$$
$$-\frac{1}{q_i^* + 1/\tilde{\gamma}_i} + \lambda \bar{I}_i + \mu - \nu_i = 0 \tag{29}$$

where $\lambda = \frac{\ln 2}{\Delta f}\lambda'$, $\mu = \frac{\ln 2}{\Delta f}\mu'$ and $\nu_i = \frac{\ln 2}{\Delta f}\nu_i'$.

Considering the condition $\nu_i' q_i^* = 0$, we can remove ν_i from (29) and rewrite it as:

$$q_i^* \geq 0 \quad \forall i \in V, \tag{30}$$
$$\sum_{i\in V} q_i^* \bar{I}_i = I_{th}, \tag{31}$$
$$\sum_{i\in V} q_i^* = Q, \tag{32}$$
$$\frac{1}{q_i^* + 1/\tilde{\gamma}_i} = \lambda \bar{I}_i + \mu. \tag{33}$$

Substituting q_i^* from (33) into (31) and (32), we get the Lagrange multipliers, μ and λ. Considering that (30) should be equal or greater than zero, we derive q_i^* as:

$$q_i^* = \max\left\{0, \frac{1}{(\lambda + \mu \bar{I}_i)} - \frac{1}{\tilde{\gamma}_i}\right\} \quad \forall i \in V \tag{34}$$

and this ends the proof.

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