A Rate-Splitting Based Bound-Approaching Transmission Scheme for the Two-User Symmetric Gaussian Interference Channel with Common Messages

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Abstract. *This paper is concerned with a rate-splitting based transmission strategy for the two-user symmetric Gaussian interference channel that contains common messages only. Each transmitter encodes its common message into multiple layers by multiple codebooks that are drawn from one separate code book, and transmits the superposition of the messages corresponding to these layers; each receiver decodes the messages from all layers of the two users successively. Two schemes are proposed for decoding order and optimal power allocation among layers respectively. With the proposed decoding order scheme, the sum-rate can be increased by rate-splitting, especially at the optimal number of rate-splitting, using average power allocation in moderate and weak interference regime. With the two proposed schemes at the receiver and the transmitter respectively, the sum-rate achieves the inner bound of HK without time-sharing. Numerical results show that the proposed optimal power allocation scheme with the proposed decoding order can achieve significant improvement of the performance over equal power allocation, and achieve the sum-rate within two bits per channel use (bits/channel use) of the sum capacity.*

Keywords

Interference channel, rate-splitting, power control.

1. Introduction

An Interference Channel (IC) models the situation where *N* independent transmitters try to communicate their separate information to the *N* different receivers via a common channel as initiated in [1]. It has been well known that the very strong IC with additive white Gaussian noise (AWGN) can achieve the rate as high as that without interference [2]. The capacity of strong IC was also obtained in [3]-[4]. The sum-rate capacity for ICs with low crosstalk coefficients and transmission powers can be achieved by treating the interference as noise [5]-[7]. However the rate regions of moderate and weak ICs remain an open problem for several decades in multi-user information theory. The best known achievability strategy for the remaining unsolved cases was proposed by Han and Kobayashi (HK) in [8], and it contains the significant idea of rate-splitting that divides the transmitted message into two parts: a common part decodable by both receivers, and a private part decodable only by the intended receiver. In [9], a new upper bound of sum-rate was proposed by Etkin, Tse and Wang (ETW) to show that with the HK-type transmission scheme, the rates within 1 b/s/Hz of the capacity can be achieved for the two-user Gaussian IC. For fairness, HK utilized the time-sharing scheme to increase the individual rates of both users as the sum-rate of IC achieving the maximum value.

The model and results of ICs could be used in the verge of modern cellular networks which are becoming increasingly interference limited as cell sizes shrink to accommodate a growing number of users, i.e. in cellular system with the frequency reuse-1, the cell edge users are always interfered by the cell edge users from other cells. In order to improve the rates of users in the cell edge as well as the spectrum efficiency, we must reduce the inter-cell interferences. Therefore, the efficient interference management to reduce the effect of interference is an important issue in realistic ICs. The available interference management methods can be roughly classified into three categories: *Interference Alignment* [10], *Interference Cancellation* and *Power Control*.

In practical communications, the above methods of interference managements can be used in conjunction with each other. So far, extensive research efforts [11]-[18] have been devoted to dynamic spectrum management, power allocation in transmitter and decoding order in receiver. Both power allocation and decoding order were considered in [14] to minimize the total power consumption. A greedy decoding order algorithm was developed in [15] for a *K*-user memoryless IC, where each receiver decoded a subset of

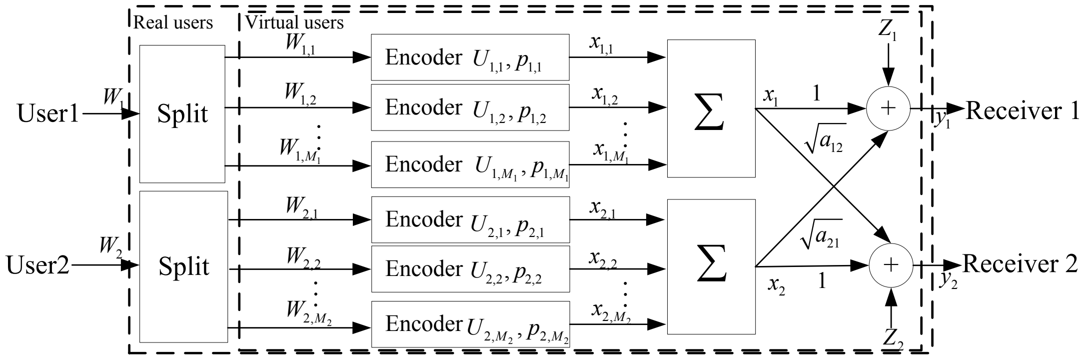

Fig. 1. The proposed rate-splitting model for Gaussian IC-CM.

all transmitters sequentially before decoding the data of the designated transmitter. Two algorithms determining the decoding order for rate-splitting with partial interference decoded were proposed in [17], which can improve the sum-rate as the number of rate-splitting increases. In [18], a rate-splitting based scheme was proposed, in which all interferences were decoded successively, and an iterative multiple waterlevels water-filling algorithm was introduced to optimize the power allocation. It was shown that this scheme can achieve the near-optimal performance.

In this paper, we attempt to combine power control and interference cancellation with rate-splitting technique to increase the sum-rate of two-user symmetric Gaussian IC with common messages (IC-CM). In the IC-CM, as in the IC (without private message), each transmitter has the message which it needs to its corresponding receiver decodable by both receivers. For symmetric Gaussian IC-CM, we generalize the scheme of HK to split the common message of each transmitter into more than two layers, each with a power determined by an *optimal fixed power allocation* (OFPA) transmission scheme. At the receiver, a *successive total-interference cancellation* (STIC) decoding order scheme is also proposed to decode all common messages (containing desired message and interference) successively. In moderate and weak IC-CM, we will show that with the use of OFPA transmission scheme, the sum-rate achieves the inner bound of HK for the symmetric Gaussian IC-CM, when the transmitted power is equal to the total power of the OFPA scheme. Finally, numerical results are provided for performance comparisons of OFPA scheme, average power allocation scheme and the inner bound of HK without time-sharing.

2. System Model

2.1 Two-User Symmetric Gaussian IC-CM

This paper considers the two-user Gaussian IC-CM, as shown in Fig. 2. The input-output relationship can be described by

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \mathbf{H} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} \tag{1}$$

where $x_n, n = 1,2$, denotes the input which can be decoded by each user, $y_n, n = 1,2$, denotes the output of user n, and Z_n is the real Gaussian noise with zero mean and variance of unit. The input x_n is subject to a power constraint P_n. $\mathbf{H}$ is the normalized channel-gain-matrix, i.e.,

$$\mathbf{H} = \begin{bmatrix} 1 & \sqrt{a_{2,1}} \\ \sqrt{a_{1,2}} & 1 \end{bmatrix} \tag{2}$$

where $\sqrt{a_{n,k}}$ denotes the channel crosstalk coefficient from user n to user k ($n \neq k$). In this paper, we assume that each receiver knows the codebooks of all users.

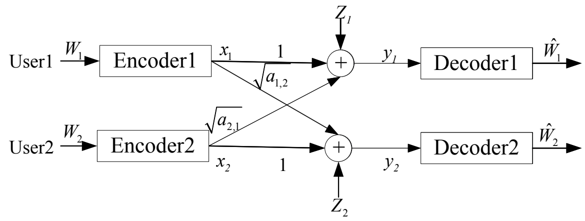

Fig. 2. Standard N-user Gaussian IC-CM.

2.2 Model of Rate-Splitting

Assume that the real user n is split into M_n virtual users indexed by $U_{n,m}, m \in \{1,...,M_n\}, n \in \{1,2\}$ as shown in Fig. 1. Let $P_{U_{n,m}}$ denote the power of virtual user $U_{n,m}$ with constraint $\sum_{m=1}^{M_n} P_{U_{n,m}} \leq P_n$. Therefore, the common messages from real user constitute the set of messages of virtual users defined as

$$x_n = \sum_{m=1}^{M_n} x_{n,m} \tag{3}$$

where $x_{n,m}$ is the message of virtual user $U_{n,m}, n \in \{1,2\}$, $m \in \{1,2,\cdots,M_n\}$. The rate of user n is

$$R_n = \sum_{m=1}^{M_n} R_{U_{n,m}} \tag{4}$$

where $R_{U_{n,m}}$ defines the rate of virtual user $U_{n,m}$.

3. STIC Decoding Order Scheme

It is worth noting that the interferences influencing the virtual user $U_{n,m}$ are not only from other real user's virtual users, but also from other virtual users of real user n. In the

STIC scheme, the receiver n has to decode all messages that were received by successive interference cancellation (SIC). Therefore, compared with partial interference cancellation [15], we refer to the proposed scheme as STIC. Assume that each real user contains the same number of virtual users, M, the two messages of virtual users which come from the different real user constitute a layer at each receiver, and be decoded in the manner of layer by layer at both receivers. At each layer, all messages are decoded according to the designed order. To begin with, we formalize a definition to clarify the illustration in sequel.

Definition: The decoding order of receiver n is denoted by a $M \times 2$ matrix,

$$\boldsymbol{\pi}^n = (\boldsymbol{\pi}^{[n]}(1), \boldsymbol{\pi}^{[n]}(2), \cdots, \boldsymbol{\pi}^{[n]}(M))^T \tag{5}$$

where $\boldsymbol{\pi}^{[n]}(i) = [\pi^{[n]}(i,1), \pi^{[n]}(i,2)]^T, i \in \{1, \cdots, M\}$, is the decoding order of receiver n during decoding the messages of layer i. Receiver n first decodes the message of virtual user $\pi^{[n]}(1,1)$, then that of virtual user $\pi^{[n]}(1,2)$, and so forth, until the message of virtual user $\pi^{[n]}(M,2)$ is decoded .

At receiver n, the set $\mathbf{D}^n$ is defined as the set of messages that have been decoded. In the two-user IC-CM, the STIC scheme is described as follows.

STIC scheme

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1. **Initialization**

   $\boldsymbol{\pi}^n$: the decoding order of receiver n;
   $\mathbf{D}^n$: the set which contains the messages that have been decoded already.

2. **for all** n such that $n \in \{1, 2\}$

3. **for all** i such that $i \in \{1, 2, \ldots, M\}$

4. **Determine the decoding order**

   Determine the message which should be decoded in this layer,
   $m_n^* = \arg \max_{m \in \{1,2,\cdots,M\}, x_{n,m} \notin \mathbf{D}^n} I(Y_n; x_{n,m} | \mathbf{D}^n)$;
   $m_{\bar{n}}^* = \arg \max_{m \in \{1,2,\cdots,M\}, x_{\bar{n},m} \notin \mathbf{D}^n} I(Y_n; x_{\bar{n},m} | \mathbf{D}^n)$
   where $n, \bar{n} \in \{1, 2\}$, $\bar{n} \neq n$.

5. **Update**

   $\boldsymbol{\pi}^n$: update the decoding order of receiver n,
   $\pi^{(n)}(i,1) = U_{n,m_n^*}$, $\pi^{(n)}(i,2) = U_{\bar{n},m_{\bar{n}}^*}$;
   $\mathbf{D}^n$: update the set $\mathbf{D}^n$, $\mathbf{D^n} = \mathbf{D^n} \bigcup \{x_{n,m_n^*}, x_{\bar{n},m_{\bar{n}}^*}\}$.

6. **end for**

7. **end for**

Let $\theta_{U_{n,m}}^{[n]}$ denote the position of virtual user $U_{n,m}$ in $\boldsymbol{\pi}^n$. It can be obtained that

$$\begin{aligned} &\pi^{[n]}(\theta_{U_{n,m}}^{[n]}, 1) = U_{n,m}, \pi^{[n]}(\theta_{U_{\bar{n},m}}^{[n]}, 2) = U_{\bar{n},m}, \\ &\pi^{[\bar{n}]}(\theta_{U_{n,m}}^{[\bar{n}]}, 2) = U_{n,m}, \pi^{[\bar{n}]}(\theta_{U_{\bar{n},m}}^{[\bar{n}]}, 1) = U_{\bar{n},m} \end{aligned} \tag{6}$$

where $n, \bar{n} \in \{1, 2\}, \bar{n} \neq n$ and $m \in \{1, 2, \cdots, M\}$.

4. Power Allocation

In this section, we propose two schemes for power allocation based on the STIC. One is the average power allocation scheme, another is the OFPA scheme. We also present two theorems to show the relation between those two schemes and rate-splitting. Consider an IC-CM, where each real user has the same power allocation scheme, i.e.,

$$P_{U_{1,m}} = P_{U_{2,m}} = p_m, m \in \{1, 2, \cdots, M\}, \tag{7}$$

each receiver of symmetric IC-CM has the same decoding order. Therefore, the two messages of virtual users that have the same index in different real users are arranged in the same layer of each receiver, i.e.,

$$\theta_{U_{n,m}}^{[n]} = \theta_{U_{\bar{n},m}}^{[n]} = \theta_{U_{\bar{n},m}}^{[\bar{n}]} = \theta_{U_{n,m}}^{[\bar{n}]} = \theta_m. \tag{8}$$

Let us begin with introducing two lemmas to clarify the theorems in sequel. Denote $\gamma(x) \overset{\Delta}{=} \frac{1}{2}\log_2(1 + x)$.

Lemma 1: According to the HK scheme without time-sharing, the maximum sum-rate achievable of two-user symmetric Gaussian IC-CM with moderate and weak interference is

$$R_{sum} = \sum_{n=1}^{2} R_n \leqslant \gamma(P + aP) \tag{9}$$

where P denotes the power of transmitted, and a denotes the channel cross talk coefficient that satisfies $\frac{\sqrt{1+4P}-1}{2P} \leq a \leq 1$.

Proof: According to the HK scheme without time-sharing, the sum-rate of two-user symmetric Gaussian IC-CM must satisfy the following constraints,

$$R_1 + R_2 \leqslant 2\gamma(P), \tag{10}$$

$$R_1 + R_2 \leqslant 2\gamma(aP), \tag{11}$$

$$R_1 + R_2 \leqslant \gamma(P + aP) \tag{12}$$

where inequality (10) corresponds to the individual rate constraint of decoding the common messages at the desired receiver, inequality (11) corresponds to the individual rate constraint of decoding the common messages at the undesired receiver and inequality (12) corresponds to the sum-rate constraint of jointly decoding both common messages at receiver 1 and receiver 2.

Comparing inequalities (10) and (11), we can see that the bound in (11) is more tight than the bound in (10) which is always true for $0 < a < 1$. Then comparing inequalities

(11) and (12), we can see that the bound in (12) is more tight than the bound in (11) if

$$1+P+aP \leqslant (1+aP)^2. \quad (13)$$

Since we study the sum-rate achievable in the moderate and weak regime, the transmitted power should satisfy $\frac{\sqrt{1+4P}-1}{2P} \leq a \leq 1$, refer to [5]-[7]. Therefore, the inequality (9) is always true for two-user symmetric Gaussian IC-CM with moderate and weak interference. □

Lemma 2: Consider a two-user symmetric Gaussian IC-CM, where the channel crosstalk coefficient is $\sqrt{a}$ and two users are assumed to have the same power P. Based on STIC decoding order, the achievable rate of user n satisfies

$$R_1 = R_2 = R_n = \sum_{m=1}^{M} R_{U_{n,m}} \leq \sum_{m=1}^{M} \min\{r_{U_{n,m}}, r'_{U_{n,m}}\} \leq \frac{\gamma(P+aP)}{2} \quad (14)$$

where

$$\begin{cases} r_{U_{n,m}}(M,m,\mathbf{P}) = \gamma\left(\frac{P_{U_{n,m}}}{1+\sum\limits_{i=\theta^{[n]}_{U_{n,m}}+1}^{M} P_{\pi^{[n]}(i,1)} + \sum\limits_{i=\theta^{[n]}_{U_{\bar{n},m}}}^{M} P_{\pi^{[n]}(i,2)}a}\right) \\ r'_{U_{n,m}}(M,m,\mathbf{P}) = \gamma\left(\frac{P_{U_{n,m}}a}{1+\sum\limits_{i=\theta^{[\bar{n}]}_{U_{n,m}}+1}^{M} P_{\pi^{[\bar{n}]}(i,2)}a + \sum\limits_{i=\theta^{[\bar{n}]}_{U_{\bar{n},m}}+1}^{M} P_{\pi^{[\bar{n}]}(i,1)}}\right) \end{cases}$$

n, $\bar{n} \in \{1,2\}$, $n \neq \bar{n}$ and $\mathbf{P}$ denotes the power allocation schemes.

Proof: Since each user of symmetric Gaussian IC-CM has the same power allocation and the same decoding order, the rates of two users are equal, i.e., $R_1 = R_2 = R_n$. The symbol $r_{U_{n,m}}$ represents the maximum rate with which the virtual user $U_{n,m}$ transmits to receiver n, and the symbol $r'_{U_{n,m}}$ represents the maximum rates with which the virtual user $U_{n,m}$ transmit to receiver $\bar{n}$. In order to decode the message of virtual user $U_{n,m}$ at both receivers correctly, the achievable rate of virtual user $U_{n,m}$ is bounded by

$$R_{U_{n,m}} \leq \min\{r_{U_{n,m}}, r'_{U_{n,m}}\}. \quad (16)$$

Since $\sum_{m=1}^{M} r_{U_{n,m}}(M,m) + \sum_{m=1}^{M} r'_{U_{n,m}}(M,m)$ satisfies (15), where (a) is due to (8) and (b) holds since each real user has the same power allocation as (7) and the two virtual users which have the same power are allocated in the same layer of decoding order of the two receivers, it can be obtained that

$$\sum_{m=1}^{M} r_{U_{n,m}}(M,m,\mathbf{P}) \text{ or } \sum_{m=1}^{M} r'_{U_{n,m}}(M,m,\mathbf{P}) \leq \frac{\gamma(aP+P)}{2}. \quad (17)$$

Therefore,

$$\sum_{m=1}^{M} \min\{r_{U_{n,m}}(M,m,\mathbf{P}), r'_{U_{n,m}}(M,m,\mathbf{P})\} \leq \frac{\gamma(P+aP)}{2}. \quad (18)$$

We have equality in (18) if

$$r_{U_{n,m}}(M,m,\mathbf{P}) = r'_{U_{n,m}}(M,m,\mathbf{P}), m \in \{1,2,\cdots,M\}. \square \quad (19)$$

4.1 Average Power Allocation

Since each virtual user has been allocated the same power, we can denote the decoding order $\theta^{[\cdot]}_{U_{n,m}}$ of $U_{n,m}$ by the message order m at both receivers based on STIC, i.e.,

$$\theta^{[n]}_{U_{n,m}} = \theta^{[\bar{n}]}_{U_{n,m}} = \theta^{[n]}_{U_{\bar{n},m}} = \theta^{[\bar{n}]}_{U_{\bar{n},m}} = m. \quad (20)$$

Therefore, the maximum rate of each user has to satisfy

$$\begin{aligned} R_1(M) &= R_2(M) = R_{sum}(M)/2 \\ &= \max_{M} \sum_{m=1}^{M} \min\{r_{U_{n,m}}, r'_{U_{n,m}}\}. \end{aligned} \quad (21)$$

where

$$\begin{cases} r'_{U_{n,m}}(M,m) = \gamma\left(\frac{aP}{M+(M-m)P+(M-m)aP}\right) \\ r_{U_{n,m}}(M,m) = \gamma\left(\frac{P}{M+(M-m)P+(M-m+1)aP}\right). \end{cases} \quad (22)$$

We will find the M which maximizes the rates of users as follows.

$$\begin{aligned} \sum_{m=1}^{M} r_{U_{n,m}} + \sum_{m=1}^{M} r'_{U_{n,m}} &= \sum_{m=1}^{M} \gamma\left(\frac{P_{U_{n,m}}}{1+\sum\limits_{i=\theta^{[n]}_{U_{n,m}}+1}^{M} P_{\pi^{[n]}(i,1)} + \sum\limits_{i=\theta^{[n]}_{U_{\bar{n},m}}}^{M} P_{\pi^{[n]}(i,2)}a}\right) + \gamma\left(\frac{P_{U_{n,m}}a}{1+\sum\limits_{i=\theta^{[\bar{n}]}_{U_{n,m}}+1}^{M} P_{\pi^{[\bar{n}]}(i,2)}a + \sum\limits_{i=\theta^{[\bar{n}]}_{U_{\bar{n},m}}+1}^{M} P_{\pi^{[\bar{n}]}(i,1)}}\right) \\ &\overset{(a)}{=} \sum_{m=1}^{M} \gamma\left(\frac{P_{U_{n,m}}}{1+\sum\limits_{i=\theta_m+1}^{M} P_{\pi^{[n]}(i,1)} + \sum\limits_{i=\theta_m}^{M} P_{\pi^{[n]}(i,2)}a}\right) + \gamma\left(\frac{P_{U_{n,m}}a}{1+\sum\limits_{i=\theta_m+1}^{M} P_{\pi^{[\bar{n}]}(i,2)}a + \sum\limits_{i=\theta_m+1}^{M} P_{\pi^{[\bar{n}]}(i,1)}}\right) \\ &= \frac{1}{2}\sum_{m=1}^{M} \log\left(\frac{1+\sum\limits_{i=\theta_m}^{M} P_{\pi^{[n]}(i,1)} + \sum\limits_{i=\theta_m}^{M} P_{\pi^{[n]}(i,2)}a}{1+\sum\limits_{i=\theta_m+1}^{M} P_{\pi^{[n]}(i,1)} + \sum\limits_{i=\theta_m}^{M} P_{\pi^{[n]}(i,2)}a}\right) + \log\left(\frac{1+\sum\limits_{i=\theta_m}^{M} P_{\pi^{[\bar{n}]}(i,2)}a + \sum\limits_{i=\theta_m+1}^{M} P_{\pi^{[\bar{n}]}(i,1)}}{1+\sum\limits_{i=\theta_m+1}^{M} P_{\pi^{[\bar{n}]}(i,2)}a + \sum\limits_{i=\theta_m+1}^{M} P_{\pi^{[\bar{n}]}(i,1)}}\right) \\ &\overset{(b)}{=} \frac{1}{2}\log(1+\sum_{i=1}^{M} p_i + \sum_{i=1}^{M} p_i a) = \gamma(P+Pa) \end{aligned} \quad (15)$$

Theorem 1: Consider a two-user symmetric Gaussian IC-CM with moderate and weak interference, i.e., $\sqrt{a_{1,2}} = \sqrt{a_{2,1}} = \sqrt{a}$ $(\frac{\sqrt{1+4P}-1}{2P} \le a \le 1)$, $P_1 = P_2 = P$. Using the average power allocation based on STIC, there are M^* virtual users,

$$M^* = \frac{a^2 P}{1-a}, \tag{23}$$

which makes the $R_n(M^*)$ achieve maximum value approximately as P approaches infinite.

Proof: According to Lemma 2, when M satisfies the set of equation

$$r'_{U_{n,m}}(M,m) = r_{U_{n,m}}(M,m), m \in \{1,2,\cdots,M\}, \tag{24}$$

the sum-rate approaches the maximum value $\gamma(P+aP)$. However the values of M which satisfy these M equations are different. Since

$$r_{U_{n,M}}(M,M) > r_{U_{n,i}}(M,i), i \in \{1,2,\cdots M-1\}, \tag{25}$$

we use M^* which satisfies the Mth equation,

$$r_{U_{n,M^*}}(M^*,M^*) = r'_{U_{n,M^*}}(M^*,M^*), \tag{26}$$

to make sum-rate approach maximum value. Then, $M^* = \frac{a^2 P}{1-a}$. According to (22), when P tends to infinite,

$$r_{U_{n,M}}(M,M) \gg r_{U_{n,i}}(M,i), i \in \{1,2,\cdots M-1\} \tag{27}$$

and

$$R_n(M) \approx \min(r_{U_{n,M}}(M,M), r'_{U_{n,M}}(M,M)). \tag{28}$$

Therefore, $R_n(M^*)$ achieve the maximum value approximately as P approaches infinite. Since the number of virtual users, M, should more than one, so a should be restricted in $[\frac{\sqrt{1+4P}-1}{2P}, 1]$. □

4.2 Optimal Fixed Power Allocation

Given the number of virtual users M, the following theorem states the OFPA scheme. We will show that the OFPA scheme is optimal and it can achieve the maximum sum-rate based on STIC.

Theorem 2: Consider a two-user symmetric Gaussian IC-CM with moderate and weak interference that the virtual users' number M for each real user, i.e., $\sqrt{a_{1,2}} = \sqrt{a_{2,1}} = \sqrt{a}$ $(\frac{\sqrt{1+4P^*}-1}{2P^*} \le a \le 1)$. When each virtual user has been assigned the power by using the OFPA scheme

$$\left\{\frac{1-a}{a^2}, \frac{1-a}{a^4}, \cdots, \frac{1-a}{a^{2M}}\right\} \tag{29}$$

based on STIC, the sum-rate of two-user achieves the maximum value

$$R^*_{sum} = \gamma(P^* + aP^*), \tag{30}$$

when the total transmitted power of each real user satisfies

$$P^* = \frac{1-a}{a^2}\left[1 + \frac{1}{a^2} + \frac{1}{a^4} + \cdots + \frac{1}{a^{2M-2}}\right]. \tag{31}$$

Proof: Given the number of virtual users, M, for each user, the maximum achievable rate of each user is

$$\begin{aligned} R_1(\mathbf{P}) &= R_2(\mathbf{P}) = R_{sum}(\mathbf{P})/2 \\ &= \max_{\mathbf{P}} \sum_{m=1}^{M} \min\{r_{U_{n,m}}(m,\mathbf{P}), r'_{U_{n,m}}(m,\mathbf{P})\} \end{aligned} \tag{32}$$

where
$$\begin{cases} r_{U_{n,m}}(m,\mathbf{P}) = \gamma\left(\dfrac{P_{U_{n,m}}}{1+\sum\limits_{i=\theta^{[n]}_{U_{n,m}}+1}^{M} P_{\pi^{[n]}(i,1)} + \sum\limits_{i=\theta^{[n]}_{U_{\bar{n},m}}}^{M} P_{\pi^{[n]}(i,2)}a}\right) \\ r'_{U_{n,m}}(m,\mathbf{P}) = \gamma\left(\dfrac{P_{U_{n,m}}a}{1+\sum\limits_{i=\theta^{[\bar{n}]}_{U_{n,m}}+1}^{M} P_{\pi^{[\bar{n}]}(i,2)}a + \sum\limits_{i=\theta^{[\bar{n}]}_{U_{\bar{n},m}}+1}^{M} P_{\pi^{[\bar{n}]}(i,1)}}\right) \end{cases}$$

where M is given and $\mathbf{P}$ denotes the power allocation schemes. It can be seen that the sum-rate achieve the maximum value along with the rate of each user achieve the maximum value.

According to Lemma 2, in order to obtain the maximum sum-rate, the power should be distributed to each virtual user such that

$$\begin{aligned} &\gamma\left(\frac{P_{U_{n,m}}}{1+\sum\limits_{i=\theta^{[n]}_{U_{n,m}}+1}^{M} P_{\pi^{[n]}(i,1)} + \sum\limits_{i=\theta^{[n]}_{U_{\bar{n},m}}}^{M} P_{\pi^{[n]}(i,2)}a}\right) \\ &= \gamma\left(\frac{P_{U_{n,m}}a}{1+\sum\limits_{i=\theta^{[\bar{n}]}_{U_{n,m}}+1}^{M} P_{\pi^{[\bar{n}]}(i,2)}a + \sum\limits_{i=\theta^{[\bar{n}]}_{U_{\bar{n},m}}+1}^{M} P_{\pi^{[\bar{n}]}(i,1)}}\right) \end{aligned} \tag{33}$$

where $n \in \{1,2\}, m \in \{1,2,\cdots,M\}$. In order to satisfy (33), power allocation should be as follows.

- For $m = M$:

$$\gamma\left(\frac{P_{U_{n,M}}}{1+P^*_{\pi^{[n]}(M,2)}a}\right) = \gamma(P_{U_{n,M}}a), \tag{34}$$

$$P^*_{\pi^{[n]}(M,2)} = \frac{1-a}{a^2}. \tag{35}$$

- For $m = M-1$:

$$\begin{aligned} &\gamma\left(\frac{P_{U_{n,M-1}}}{1+P_{\pi^{[n]}(M,2)}a + P_{\pi^{[n]}(M-1,2)}a + P_{\pi^{[n]}(M,1)}}\right) \\ &= \gamma\left(\frac{P_{U_{n,M-1}}a}{1+P_{\pi^{[\bar{n}]}(M,1)} + P_{\pi^{[\bar{n}]}(M,2)}a}\right). \end{aligned} \tag{36}$$

Since (7) and (8), the virtual users which are allocated in the same layer of the decoding order have the same power, i.e., $P_{\pi^{[n]}}(M,1) = P_{\pi^{[n]}}(M,2) = P_{\pi^{[\bar{n}]}}(M,1) =$

$P_{\pi^{[\bar{n}]}}(M,2)$. Therefore,

$$P^*_{\pi^{[n]}(M-1,2)} = \frac{1-a}{a^4}. \quad (37)$$

$$\vdots$$

- For m=1:

$$\gamma\left(\frac{P_{U_{n,m}}}{1+\sum_{i=2}^{M} P_{\pi^{[n]}(i,1)} + \sum_{i=1}^{M} P_{\pi^{[n]}(i,1)}a}\right)$$
$$= \gamma\left(\frac{P_{U_{n,m}}a}{1+\sum_{i=2}^{M} P_{\pi^{[\bar{n}]}(i,2)}a + \sum_{i=2}^{M} P_{\pi^{[\bar{n}]}(i,1)}}\right) \quad (38)$$

$$P^*_{\pi^{[n]}(1,2)} = \frac{1-a}{a^{2M}}. \quad (39)$$

Therefore the optimal total-power for each user is

$$\sum_{m=1}^{M} P^*_{U_{\bar{n},m}} = \frac{1-a}{a^2}\left(1+\frac{1}{a^2}+\cdots+\frac{1}{a^{2M-2}}\right). \quad (40)$$

When $P^*_{U_{n,M}} \leq P^*$, the OFPA scheme can improve the sum-rate, which is similar to the case indicated by Theorem 1. Therefore, a should be restricted in $[\frac{\sqrt{1+4P}-1}{2P}, 1]$. □

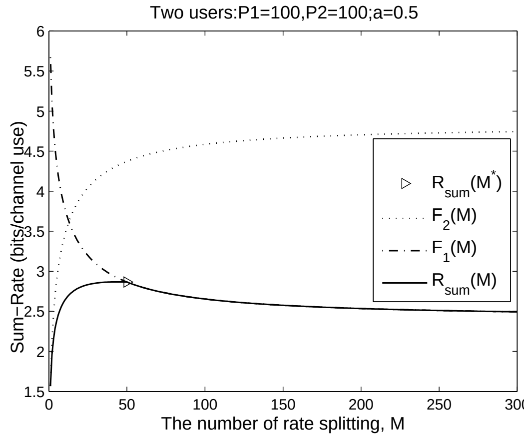
By Theorem 2, the maximum sum-rate of two-user symmetric Gaussian IC-CM can be achieved coinciding with the inner bound of HK based on STIC decoding order as the rates of two users achieving the maximum value equally.

5. Numerical Examples

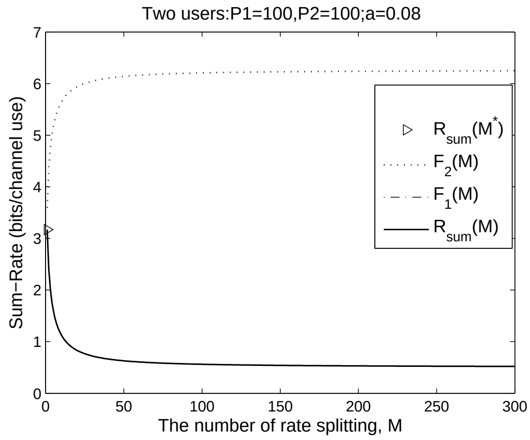
In this section, we present numerical results to illustrate those two theorems. For the two-user symmetric Gaussian IC-CM under consideration, we assume that both transmitters are subject to the same power constraint $P_1 = P_2 = P = 100$. We define

$$\begin{cases} F_1(M) & = \sum_{m=1}^{M} r'_{U_{n,m}}(M,m), \\ F_2(M) & = \sum_{m=1}^{M} r_{U_{n,m}}(M,m). \end{cases} \quad (41)$$

Using the average power allocation scheme, two examples are given in Fig. 3: (a) $a = 0.5$ and (b) $a = 0.08$. From Fig. 3(a), we can see that the inflection point of sum-rate is approximate to the maximum rate when $M = M^* = 50$. For this region $a > \frac{\sqrt{1+4P}-1}{2P}$, decoding interference can increase the sum-rate. In case of $a < \frac{\sqrt{1+4P}-1}{2P}$, the maximum sum-rate can be achieved when the number of rate-splitting is less than 1 as shown in Fig. 3(b). Therefore, the STIC decoding order based on rate-splitting is not optimal for the two-user symmetric Gaussian IC-CM where the square of channel crosstalk coefficient is less than the threshold $\frac{\sqrt{1+4P}-1}{2P}$. It is consistent with the findings of [5]-[7] where it was shown that treating the interference as noise achieves capacity for the further restricted region: $P \leq \frac{1-2\sqrt{a}}{2a\sqrt{a}}$.



(a) $a > \frac{\sqrt{1+4P}-1}{2P}$



(b) $a \leq \frac{\sqrt{1+4P}-1}{2P}$

Fig. 3. The sum-rates of two-user symmetric Gaussian IC-CM versus the number of rate-splitting of STIC scheme.

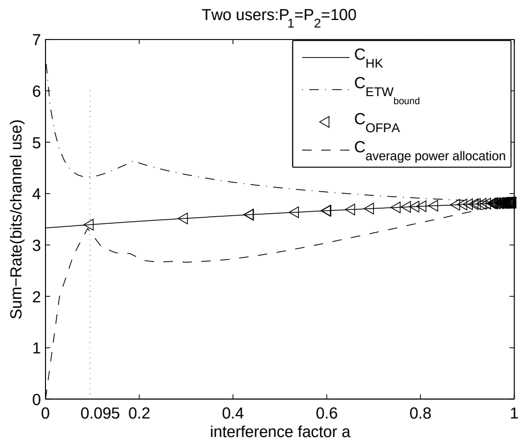


Fig. 4. The achievable maximum sum-rates in the two-user symmetric Gaussian moderate and weak IC-CM.

The OFPA scheme is simulated and the average power allocation scheme, the upper bound of ETW and HK scheme without time-sharing are also shown in Fig. 4. Comparing with the upper bound of ETW, we can seen that the sum-rate of OFPA scheme can achieve within two bits/channel use of the capacity, when the interference is moderate and weak ($\frac{\sqrt{1+4P}-1}{2P} \leq a \leq 1$). Comparing with average power allocation scheme, we can see that the sum-rate of OFPA scheme is superior to that of the average power allocation scheme. When the transmitted power satisfies the total power of OFPA scheme, the sum-rate of OFPA scheme approaches the HK bound without time-sharing.

6. Conclusion

This paper proposed an optimal fixed power allocation scheme (OFPA) and a successive total-interference cancellation decoding order (STIC) with the use of rate-splitting on the two-user Gaussian IC-CM. We showed that the sum-rate based rate-splitting can be increased by average power allocation with moderate and weak interference. However, since decoding interference does not improve the overall system throughput if the interference level below certain thresholds, thus the scheme is not optimal in the low-interference regime. It is similar to the findings of [5]-[7] where it was shown that treating the interference as noise achieves the sum capacity in the low-interference regime. We also showed that the proposed scheme based on STIC can improve the sum-rate to approach the inner bound of HK when the transmitted power satisfies the total power of OFPA scheme.

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