Graph Cuts based Image Segmentation using Fuzzy Rule Based System

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Abstract. This work deals with segmentation of the gray scale, color and texture images using graph cuts. From an input image, a graph is constructed using intensity, color and texture profiles of the image simultaneously (i.e., intensity and texture for gray scale images and color and texture for color images). Based on the nature of image, a fuzzy rule based system is designed to find the weight that should be given to a specific image feature during graph development. The graph obtained from the fuzzy rule based weighted average of different image features is further used in normalized graph cuts framework. The graph is iteratively bi-partitioned through the normalized graph cuts algorithm to get optimum partitions resulting in segmented image. The Berkeley segmentation database is used to test our algorithm and the segmentation results are evaluated through probabilistic rand index, global consistency error, sensitivity, positive predictive value and Dice similarity coefficient. It is shown that the presented segmentation method provides effective results for most types of images.

Keywords
Segmentation, normalized graph cuts, fuzzy rule based system.

1. Introduction

Image segmentation divides an image into meaningful pieces or segments with perceptually same features and properties. The aim is simplification and representation of the image to make it more meaningful and easier to analyze. It is an important step in high level processing techniques (like object detection and recognition) and an important research area [1]-[16].

Several techniques of image segmentation exist in the literature (including histogram [1], edge based [2], data clustering [4], watershed transformation [6], mean shift [7] and graph cuts [8]). Sometimes different techniques are merged to get better results [9]. Histogram thresholding [1] works effectively for monochrome images but for color images, the scenario is different (where multi-thresholding for RGB histograms becomes challenging). The boundaries formed using edge detection based methods [2] are not necessarily closed and the results vary where regions are fused together. In data clustering [4], the edge information and spatial structure is not well-looked-after, furthermore it is biased towards ellipsoidal clusters. In case of watershed transformation [6] and mean shift [7], a large number of small regions are produced (as these are unsupervised segmentation methods having a priori knowledge of number and size of segments), therefore a kind of region merging algorithm is applied to cater to this effect.

So far, one of the most promising approaches for image segmentation is based on the graph cuts [8]. Graph based methods help in image perceptual grouping and organization using image features and spatial information. The input image is converted into an undirected graph with image pixels as its nodes and edge weighting is made by taking into account the similarity or dissimilarity between image pixels. Then a graph partitioning algorithm is applied to partition the graph. Existing partition methods include ratio cuts, average cuts, minimum cuts and normalized cuts [10]-[12]. The normalized cuts method was proposed [12], [13] to solve the perceptual grouping problem for optimal segmentation. Resultantly, segmented image is obtained from the partitioned graph.

In related work, there are some techniques that use different image features (i.e., color and texture) and graph cuts for color-texture segmentation [14]-[18]. In papers [14], [16], the foreground is extracted from background using color and texture features of image and minimum cuts algorithm as graph cuts. Texture feature extraction is made by textons computation in [14] and SIFT (scale-invariant feature transform) in [16] while color feature extraction is done by using RGB color model in both. Here, color model to be chosen is important. RGB does not provide a good human perception of colors which should be considered while using graph cuts. Also, minimum cuts algorithm does not work for isolated nodes in graph and it considers them as a separate segment which is not desirable [13]. In another method [15], watershed algorithm is used to find a large number of small regions and these regions are used to construct a graph.
with these regions as its nodes. Then graph cuts algorithm is used to get the segmented image with K-means algorithm for quantization. Merging different segmentation algorithms is effective but computationally expensive. In papers [17], [18], minimum cuts algorithm is used which does not cater the effect of isolated nodes in the graphs. In our method, Lab color space is used for better human perception of colors as it is specifically designed for this purpose. Also the normalized graph cuts algorithm is used to cater the effect of isolated nodes which is ignored by minimum cuts algorithm, and for optimum partitioning of graph.

Segmentation using the graph cuts is highly dependent on optimum calculation of weights on the edges in graph development. Different methods have been discussed in this regard [13] but still some improvements are required to accomplish this essential step. With in view, edge weighting is made by considering brightness, color and texture profiles of an image simultaneously (i.e., brightness and texture for gray scale images and color and texture for color images). The main focus is to get the knowledge about how much a specific image feature should be involved in building the optimized graph. Simple averaging may not give the required results. Therefore, a fuzzy rule based system [19] is used to get the knowledge of how much a specific image feature should be involved in building the optimized graph. The proposed image segmentation scheme can be embedded into high level processing techniques (like object classification and recognition) for better results. The Berkeley segmentation database [20] is used to experiment with our algorithm. Different evaluation measures like probabilistic rand (PR) index [21], global consistency error (GCE) [22], sensitivity, positive predictive value (PPV) and Dice similarity coefficient [23] are used to quantitatively evaluate the segmentation results.

2. Graph Development using Fuzzy Rule Based System

The input image is first transformed into a weighted undirected graph \( G(V,E) \) where \( V = \{v_1,v_2,v_3, \ldots ,v_m\times n \} \) is the set of nodes and the set of edges between nodes is represented by \( E \). The graph \( G(V,E) \) is internally represented by an affinity matrix or similarity matrix \( W \) that contains weights on all the edges in the graph. Different image features help in finding the degree of similarity between neighboring pixels to construct the similarity matrix \( W \). A vital step in the graph cuts based method is the optimum computation of similarity matrix as graph partitioning part totally counts on it. Local image features like brightness, color and texture can be modeled and formulated to approximate the likelihood of neighboring pixels to belong to a common segment. Different categories of images can have different types of local features in rich amount. For example, in gray scale images like medical images, the intensity characteristics cover most of the information and in natural images, the color feature is dominant. Choice of proper color space is an important factor while choosing among different color spaces like RGB, HSV and Lab etc. Other than gray level or color information, texture is considered to be an important feature that provides strong base to interpret images.

2.1 Image Feature Models

Now we define the image feature models using brightness, color and texture for development of similarity matrix.

2.1.1 Brightness

Considering the brightness feature of pixels and their spatial locations [13], intermediate brightness similarity matrix \( W_b \) can be calculated as:

\[
W_b(v_i,v_j) = \exp \left[ -\frac{\| I_{v_i} - I_{v_j} \|_2^2}{\alpha} - \frac{\| S_{v_i} - S_{v_j} \|_2^2}{\beta} \right],
\]

(1)

otherwise, \( W_b(v_i,v_j) = 0 \). In (1), \( \| I_{v_i} - I_{v_j} \|_2 \) and \( \| S_{v_i} - S_{v_j} \|_2 \) are the Euclidean distances in intensity and spatial domains respectively. \( I_{v_i} \) is the intensity value while \( S_{v_i} \) is the spatial location of node \( v_i \). \( \| \cdot \|_2 \) is the Euclidean norm. \( \alpha, \beta \in (0,\infty) \) are the free factors to adjust the gray level and position impact on calculation of the weights. \( \Re \) controls the influence of the number of local vertices taking part in weight calculation.

2.1.2 Color

For optimal segmentation based on color feature, the perceptual color differentiation and Euclidean distance in color space should be linked together [9]. Lab color space has been specifically designed to keenly approximate the human vision perception. \( L \) component, which is lightness, matches the human perception of brightness. \( a \) and \( b \) are the chromaticity coordinates. The intermediate color similarity matrix \( W_c \) using Lab color space can be calculated as:

\[
W_c(v_i,v_j) = \exp \left[ -\frac{\| Z_{v_i} - Z_{v_j} \|_2^2}{\alpha} - \frac{\| S_{v_i} - S_{v_j} \|_2^2}{\beta} \right],
\]

(2)

otherwise, \( W_c(v_i,v_j) = 0 \). Here \( Z_{v_i} = \{L(v_i), a(v_i), b(v_i)\} \) is the color feature vector for node \( v_i \).

2.1.3 Texture

In both image categories (natural color images and gray scale), texture holds a substantial information to analyze the image. Julesz [24] introduced the term texton. Textons are used widely to analyze the images. Windowed texton histograms are compared to calculate texture similarities. Window for pixel \( v_i \) is represented by \( J(v_i) \) centered at pixel \( v_i \). There are \( K \) number of bins per histogram, one for each texton channel. The number of pixels in texton channel \( k \) that fall inside the window \( J(v_i) \) is used to compute the value of
It can be formulated as:

\[ h_t(k) = \sum_{j \in J(v)} I[T(v_j) = k] \] (3)

where \( T(v_j) \) gives the texton assigned to pixel \( v_j \) and \( I[\cdot] \) is the indicator function. The pairwise difference between two histograms \( h_t \) and \( h_j \) at pixels \( v_t \) and \( v_j \) respectively is calculated as:

\[ \chi^2(h_t, h_j) = \frac{1}{2} \sum_{k=1}^{K} \frac{[h_t(k) - h_j(k)]^2}{h_t(k) + h_j(k)} \] (4)

where \( h_t \) and \( h_j \) are the two histograms. Now the intermediate texture similarity matrix \( W_t \) can be calculated as:

\[ W_t(v_i, v_j) = \exp \left[ -\chi^2(h_t, h_j)/\gamma \right] \] (5)

where \( \gamma \in (0, \infty) \) is a free factor adjusting the effect of texture on calculation of the weights. If the histograms \( h_t \) and \( h_j \) are very different, weight \( W_t(v_i, v_j) \) is small due to large \( \chi^2 \).

### 2.2 Weighted Average of Image Features

The above feature models provide the intermediate similarity matrices estimating the similarity between neighboring pixels. Each model estimates some dominant feature information about the nature of image. This can be done by taking mean value of the intermediate similarity matrices. The mean value will be the closer to zero the higher the involvement of that specific feature. These values are the inputs of fuzzifier where crisp values are transformed into fuzzy values. Gaussian fuzzifier (GF) [19] is used as it has advantage over other fuzzifiers in terms of accuracy and efficiency. Membership function (MF) of GF to map \( x^a \) to fuzzy set, can be represented mathematically as follows:

\[ \mu_F(x) = e^{-\left(\frac{x-x_1}{\sigma_1}\right)^2 \ast \ldots \ast e^{-\left(\frac{x-x_N}{\sigma_N}\right)^2}} \] (7)

where \( \sigma_i \) are the positive parameters, \( N \) represents number of linguistic terms used and \( \ast \)-norm \( \ast \) is usually selected as algebraic product. A collection of linguistic values (i.e., \( H: \) high, \( M: \) medium, \( L: \) low and \( S_{c_i}: \) small, \( MS_{c_i}: \) medium small, \( M_{c_i}: \) medium, \( ML_{c_i}: \) medium large, \( L_{c_i}: \) large) is given to each of the input and output variables. The input and output Gaussian MFs are shown in Fig. 2.

![Fig. 1. Similarity or weight matrix calculation.](image)

The intermediate similarity matrices \( W_b, W_c \) and \( W_t \) can be analyzed to gauge which feature would provide more information about the nature of image. This can be done by taking mean value of the intermediate similarity matrices. The mean value will be the closer to zero the higher the involvement of that specific feature. These values are the inputs of fuzzifier where crisp values are transformed into fuzzy values. Gaussian fuzzifier (GF) [19] is used as it has advantage over other fuzzifiers in terms of accuracy and efficiency. Membership function (MF) of GF to map \( x^a \) to fuzzy set, can be represented mathematically as follows:

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![Fig. 2. (a), (b) and (c) Gaussian MFs of linguistic values for inputs. (d) Gaussian MF of linguistic values for output.](image)

The calculation of constants \( c_p \), \( c_t \) and \( c_c \) depends on the involvement of brightness, color and texture in the image accordingly. This knowledge is imprecise since it does not state how much these different image features are involved in the image and what should be the values of constants. In this case, a fuzzy rule based system can be viewed for approximation. The fuzzy rule based system is interpreted in linguistic terms (i.e., \( small, medium, large \)) and provides high interpretability as well as high accuracy. The process of feature extraction, constants calculation using fuzzy rule based system and weighted averaging is shown in Fig. 1.
are nine cells generated by three antecedent linguistic values (i.e., \( H: \) high, \( M: \) medium, \( L: \) low) on each axis of two dimensional space. Each of the nine cells correspond to nine fuzzy IF-THEN rules. Fig. 3 shows the decision table and fuzzy IF-THEN rules. For certain input, any of the nine rules can be fired. If multiple rules are fired, inference engine will look after the processing. In the inference engine, all rules to be fired are combined first to get a single fuzzy relation, viewed as a single fuzzy IF-THEN rule which is then used to get the output. Accepting the point of view that all rules are independent conditional statements, Mamdani inference engine is used to combine the rules. Mathematically:

\[
Q_M = \bigcup_{r=1}^{M} R_r^{(a)}
\]

where \( Q_M \) is the resultant fuzzy relation, \( M \) is the number of rules to be fired.

The fuzzy output from inference engine is transformed back into crisp value using center average defuzzifier [19]. As fuzzy output set is union of \( M \) fuzzy sets, a better approximation is weighted average of centers of \( M \) fuzzy sets with the weights \( w_r \) equal to heights of corresponding fuzzy sets. It can be written as:

\[
c^*_t = \frac{\sum_{r=1}^{M} c^*_t w_r}{\sum_{r=1}^{M} w_r}
\]

where \( c^*_t \) is the final crisp output. It is computationally simple and small changes in \( c^*_t \) and \( w_r \) result in small change in \( c^*_t \).

3. **Graph Partitioning using Normalized Graph Cuts Framework**

Finalized similarity matrix \( W \) through weighted average of intermediate similarity matrices \( W_b, W_r \) and \( W_t \) is used to build the required graph \( G(V,E) \). This graph is then partitioned using normalized graph cuts framework [12],[13]. The normalized graph cuts framework [12],[13]

\[
(D - W)y = \lambda Dy
\]

where \( D \) is the diagonal matrix calculated as \( D(v_i, v_j) = \sum_j w(v_i, v_j) \) and \( y = \{a,b\}^{n \times n} \) is an indicator vector to indicate the identity of pixels towards their group. Here \( y_{vi} = a \) if \( v_i \in V_1 \) and \( y_{vj} = b \) if \( v_j \in V_2 \). \( \lambda \) represents the eigenvalues which give eigenvectors to partition the graph. Using the second smallest eigenvector, an optimized partition is calculated and graph is partitioned into two parts. This process of bi-partitioning the graph, continues iteratively if segmented graph needs to be subdivided. From these graph partitions, the segmented image is obtained.

The proposed method of graph development through weighted average of different image features using fuzzy rule based system and graph partitioning through normalized graph cuts framework can be summarized as follows:

1. Given an image, extract the image features using different feature models as follows:
   (a) Use the image intensity profile to calculate \( W_b \) from (1).
   (b) Use Lab color model to find \( W_c \) from (2).
   (c) Use the texton based texture analysis method to compute \( W_t \) from (4) and (5).

2. Calculate the constants \( c_b, c_c \) and \( c_t \) using fuzzy rule based system as follows:
   (a) Transform the crisp inputs into fuzzy inputs using GF with three linguistic terms from (7).
   (b) Apply the fuzzy IF-THEN rules to get the output.
   (c) Combine the rules to be fired using Mamdani inference engine from (8) to get a single fuzzy relation.
   (d) Transform the fuzzy output into crisp output using center average defuzzifier from (9).
   (e) Get the constants \( c_b, c_c \) and \( c_t \).

3. Calculate the weighted average of \( W_b, W_c \) and \( W_t \) to obtain final similarity matrix \( W \) from (6).

4. Partition the graph \( G(V,E) \) using the normalized graph cuts framework as follows:
   (a) Use the similarity matrix \( W \) to find the diagonal matrix \( D \) [13].
   (b) To get eigenvectors with smallest eigenvalues, solve the generalized eigenvector system using (10).
   (c) Bi-partition the graph using eigenvector with second smallest eigenvector.
(d) Decide if current partition should be subdivided. If yes, repartition the segments iteratively to get final result.

5. Get segmented image from the partitioned graph.

4. Quantitative Evaluation

Recently, the development of new image segmentation techniques has inspired the requirement of evaluating these methodologies. From one individual to another, image perception is inconsistent resulting in multiple solutions so we need some techniques that compare the results with different manual segmentations (ground-truth). PR index [21], GCE [22], sensitivity, PPV and Dice similarity coefficient [23] methods are used to evaluate our segmentation algorithm. These methods compare the segmentation results with ground-truth images and provide a measure of similarity/dissimilarity to evaluate the segmentation algorithm in an effective way.

4.1 PR Index

PR index is a generalization to rand index. Rand index [25] measures the agreement of a segmentation result with a given ground-truth. It compares the two segmentations (i.e., segmentation result and ground-truth) by considering pairwise label relationship. Let the two sets be $S$ and $S'$ with label assignment $l_i$ and $l'_i$ respectively for $N$ points $X = x_i$, where $i = 1, 2, \ldots, N$. Rand index is defined as ratio of number of pixel pairs having same label relationship in $S$ and $S'$. It can be represented as:

$$R(S, S') = \frac{2}{N(N-1)} \sum_{i<j} [I(l_i = l'_j, l_i \neq l'_j) + I(l_i \neq l'_j, l_i = l'_j)]$$

where $i \neq j$ and $I$ is the identity function while the denominator represents all possible unique pixel pairs in a data set of $N$ points. The number of unique labels can be different and same (special case) in $S$ and $S'$. This measure varies from 0 to 1, where 1 represents that $S$ and $S'$ are identical and 0 represents the total dissimilarity. Let us denote a set of ground-truth segmentations $\{S_1, S_2, \ldots, S_K\}$ of an image $X = \{x_1, x_2, \ldots, x_N\}$, where $N$ is total number of pixels. Let $S_{test}$ be the segmentation result to be compared with ground-truth segments. Let $l_i$ be the label of point $x_i$ in segmentation $S_{test}$ and $l'_i$ in the ground-truth segmentations $S_k$. Each label $l'_i$ and $l'_i$ can take values in a discrete set of size $L_k$ and $L_{test}$ respectively. In a scenario where each human observer gives the segmentation $S_k$ in form of binary numbers $I(l'_i = l'_j)$ for each pair of pixels $i$ and $j$. The set of ground-truth segmentations defines a Bernoulli distribution over this number giving a random variable with expected value $p_{ij}$. The PR index is then defined as:

$$PR(S_{test}, \{S_k\}) = \frac{2}{N(N-1)} \sum_{i,j} [c_{ij}p_{ij} + v(1-c_{ij})(1-p_{ij})]$$

(12)

where $i < j$ and $c_{ij} = I(l'_i = l'_j)$ represents the event of pixels $i$ and $j$ having same labels in $S_{test}$ and $p_{ij} = \sum_k L_k - 1 I(l'_i = l'_j)$ is the probability of $i$ and $j$ having the same label across $S_k$. This measure varies from 0 to 1, where 1 represents maximum similarity between $S_{test}$ and ground-truth segmentations and 0 represents no similarity. Since $c_{ij} \in \{0, 1\}$, equation (12) can be written as:

$$PR(S_{test}, \{S_k\}) = \frac{2}{N(N-1)} \sum_{i,j} \left[p_{ij}^c(1 - p_{ij})^{1-c_{ij}}\right]$$

(13)

where $i < j$. In (13), $p_{ij}^c(1 - p_{ij})^{1-c_{ij}}$ represents the likelihood of pixels $i$ and $j$ taking values $l'_i$ and $l'_j$ under the defined Bernoulli distribution. The computational complexity of PR index is $O(KN + \sum_k L_k)$ which is only linear in $N$.

4.2 GCE

This evaluation measure [22] is related to consistency among segmentations. GCE is designed to be tolerant to refinement. Let $S$ and $S'$ be the two segmentations to be evaluated through consistency error measure. Error measure lies in the range 0 to 1, where 0 represents no error. Segments $S$ and $S'$ are considered for a given pixel $p_i$. The pixel lies in the area of refinement if one of the segments is a proper subset of the other. Otherwise, two regions overlap in an inconsistent way and corresponding error is calculated. Set difference is denoted by $n$ and $|x|$ for the cardinality of set $x$. For $R(S, p_i)$ being a set of pixels that correspond to a region in segmentation $S$ containing pixel $p_i$, the local refinement error is given as:

$$E(S, S', p_i) = \frac{|R(S, p_i) \cap R(S', p_i)|}{|R(S, p_i)|}.$$  

(14)

This local error measure is asymmetric. It encodes only one directional refinement measure. $E(S, S', p_i)$ is approximately zero when $S$ is a refinement of $S'$ but not vice versa. This local refinement can be considered in each direction. The method used to combine the values for the entire image into an error measure is called GCE which forces all the local refinements to be in the same direction. GCE is defined as:

$$GCE(S, S') = \frac{1}{N} \min \left\{ \sum_i E(S, S', p_i) \right\}$$

(15)

where $N$ is the total number of pixels.

4.3 Sensitivity, PPV and Dice Similarity Coefficient

The segments obtained after segmentation process can be validated and evaluated by comparing them to manual segmentation or ground-truth using binary classifiers. Sensitivity $S$ and PPV are the measures used to evaluate the
segmentation results by calculating number of true positive $TP_n$, false positive $FP_n$ and false negative $FN_n$ voxels. Using these voxels, $S$ and PPV is given as:

$$S = \frac{TP_n}{TP_n + FN_n},$$

(16)

$$PPV = \frac{TP_n}{TP_n + FP_n}.$$  

(17)

Dice similarity coefficient $D$ [23] compares the two sets, in our case one set is a segment from test image and other from ground-truth or manual segmentation. Let these two sets be $A$ and $B$, then $D$ is calculated as:

$$D = \frac{2|A \cap B|}{|A| + |B|}$$

(18)

where $|\cdot|$ represents the function that provides area of the segment.

5. Results and Discussion

A variety of gray scale and natural color images are used to test the performance of proposed algorithm. The Berkeley segmentation database [20] with 500 images of size 481 x 321 and their ground-truth or manual segmentations is used to test our algorithm. Simulations are performed using C programming language and Matlab. To compute the intermediate weight matrices, Euclidean distance in spatial domain or connection radius of graph is taken as $\sqrt{2}$ or $\Re = \sqrt{2}$. This indicates that we are considering only immediate neighbor pixels to calculate the graph due to their major contribution in describing similarity/dissimilarity between pixels. Parameters $\alpha$, $\beta$ and $\gamma$ are taken as unity.

Fig. 4(a)-(f) shows the test images used to experiment on the proposed algorithm. The segmentation results through proposed algorithm are shown in Fig. 4(g)-(l). For comparison, segmentation results using generic normalized graph cuts which uses individual image feature for graph development (i.e., brightness $W_b$ and color $W_c$ for gray scale and color images accordingly), are shown in Fig. 4(m)-(r). We can observe that the results are not impressive as there is some involvement of texture, not significant but ignored. In same manner, if we only take $W_t$ texture feature and ignore brightness $W_b$ or color $W_c$ which holds major image information, segmentation results are not even considerable as shown in Fig. 4(s)-(x). Fig. 5 shows some more seg-
Fig. 5. Image segmentation results through proposed method of weighted average of different image features using fuzzy rule based system and graph partitioning using graph cuts.
PR index [21], GCE [22], sensitivity, PPV and Dice coefficient [23] are used for quantitative evaluation of proposed algorithm. These methods use the ground-truth segmentations to evaluate the segmentation results. Fig. 7 shows ground-truth segmentations for images depicted in Fig. 4(d) and Fig. 4(e). PR index and GCE distribution for proposed method and its comparison with generalized normalized cuts method using individual image feature (i.e., color/brightness for color/gray level images and texture for texture images) are shown in Fig. 6(a)-(b).

Here we can see that through PR index most of the image percentage for proposed method is close to 0.8 and 0.9 index value which shows good segmentation results. Most of the GCE distribution for the proposed method is close to zero which indicates very low error. PR index and GCE distribution is not up to the mark specially in case that only texture feature is used whereas PR distribution is good but not impressive in case of color/brightness feature. PR index and GCE is calculated for images shown in Fig. 4(a)-(f) and comparison of proposed method and normalized cuts is presented in Fig. 6(c)-(d).

The sensitivity, PPV and Dice coefficient [23] for every segment is calculated for image segmentation results shown in Fig. 5. The segmented images are labeled in numbers as shown in Fig. 8. The mean value along with the standard deviation (Mean±Std) for S, PPV and D is calculated for every segment in the segmented image using multiple manual seg-

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**Fig. 6.** (a)-(b) PR index and GCE distribution of proposed method and generalized normalized cuts method [12], [13]. (c)-(d) PR index and GCE measure for images depicted in Fig. 4(a)-(f) using proposed method and normalized cuts method [12], [13].

**Fig. 7.** Ground-truth segmentations for the images shown in Fig. 4(d) and Fig. 4(e).
<table>
<thead>
<tr>
<th>Segmented Image</th>
<th>Evaluation Measure</th>
<th>Seg # 1</th>
<th>Seg # 2</th>
<th>Seg # 3</th>
<th>Seg # 4</th>
<th>Seg # 5</th>
<th>Seg # 6</th>
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<td>1</td>
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<td></td>
<td>D</td>
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Fig. 8. Evaluation of image segmentation results using sensitivity, positive predictive value and dice coefficient for proposed method.
mentations as shown in Fig. 8. We can see that most of the evaluation measures range between 0.85 and 0.95 for S, PPV and D which shows a good segmentation.

6. Conclusion

In this work a new approach for image segmentation of gray scale, color and texture images using normalized graph cuts framework is proposed. A graph representing the measure of likelihood between surrounding pixels is constructed using brightness, color and texture profiles of image at the same time (i.e., brightness and texture for gray scale images and color and texture for color images). A fuzzy rule based system provides the information how much a specific feature is involved in image based on the nature of image. This graph is further used in normalized graph cuts framework to recursively partition the graph. The graph partitions obtained are then used to get the segmented image. Proposed algorithm is tested on the Berkeley segmentation database and is evaluated through probabilistic rand index, global consistency error, sensitivity, positive predictive value and Dice coefficient. Present method provides effective results for most types of images.

References