

# A New Low Complexity Uniform Filter Bank Based on the Improved Coefficient Decimation Method

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**Abstract.** *In this paper, we propose a new uniform filter bank (FB) based on the improved coefficient decimation method (ICDM). In the proposed FB's design, the ICDM is used to obtain different multi-band frequency responses using a single lowpass prototype filter. The desired subbands are individually obtained from these multi-band frequency responses by using low order frequency response masking filters and their corresponding ICDM output frequency responses. We show that the proposed FB is a very low complexity alternative to the other FBs in literature, especially the widely used discrete Fourier transform based FB (DFTFB) and the CDM based FB (CDFB). The proposed FB can have a higher number of subbands with twice the center frequency resolution when compared with the CDFB and DFTFB. Design example and implementation results show that our FB achieves 86.59% and 58.84% reductions in resource utilizations and 76.95% and 47.09% reductions in power consumptions when compared with the DFTFB and CDFB respectively.*

## Keywords

FIR filter bank, flexibility, improved coefficient decimation method, low complexity.

## 1. Introduction

Finite impulse response (FIR) filters and filter banks (FBs) are widely used in digital signal processing and are preferred over their infinite impulse response counterparts due to their inherent stability and linear phase characteristics. There are two basic types of FBs – analysis FBs and synthesis FBs [1]. An  $M$ -channel analysis FB is a set of analysis filters which splits an input signal into  $M$  subband signals. Similarly, an  $M$ -channel synthesis FB consists of  $M$  synthesis filters, which combine  $M$  signals (possibly from an analysis FB) into a reconstructed signal. Analysis and synthesis FBs are widely used in multirate signal processing applications such as subband coding and digital transmultiplexers [1]. Analysis FBs are employed in wireless communication base station receivers for channelization purposes, i.e., extraction of radio frequency channels

from the wideband input frequency range. In the wireless communication technologies such as cognitive radios (CRs), analysis FBs are used to perform two critical tasks – channelization, and spectrum sensing, wherein the presence and/or absence of radio channels in the input signals is to be detected [2], [3]. In FB based spectrum sensing, the wideband input frequency range is split into subbands using analysis FBs and the presence of signals in them is then detected using techniques such as energy detection. In [4], the use of FBs for spectrum sensing in CRs is studied, and it is shown that the discrete Fourier transform (DFT) based FB (DFTFB) can be used for efficient realization of an energy detector based spectrum sensing scheme. In resource constrained applications such as battery-powered mobile CR handsets, low complexity FB implementations are desired to ensure efficient utilization of the limited available resources. In our work presented in this paper, we have tried to address this research problem of obtaining low complexity analysis FBs for channelization and spectrum sensing in CRs.

The DFTFB [1] is widely used as an analysis FB for uniform channelization. The DFTFB comprises of a poly-phase lowpass prototype filter and the inverse DFT (IDFT) operation to obtain the desired uniform subbands. A major disadvantage of the DFTFB is that the locations of the subbands are fixed with a center frequency resolution of  $2\pi/M$  for an  $M$ -channel DFTFB. This results in a fixed channel stacking that limits the flexibility of the DFTFB. A modulated FB termed Goertzel filter bank (GFB) based on the Goertzel algorithm was proposed in [5] to overcome the fixed channel stacking problem of the DFTFB. But the GFB has a high implementation complexity due to the Goertzel algorithm used in it for DFT computation.

In [6], a coefficient decimation method (CDM) was proposed for obtaining low complexity and reconfigurable finite impulse response (FIR) filters with variable frequency responses, using a single lowpass modal (initial prototype) filter. Two coefficient decimation operations, one to obtain different multi-band frequency responses (called CDM-I) and another to obtain variable lowpass frequency responses (called CDM-II) were proposed. A CDM based FB (called CDFB) is proposed in [7], and is shown to be a low complexity alternative to the other FBs, especially the DFTFB. The CDFB employs the CDM, spectral subtraction, complementary frequency response

operation and frequency response masking filters to obtain the desired subbands [7]. In [8], a CDM-II based reconfigurable filter is used to realize an energy detector based serial spectrum sensing scheme for CRs. The spectrum sensing scheme in [8] shows a lower complexity than the DFTFB based spectrum sensing approach [4], but due to increased delay of the serial sensing scheme proposed in it, it is mainly applicable in scenarios where the channel distribution in the input signal is not changing rapidly.

We recently proposed a modified coefficient decimation method (MCDM) [9] to obtain reconfigurable FIR filters with enhanced frequency response flexibility and twice center frequency resolution when compared to the conventional CDM [6]. Two coefficient decimation operations can be performed using our MCDM, one to obtain different multi-band frequency responses (termed as MCDM-I) and another to obtain variable highpass frequency responses (termed as MCDM-II) [10]. Based on the combination of our MCDM-II and the conventional CDM-II (the combined method is termed as improved coefficient decimation method II, abbreviated as ICDM-II), we have proposed a new FB (termed as ICDM-II based FB) in [10] that can be used for uniform as well as non-uniform channelization applications. In the ICDM-II based FB, the desired subbands are obtained by the spectral subtraction of the resultant frequency responses after performing ICDM-II operations on the modal filter using different decimation factor values. The ICDM-II based FB has two constraints involved in its design [10]:

1. Least Common Multiple (LCM) constraint of modal filter order: The modal filter has to be designed such that its order is a multiple of the LCM of the distinct decimation factors involved. This ensures that the resultant filters after ICDM-II operations have integer valued group delays, which is a necessary condition for performing spectral subtraction. If the number of desired subbands increases, the LCM constraint would impose the requirement of a high order modal filter, as the required number of decimation factors and hence their LCM value will also be large in that case. This can significantly increase the complexity of the ICDM-II based FB.
2. Transition band width (TBW) constraint: In the ICDM-II operations, if the decimation factor is  $M$ , the TBW of the lowpass or the highpass filter obtained after coefficient decimation becomes  $M$  times that of the modal filter. Therefore, in the ICDM-II based FB, the modal filter has to be designed with a considerably narrower TBW and consequently with an increased filter order such that all the subbands obtained using ICDM-II operations have their TBWs within the desired specifications. If the required decimation factor is large, it will result in the requirement of a significantly high order modal filter. This will increase the implementation complexity of the ICDM-II based FB, resulting in high hardware resource utilization and power consumption.

In this paper, we propose a new uniform FB which employs the combination of our MCDM-I and the conventional CDM-I (the combined method is termed as improved coefficient decimation method I, abbreviated as ICDM-I) to obtain the desired subbands. In the proposed FB design technique, low order wide-TBW frequency response masking filters [11] are used to individually extract the desired subbands from the multi-band frequency responses obtained after performing ICDM-I operations on the modal filter. As neither spectral subtraction nor ICDM-II operations are required to be performed, the LCM and TBW constraints are not present in the proposed FB design technique. It can be noted that ICDM-II can be used to design uniform as well as non-uniform FBs [10]. The ICDM-II based FB design technique in [10] is an efficient method to obtain low complexity non-uniform FBs. But to obtain a uniform FB, the ICDM-I based FB design technique proposed in this paper will be more efficient than the ICDM-II based FB design technique in [10] due to the absence of the LCM and TBW constraints in the former. Thus the proposed FB will have a lower complexity than the ICDM-II based FB [10].

The proposed FB is henceforth termed as ICDM-I based FB in this paper. We show that our ICDM-I based FB is a very low complexity alternative to the other uniform FBs in literature, and shows a significantly higher flexibility than other FBs in terms of the possible number and locations of its constituent subbands. The rest of the paper is organized as follows: Section 2 presents the mathematical formulation and design procedure of the proposed ICDM-I based FB. Section 3 presents a design example, implementation results and comparisons of the proposed ICDM-I based FB with other FBs. Section 4 has our conclusions.

## 2. Proposed ICDM-I based Filter Bank

### 2.1 Mathematical Formulation

In the conventional CDM [6], if the coefficients of a lowpass modal filter are decimated by  $M$ , i.e., if every  $M^{\text{th}}$  coefficient is retained and the others are replaced by zeros, an FIR filter with a multi-band uniform subband bandwidth (BW) frequency response is obtained. The center frequency locations of the subbands in the resultant frequency response are given by  $2\pi k/M$ , where  $k$  is an integer ranging from 0 to  $(M - 1)$ . If  $H(e^{j\omega})$  denotes the Fourier transform of the modal filter coefficients, then the Fourier transform of the resulting filter's coefficients is given by

$$H'(e^{j\omega}) = \frac{1}{M} \sum_{k=0}^{M-1} H(e^{j(\omega - \frac{2\pi k}{M})}). \quad (1)$$

This operation is called CDM-I and its mathematical derivation is given in [7]. After performing CDM-I by decimation factor  $M$ , if all the retained coefficients in the resultant

filter are grouped together by discarding the intermittent zeros, a lowpass frequency response is obtained with its passband and transition band widths  $M$  times that of the modal filter. This operation is called CDM-II [7].

In the new coefficient decimation operation proposed by us in [9], if the coefficients of the modal filter are decimated by a factor  $M$ , every  $M^{\text{th}}$  coefficient is retained and the sign of every alternate retained coefficient is reversed. All other filter coefficients are replaced by zeros. This operation gives an FIR filter with a multi-band uniform subband BW frequency response with the center frequency locations of the subbands given by  $(2k + 1)\pi/M$ , where  $k$  is an integer ranging from 0 to  $(M - 1)$ . If  $H(e^{j\omega})$  denotes the Fourier transform of the modal filter coefficients, then the Fourier transform of the resulting filter's coefficients is given by

$$H'(e^{j\omega}) = \frac{1}{M} \sum_{k=0}^{M-1} H\left(e^{j\left(\omega - \frac{\pi(2k+1)}{M}\right)}\right). \quad (2)$$

The mathematical derivation of this operation which was shown in our preliminary work [9] is given below for completion.

Let the modal filter coefficients be denoted by  $h(n)$ , and the modified coefficients be denoted by  $h'(n)$ . Let

$$h'(n) = h(n)d_M(n) \quad \text{for } n = 0, 1, 2, \dots \quad (3)$$

where  $d_M(n)$  denotes a function that performs the operation of retaining every  $M^{\text{th}}$  filter coefficient and performing the appropriate sign changes. It can be represented as

$$\begin{aligned} d_M(n) &= 1 & \text{for } n = mM, m = 0, 2, 4, 6, \dots \\ &= -1 & \text{for } n = pM, p = 1, 3, 5, 7, \dots \\ &= 0 & \text{otherwise} \end{aligned} \quad (4)$$

From (4), it can be noted that  $d_M(n)$  is a periodic function with a period  $2M$ . Its Fourier series expansion is given by

$$d_M(n) = \frac{1}{2M} \sum_{k=0}^{2M-1} D(k) e^{j\frac{2\pi kn}{2M}} \quad (5)$$

where  $D(k)$  represents complex Fourier series coefficients which are given by

$$D(k) = \sum_{n=0}^{2M-1} d_M(n) e^{-j\frac{2\pi kn}{2M}}. \quad (6)$$

From (4) and (6), we can derive that  $D(k) = 0, 2, 0, 2, 0, 2, \dots$  for any value of  $M$ , for  $k = 0$  to  $(2M - 1)$ . Using this observation and substituting for  $D(k)$ , we can rewrite (5) as

$$d_M(n) = \frac{1}{M} \sum_{k=0}^{M-1} e^{j\frac{\pi(2k+1)n}{M}} \quad (7)$$

The Fourier transform of the modified coefficients is

$$H'(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h'(n) e^{-j\omega n} \quad (8)$$

Substituting (3) and (7) in (8), we get

$$H'(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n) \left( \frac{1}{M} \sum_{k=0}^{M-1} e^{j\frac{\pi(2k+1)n}{M}} \right) e^{-j\omega n} \quad (9)$$

By interchanging the summations in (9), we get

$$\begin{aligned} H'(e^{j\omega}) &= \frac{1}{M} \sum_{k=0}^{M-1} \sum_{n=-\infty}^{\infty} h(n) e^{-jn\left(\omega - \frac{\pi(2k+1)}{M}\right)} \\ \therefore H'(e^{j\omega}) &= \frac{1}{M} \sum_{k=0}^{M-1} H\left(e^{j\left(\omega - \frac{\pi(2k+1)}{M}\right)}\right) \end{aligned} \quad (10)$$

Thus, the frequency response of the modified coefficients is scaled by  $M$  and the original frequency spectrum is replicated at the locations  $(2k + 1)\pi/M$ , where  $k = 0$  to  $(M - 1)$ . This operation is termed as MCDM-I [10]. After performing MCDM-I by a decimation factor  $M$ , if all the retained coefficients in the resultant filter are grouped together by discarding the intermittent zeros, a highpass frequency response is obtained with its passband and transition band widths  $M$  times that of the modal filter. This operation is termed as MCDM-II [10].

In all the coefficient decimation operations, the stopband attenuation (SA) of the resulting filters deteriorates as the value of  $M$  increases [6], [9]. This deterioration in SA is mathematically given by

$$\delta_{s(\text{modal})} = \frac{\delta_{s(\text{final})}}{M} \quad (11)$$

where  $\delta_{s(\text{modal})}$  is the SA of the modal filter and  $\delta_{s(\text{final})}$  is the SA of the filter obtained after performing a coefficient decimation operation by  $M$  [12]. The SA deterioration problem can be addressed by overdesigning the modal filter. If  $f_p$  and  $f_s$  are the desired passband and stopband edge frequencies (normalized with respect to half of the sampling frequency),  $\delta_p$  and  $\delta_s$  are the desired passband and stopband peak ripple specifications, then the order of the desired FIR filter ( $N$ ) can be obtained using [13]

$$N = -\frac{4 \log_{10}(10\delta_p\delta_s)}{3(f_s - f_p)} - 1. \quad (12)$$

Thus, from (11) and (12), if a filter is to be coefficient decimated by  $M$  and the SA of the resulting filter is to be kept within a desired value  $\delta_s$ , the minimum order of the overdesigned modal filter can be computed using

$$N = \left[ -\frac{4 \log_{10}(10\delta_p\delta_s)}{3(f_s - f_p)} - 1 \right] + \frac{4 \log_{10} M}{3(f_s - f_p)}. \quad (13)$$

The 2nd term on the right hand side of (13) is the increase in the order of the overdesigned modal filter required to compensate the SA deterioration after coefficient decimation by  $M$ . The mathematical formulation presented in this section using (1)-(13) forms the theoretical basis of the proposed ICDM-I based FB.

## 2.2 Design Procedure

The design procedure for obtaining the proposed ICDM-I based FB is given below.

**Part A:** Design and implementation of the modal filter and the corresponding ICDM-I operations -

*Step-1:* Fix the passband and stopband edge frequency specifications of the modal filter with respect to the normalized BWs of the desired subbands.

*Step-2:* Using (1) and (2), determine the smallest set of  $M$  values and the corresponding ICDM-I operations to be performed on the modal filter, for obtaining different frequency responses containing the desired subbands. Let the maximum coefficient decimation factor be  $M_{max}$ .

*Step-3:* Corresponding to  $M_{max}$ , compute the minimum order of the modal filter required to satisfy the desired SA specifications using (13). Fix an appropriate value of the filter order and obtain the modal filter coefficients.

*Step-4:* Perform appropriate ICDM-I operations on the modal filter with all the  $M$  values identified in *Step-2* and obtain the corresponding multi-band frequency responses.

**Part B:** Design and implementation of frequency response masking filters and the corresponding ICDM-I operations -

*Step-5:* The frequency response masking approach [11] involves the use of low order wide-TBW filters to realize low complexity sharp transition band FIR filters. In the proposed FB design method, the frequency response masking approach is used to extract the desired subbands from the multi-band frequency responses obtained after performing appropriate ICDM-I operations. According to the distribution of the subbands in the multi-band frequency responses obtained in *Step-4*, identify the minimum number of frequency response masking filters required to extract the desired subbands and fix their passband and stopband edge frequency specifications. (Note: The TBWs of the masking filters should be fixed to have largest possible values according to the edge frequencies of the desired subband and its adjacent subbands in the corresponding frequency responses. From (13), it can be noted that this results in lower filter order values for the masking filters, thus minimizing their complexity.)

*Step-6:* Using (1) and (2), determine the required  $M$  values and the corresponding ICDM-I operations to be performed on the masking filters to obtain the frequency responses that can be used for individually obtaining the desired subbands.

*Step-7:* Compute the masking filters' orders using (13) and obtain the corresponding filter coefficients.

*Step-8:* Perform appropriate ICDM-I operations on the masking filters with the  $M$  values identified in *Step-6* and obtain the corresponding frequency responses.

*Step-9:* From the multi-band frequency responses obtained in *Step-4*, extract the desired subbands using the designed

masking filters and their corresponding ICDM-I output frequency responses obtained in *Step-8*.

The different stages in the proposed ICDM-I based FB are summarized in the block diagram shown in Fig. 1. It can be noted from Fig. 1 that the proposed FB is based on the ICDM-I and frequency response masking techniques. The usage of the steps given in the design procedure is illustrated in a design example presented in Section 3.1.

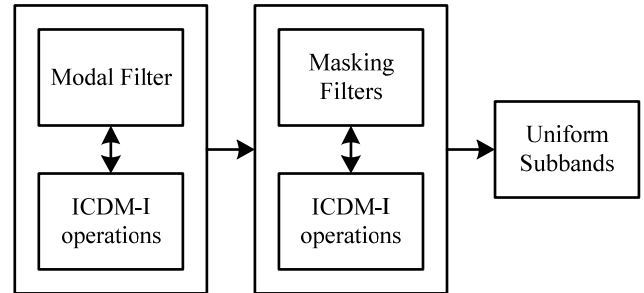


Fig. 1. Block diagram of proposed ICDM-I based FB.

## 3. Results and Analysis

### 3.1 Design Example

In this section, we present a design example of the proposed ICDM-I based FB and compare it with the other uniform FBs.

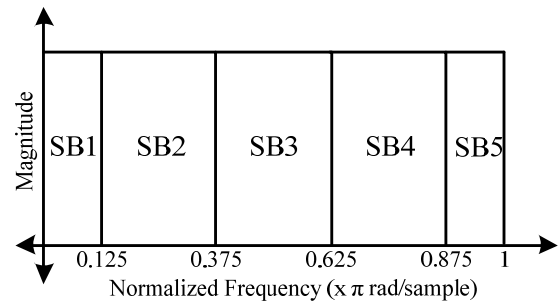


Fig. 2. Magnitude response of 8-channel DFTFB from 0 to  $f_{samp}/2$ .

Fig. 2 shows the ideal output magnitude response of an 8-channel DFTFB which contains five real subbands in the frequency range 0 to  $f_{samp}/2$ , where  $f_{samp}$  is the sampling frequency. In Fig. 2, the BWs of subbands SB1 and SB5 appear to be half of those of the other subbands because half of their BWs are located in the complex domain, i.e., the negative frequency range  $-f_{samp}/2$  to 0. Let the desired passband and stopband peak ripple specifications be 0.01 dB and -45 dB respectively. If  $f_p = 0.11$  and  $f_s = 0.14$  are the chosen passband and stopband edge frequency specifications corresponding to SB1 in Fig. 2, the order of the prototype filter required to realize the DFTFB computed using (12) is 155. The prototype filter in polyphase form is followed by an 8-point IDFT operation to obtain the five desired subbands.

We use the design procedure given in Section 2.2 to design the ICDM-I based FB for obtaining the five real subbands shown in Fig. 2, with the desired passband and stopband peak ripple specifications. The passband and stopband edge frequency specifications of the modal filter in the ICDM-I based FB are kept the same as those of the prototype filter in the DFTFB for fair comparison. Following *Step-2*, it can be noted from (1) and (2) that the frequency responses obtained by performing ICDM-I operations on the modal filter using  $M = 4$  are sufficient for obtaining all the five desired subbands. Thus  $M_{\max} = 4$ . Following *Step-3*, we choose the order of the overdesigned modal filter as 184 using (13) and obtain the filter coefficients. Fig. 3(a) shows the magnitude response of the modal filter. Following *Step-4*, ICDM-I operations are performed on the modal filter using  $M = 4$ . Fig. 3(b) and 3(c) show the corresponding output magnitude responses for CDM-I and MCDM-I respectively. The desired subbands SB1, SB3 and SB5 can be obtained from the magnitude response in Fig. 3(b), and the magnitude response in Fig. 3(c) can be used to obtain SB2 and SB4. The *Steps 5-9* in the design procedure that are performed to individually obtain the five desired subbands using masking filters are discussed below.

For the ICDM-I output magnitude responses shown in Fig. 3(b) and 3(c), two frequency response masking filters with their magnitude responses as shown in Fig. 3(d) and 3(e) are designed to extract the desired subbands. Let  $MF_1$  and  $MF_2$  denote the two masking filters. Following *Step-5*, the edge frequency specifications are chosen as  $f_p = 0.14$  (corresponding to the stopband edge frequency of SB1 in Fig. 3(b)) and  $f_s = 0.36$  (corresponding to the rising stopband edge frequency of SB3 in Fig. 3(b)) for  $MF_1$ , and  $f_p = 0.39$  (corresponding to the falling stopband edge frequency of SB2 in Fig. 3(c)) and  $f_s = 0.61$  (corresponding to the rising stopband edge frequency of SB4 in Fig. 3(c)) for  $MF_2$  respectively. The magnitude response of  $MF_1$  shown in Fig. 3(d) is used to extract SB1 from the magnitude response in Fig. 3(b). Similarly, the magnitude response of  $MF_2$  shown in Fig. 3(e) is used to extract SB2 from the magnitude response in Fig. 3(c). The magnitude responses obtained by performing MCDM-I on  $MF_1$  using  $M_1 = 1$  and  $M_2 = 2$  are shown in Fig. 3(f) and 3(g) respectively. These magnitude responses in Fig. 3(f) and 3(g) are used to extract SB3 and SB5 respectively, from the magnitude response shown in Fig. 3(b). The magnitude response obtained by performing MCDM-I on  $MF_2$  using  $M_1 = 1$  is shown in Fig. 3(h). This magnitude response in Fig. 3(h) is used to extract SB4 from the magnitude response in Fig. 3(c). The orders of  $MF_1$  and  $MF_2$  are computed as 23 and 21 respectively, using (13). The five desired subbands shown in Fig. 2 can thus be obtained using the proposed FB design technique. Using the appropriate frequency responses from those shown in Fig. 3, the corresponding steps that are performed to obtain each of the five desired subbands separately are summarized in the block diagram shown in Fig. 4. The magnitude responses of the obtained

subbands are also presented in Fig. 4. It can be noted from Fig. 4 that all the five obtained subbands satisfy the desired frequency response specifications (most importantly, stopband peak ripple = -45 dB) considered in this design example. Fig. 5 shows the impulse responses of the subbands that are obtained using the proposed method. Fig. 5(a) shows the impulses responses of subbands SB1 and SB5, whereas Fig. 5(b) shows the impulses responses of subbands SB2, SB3 and SB4, respectively.

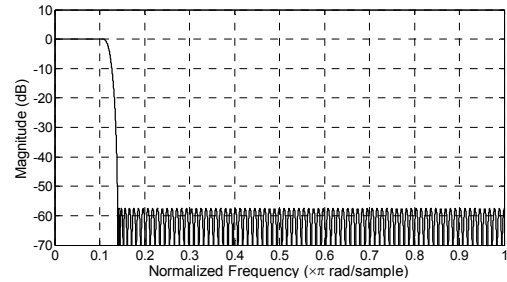


Fig. 3. (a) Magnitude response of the modal filter.

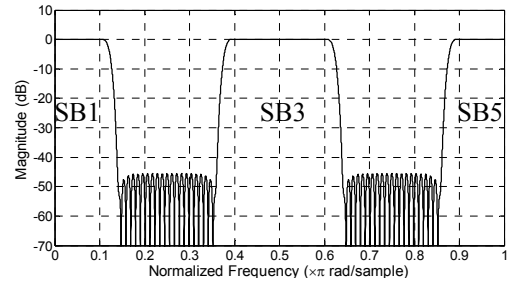


Fig. 3. (b) Magnitude response obtained by performing CDM-I on the modal filter using  $M = 4$ .

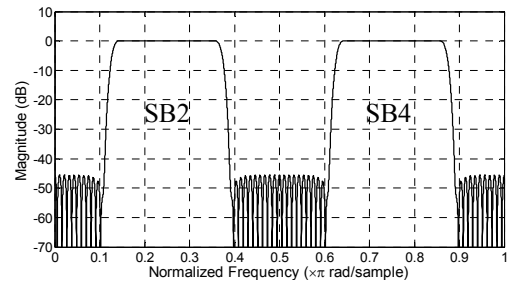


Fig. 3. (c) Magnitude response obtained by performing MCDM-I on the modal filter using  $M = 4$ .

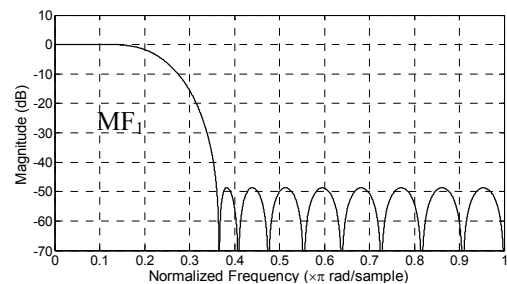


Fig. 3. (d) Magnitude response of the masking filter 1 ( $MF_1$ ).

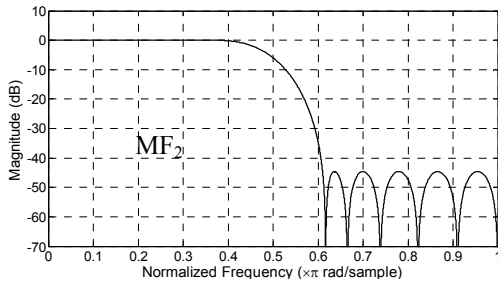


Fig. 3. (e) Magnitude response of the masking filter 2 ( $MF_2$ ).

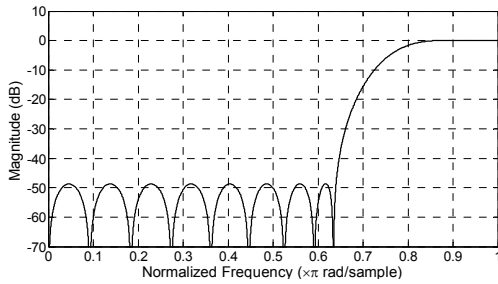


Fig. 3. (f) Magnitude response obtained by performing MCDM-I on  $MF_1$  using  $M = 1$ .

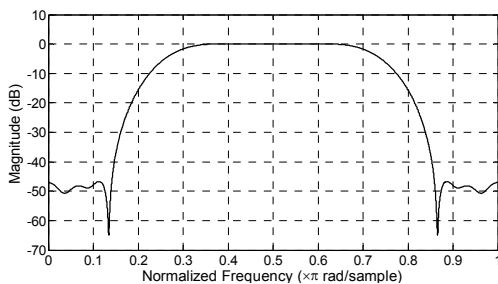


Fig. 3. (g) Magnitude response obtained by performing MCDM-I on  $MF_1$  using  $M = 2$ .

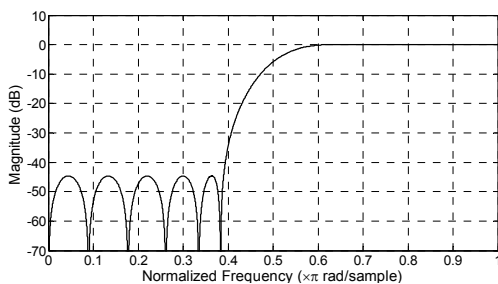


Fig. 3. (h) Magnitude response obtained by performing MCDM-I on  $MF_2$  using  $M = 1$ .

Using the design procedure in [7], a CDFB is designed for the same desired specifications. The passband and stopband edge frequency specifications of the modal filter in the CDFB are kept the same as those of the prototype filter in the DFTFB. The frequency response of the modal filter and the resultant frequency responses after performing CDM-I using  $M = 2$  and  $M = 4$  are obtained and appropriate spectral subtraction and complementary frequency response operations are performed on them to get the desired subbands [7]. Correspondingly, the order of the oversized modal filter is chosen as 184 using (13), for  $M_{\max} = 4$ . Two frequency response masking filters, each

of order 21 computed using (12), are also required in the CDFB to individually obtain the desired subbands SB2 and SB4 [7]. (Note that in this design example, the same modal filter is obtained in the CDFB and the ICDM-I based FB designs as the desired specifications and the  $M_{\max}$  values involved are coincidentally same in both the cases.)

### 3.2 Multiplication Complexity Comparison

In this section, we compare the complexity of the proposed ICDM-I based FB with other FBs. The complexity of a FB is mainly dependent on the number of multiplication operations involved in its implementation. Tab. 1 shows the number of real multiplications involved in the implementation of different FBs designed for the desired specifications discussed in Section 3.1.

	DFTFB [1]	GFB [5]	CDFB [7]	Proposed ICDM-I based FB
Prototype/modal Filter Length	156	156	185	185
Masking filter length	-	-	$2 \times 22 = 44$	$24 + 22 = 46$
Number of real multiplications for filter implementation	156	156	115	47
Number of real multiplications for DFT computation	$8 \log_2 8 = 24$	$8 \times 156 = 1248$	-	-
Total real multiplications involved in the FB	180	1404	115	47

Tab. 1. Multiplication Complexity Comparison.

The total number of real multiplications involved in the DFTFB implementation is the sum of the prototype filter length (filter order + 1) and the number of real multiplications required for an 8-point IDFT computation. We have used the radix-2 fast Fourier transform (FFT) algorithm for implementing the IDFT, and it requires  $S \log_2 S$  real multiplications for computing  $S$ -point FFT of a real input signal [14]. Thus, the number of real multiplications involved in the DFTFB is computed to be  $156 + 8 \log_2 8 = 156 + 24 = 180$ . The transposed direct-form FIR filter structure which exploits the symmetry property of filter coefficients cannot be used in the DFTFB due to the polyphase form implementation of the prototype filter. The GFB [5] shows a significantly higher multiplication complexity than the DFTFB due to the Goertzel algorithm used in it for DFT implementation. The modal filter and masking filters in the CDFB design are implemented using the transposed direct-form FIR filter structure, thus requiring only half of the total filter coefficients to be implemented [7]. Thus, the total number of real multiplications involved in the CDFB is  $\{\lceil 185 / 2 \rceil + (2 \times \lfloor 22 / 2 \rfloor)\} = 93 + 22 = 115$ . In the ICDM-I based FB design, all the desired subbands are obtained from the resultant frequency responses after performing ICDM-I operations on the modal filter using a single decimation factor  $M = 4$ . Thus, only those modal filter coefficients that are retained after performing ICDM-

I operations using  $M=4$  need to be implemented as all other coefficients are replaced by zeros. The retained filter coefficients are symmetric in nature and are implemented using the transposed direct-form FIR filter structure to exploit their symmetry property. The selective and efficient implementation of the modal filter coefficients in the proposed ICDM-I based FB significantly reduces the number of multiplication operations involved in it. Similar to the modal filter, the masking filters  $MF_1$  and  $MF_2$  are also implemented using the transposed direct-form FIR filter

structure using only those distinct coefficients that are retained after performing the corresponding ICDM-I operations. Thus, the total number of real multiplications involved in the implementation of the ICDM-I based FB is  $\{\lceil(185/4)\rceil/2 + \lceil 24/2 \rceil + \lceil 22/2 \rceil\} = \{24+12+11\} = 47$ .

From Tab. 1, it can be noted that the proposed ICDM-I based FB offers a multiplication complexity reduction of 73.89% over the DFTFB, 96.65% over the GFB and 59.13% over the CDFB.

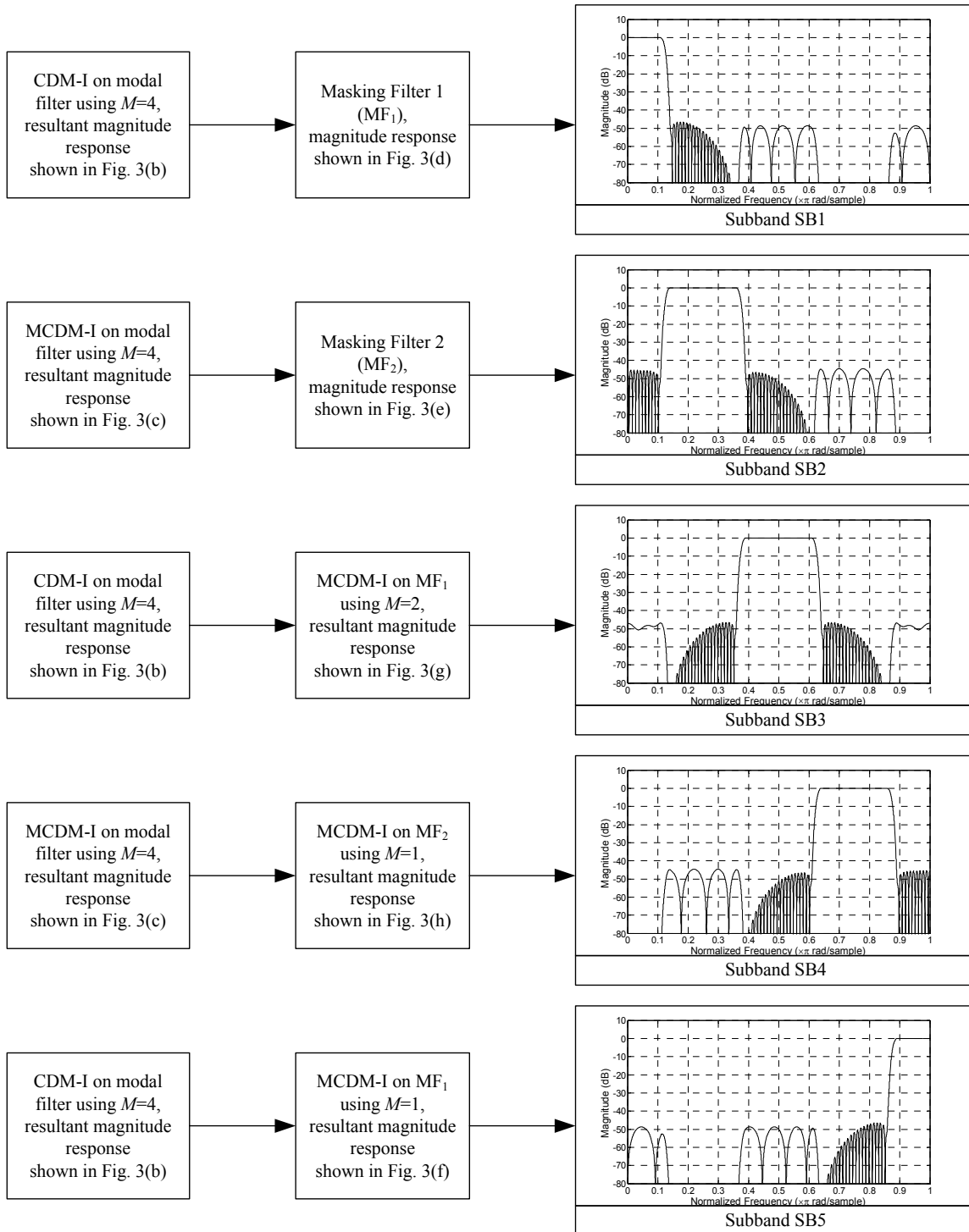


Fig. 4. Block diagram of proposed ICDM-I based FB designed in the design example.

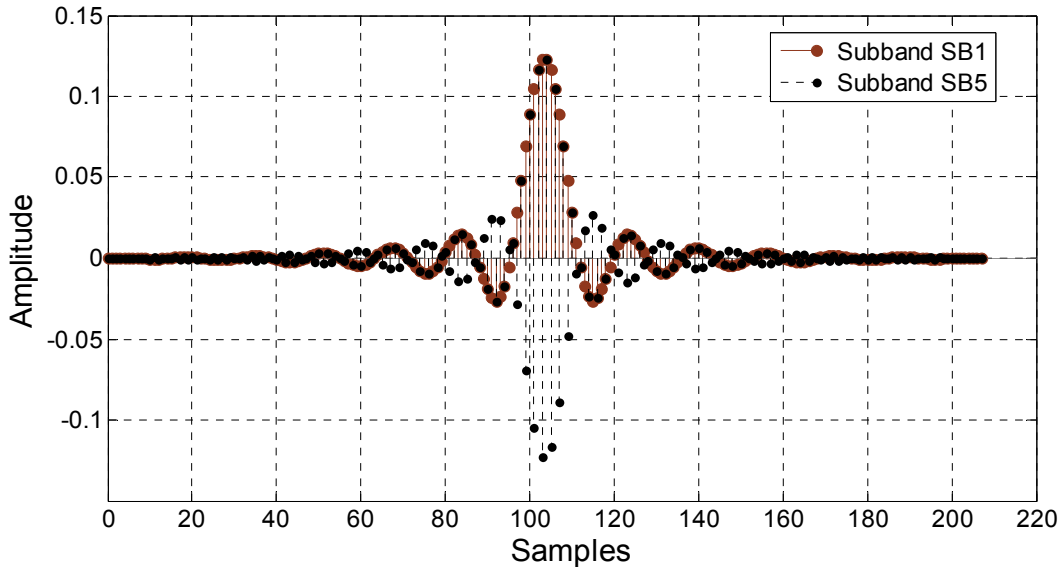


Fig. 5. (a) Impulse responses of subbands SB1 and SB5.

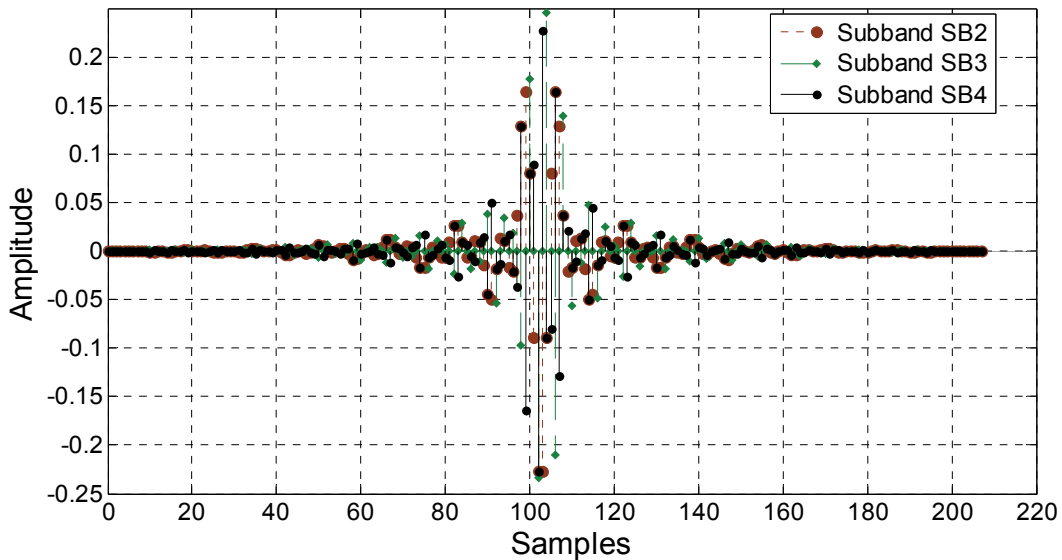


Fig. 5. (b) Impulse responses of subbands SB2, SB3 and SB4.

The multiplication complexity comparison presented in this section gives a theoretical estimate of the implementation complexities of the different FBs. The actual computational costs and resource utilizations of the different FBs are given in Section 3.4, which presents the implementation results of the different FB designs in a field programmable gate array (FPGA).

### 3.3 Flexibility Comparison

In this section, we compare the flexibility of the proposed ICDM-I based FB with other FBs in terms of the achievable number and locations of distinct subbands. From (1) and (2), it can be noted that in ICDM-I operations, the center frequency resolution in the resultant multi-band frequency responses is  $\pi/M$ , where  $M$  is the coefficient

decimation factor used. This resolution of  $\pi/M$  is obtained as locations that are both even (obtained using CDM-I) as well as odd (obtained using MCDM-I) multiples of  $\pi/M$  are achievable for the obtained center frequencies. Thus, the possible center frequency locations of subbands in the proposed ICDM-I based FB are at integral multiples of  $\pi/M$ , whereas the possible center frequency locations of subbands in the CDFB and the  $M$ -channel DFTFB are at integral multiples of  $2\pi/M$ . The proposed ICDM-I based FB thus has twice the center frequency resolution for the location of its subbands, when compared to the DFTFB and the CDFB. Also, unlike in the case of DFTFB where the value of  $M$  is fixed due to an  $M$ -point DFT implementation, different values of  $M$  corresponding to multiple decimation factors can be used in the ICDM-I based FB to obtain multiple sets of distinct center fre-



quency locations with a resolution of  $\pi/M$ . The increased center frequency resolution in the proposed ICDM-I based FB enables it to have a higher number distinctly located subbands when compared with the DFTFB and CDFB.

In the design example discussed in Section 3.1, the modal filter used in the ICDM-I based FB and CDFB designs has its normalized stopband edge frequency  $f_s = 0.14$ . Thus, values of  $M$  greater than  $\lceil(1/0.14)\rceil = 7$  will lead to aliasing and cannot be used in the coefficient decimation operations [6], [9]. From (1) and (2), it can be noted that center frequency locations obtained after performing MCDM-I using  $M = \{5, 6, 7\}$  cannot be obtained by performing CDM-I on this modal filter, as values of  $M = \{10, 12, 14\}$  are correspondingly required if CDM-I is to be performed. Thus, the use of both CDM-I as well as MCDM-I operations in the proposed ICDM-I based FB results in a significantly more number of possible distinct center frequency locations for its constituent subbands when compared with the CDFB, which merely employs the CDM-I operation. The proposed ICDM-I based FB thus has a significantly higher flexibility when compared to the DFTFB and CDFB in terms of the number of subbands that can be obtained as well as their locations.

### 3.4 Implementation Results

Tab. 2 shows the implementation results for the FB designs discussed in the design example in Section 3.1. The DFTFB, CDFB and the ICDM-I based FB designed for the desired specifications were implemented in the Xilinx xc5v1x330-1ff1760 FPGA. The GFB design was not implemented in the FPGA because of its significantly higher complexity due to the large number of multiplications involved in it, which is evident from Tab. 1.

From Tab. 2, it can be noted that the proposed ICDM-I based FB achieves 86.59% and 58.84% reductions in the number of occupied slices when compared with the DFTFB and CDFB respectively ('number of occupied slices' represents the amount of flip-flops, registers and lookup tables required to implement a FB design in the FPGA). As described in Section 3.2, the modal filter and the masking filters' coefficients are selectively implemented using the transposed direct-form FIR filter structure in the ICDM-I based FB. Thus, in spite of the use of masking filters and the overdesigned modal filter, the proposed ICDM-I based FB achieves a significant reduction in resource utilization over the DFTFB. Use of a lower number of distinct decimation factors in the coefficient decimation operations and the corresponding smaller number of computational blocks required result in a lower slice requirement for the ICDM-I based FB than the CDFB. The proposed ICDM-I based FB shows 76.95% and 47.09% reductions in power consumptions over the DFTFB and CDFB respectively. The DFTFB shows the highest power consumption because of the significantly larger number of slices utilized in it. The ICDM-I based FB has the least power consumption as it has the minimum resource utili-

zation. The timing results after the placement and routing (PAR) in the FPGA show that DFTFB is the fastest amongst the three FBs, which is expected because of the FFT technique used in it for the IDFT implementation. The proposed ICDM-I based FB has a 11.83% higher speed than the CDFB due to the lower number of multiplications involved in it, which results in a fewer number of total computational blocks required for its implementation.

	DFTFB [1]	CDFB [7]	Proposed ICDM-I based FB
No. of occupied slices	20184	6576	2707
Power (W)	0.512	0.223	0.118
Post-PAR minimum period (ns)	4.227	15.854	13.979

Tab. 2. Implementation results.

## 4. Conclusion

In this paper, we have proposed a new low complexity uniform filter bank (FB) based on the improved coefficient decimation method (ICDM), which is a combination of the conventional coefficient decimation method (CDM) and the modified coefficient decimation method (MCDM) recently proposed by us. The proposed ICDM-I based FB has twice the flexibility in terms of the possible number and locations of its subbands when compared with the DFTFB and CDFB. Design example shows that the proposed ICDM-I based FB offers a multiplication complexity reduction of 73.89% over the DFTFB, 96.65% over the GFB and 59.13% over the CDFB. Corresponding FPGA implementation results show that the proposed ICDM-I based FB offers 86.59% and 58.84% reductions in numbers of occupied slices, 76.95% and 47.09% reductions in power consumptions when compared with the DFTFB and CDFB respectively. The significant advantages in multiplication complexity, resource utilization and flexibility that are offered by the proposed ICDM-I based FB make it highly suitable for use in resource constrained applications such as portable cognitive radio handsets.

## Acknowledgements

A preliminary version of this paper has been presented at the 2012 35th International Conference on Telecommunications and Signal Processing (TSP) [9] wherein we proposed the basic idea of the modified coefficient decimation method. This paper extends the basic idea proposed in [9] to a new low complexity uniform filter bank which can be used in various digital signal processing applications.

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