Operational Trans-Resistance Amplifier Based Tunable Wave Active Filter

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Abstract. In this paper, Operational Trans-Resistance Amplifier (OTRA) based wave active filter structures are presented. They are flexible and modular, making them suitable to implement higher order filters. The passive resistors in the proposed circuit can be implemented using matched transistors, operating in linear region, making them fully integrable. They are insensitive to parasitic input capacitances and input resistances due to the internally grounded input terminals of OTRA. As an application, a doubly terminated third order Butterworth low pass filter has been implemented, by substituting OTRA based wave equivalents of passive elements. PSPICE simulations are given to verify the theoretical analysis.

Keywords
Wave active filter, OTRA, scattering parameters, ladder, tunable filter.

1. Introduction

There are many advantages of higher order filters using doubly terminated lossless ladders, like, low sensitivity to component tolerances, ample design information and design tables that can be readily applied [1]. However, inductor realization in an integrated circuit is a challenging task.

There are various techniques that circumvent these shortcomings like element replacement and operational simulation. If operational simulation is employed, Signal Flow Graphs are used to emulate the relationship between various passive elements. These are then physically realized using lossy and lossless integrators [1]. Realizing lossless integrators is difficult because of non-ideal characteristics of passive components used. Besides, floating capacitors are used in this topology, which are not very favorable in IC implementation. In the case of element replacement approach, inductors are replaced by gyrators. Although this practice leads to good results with low noise sensitivity, realizing high quality floating inductors proves to be difficult [2]. Another element replacement method using Frequency Dependent Negative Resistance (FDNR) was proposed by Bruton [1], and works well with low pass filters. LC ladder filters can also be emulated using Linear Transformation approach wherein every section of the original ladder prototype can be realized using active elements individually [3]. One drawback of this method is that it uses lossless integrators.

Apart from these approaches, the wave method [2] is also used for realizing higher order resistively terminated LC ladder filter which gives excellent results. It uses wave equivalents for different passive elements which can be readily substituted to realize a filter. In this approach, the filter realization is based on modeling the forward and reflected voltage waves. The available wave active filters [2], [4]-[10] use various active blocks such as OTA [4], current amplifier [5], CMOS cascode current mirrors [6], FPAA [7], OPAMP [2], [8], CFOA [9], and DVCCCTA [10] and operate in current [3]-[7] and voltage [2], [8]-[10] mode.

This paper presents design approach for realization of OTRA based higher order wave filter. OTRA being a current mode building block does not suffer from low slew rate and fixed gain bandwidth product [11] unlike conventional voltage mode op-amps. Additionally it has a unique feature of low impedance voltage output and is also free from the effects of parasitic capacitances and resistances at the input due to internally grounded input terminals [12], [13].

OTRA also allows the implementation of linear MOS based resistors [13], which is a huge advantage when going for IC fabrication. This property is also exploited to make the filters tunable. Although a number of tunable ladder circuits based on current-mode approach have been reported in open literature [14]-[16], they do not provide voltage output. Some of the features of the proposed work are:

- Modular structures which can be easily substituted to LC-ladder filter circuits. Provides an easy ‘ready to use’ method to realize ladders.
- Only lossy integrators are employed, which are easy to realize, and since OTRA inputs are virtually grounded, it is free from effect of parasitic elements.
It doesn’t employ passive resistors; instead it uses linear MOSFET based resistors which are voltage controlled thus making circuits electronically tunable. Section 2 elaborates on the concept of wave filter. Section 3 elaborates on how OTRA can be employed for this application. Simulation results for a third order Butterworth filter are shown in Section 4 and Section 5 concludes the paper.

2. Wave Filter Approach

The concept of wave filter is introduced in [2], [8]. This approach talks of applying scattering parameters to ladder filters. It uses voltage waves instead of power waves. Scattering matrix of a two port network is given as:

\[
\begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} = 
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
A_1 \\
A_2
\end{bmatrix}.
\] (1)

In a two port network having a series branch admittance \(Y\), as shown in Fig. 1, the scattering parameters, assuming the normalization resistance as \(R_n\), are obtained as:

\[
S_{11} = \frac{1}{2R_nY + 1},
\] (2)

\[
S_{12} = \frac{2R_nY}{2R_nY + 1},
\] (3)

\[
S_{21} = \frac{2R_nY}{2R_nY + 1},
\] (4)

\[
S_{22} = \frac{1}{2R_nY + 1}.
\] (5)

Fig. 1. Series branch admittance \(Y\).

For a series branch inductance \(L\), using (2), (3), (4) and (5), (1) reduces to

\[
B_1 = \frac{1}{(2R_n/sL) + 1} \frac{A_1}{A_2} = \frac{1}{2R_n/sL + 1} A_1,
\] (6)

\[
B_2 = \frac{2R_n/sL + 1}{2R_n/sL + 1} A_2.
\] (7)

This can be further simplified to:

\[
B_1 = \frac{1}{(1 + s\tau_L)} (A_1 - A_2),
\] (8)

\[
B_2 = \frac{1}{(1 + s\tau_L)} (A_2 - A_1).
\] (9)

In (9), \(\tau_L = L/2R_n\) is the time constant. Fig. 2 shows the symbolic representation of the wave equivalent of series branch inductor \(L\).

Fig. 2. Wave equivalent of series branch inductance \(L\), \(\tau_L = L/2R_n\).

To calculate the S-matrix for series branch capacitance \(C\), (1) reduces to:

\[
B_1 = \frac{1}{2sCR_n + 1} A_1 + \frac{2sCR}{2sCR_n + 1} A_2,
\] (10)

\[
B_2 = \frac{2sCR_n}{2sCR_n + 1} A_1 + \frac{1}{2sCR_n + 1} A_2.
\] (11)

(10) and (11) can be further simplified to:

\[
B_1 = A_2 + \frac{1}{1 + s\tau_C} (A_1 - A_2),
\] (12)

\[
B_2 = A_1 - \frac{1}{1 + s\tau_C} (A_1 - A_2).
\] (13)

In (12) and (13), \(\tau_C = 2CR_n\) is the time constant. It is observed that (8) and (9) are similar to (12) and (13) respectively, and can be obtained from each other by interchanging the output terminals \(B_1\) and \(B_2\).

This result can be generalized to show that for a series branch admittance \(Y\), its dual admittance \((Y')\) can be obtained by using the following equation [2]:

\[
y' = \frac{1}{4R_n^2 y}.
\] (14)

Accordingly the wave equivalent symbol of series branch capacitance \(C\), as shown in Fig. 3, indicates this fact.

Fig. 3. Wave equivalent of series branch capacitance \(C\), \(\tau_C = 2CR_n\).

For an inductor \(L\) connected in series with capacitance \(C\) in a series arm the wave equivalent can be obtained by
cascading the wave equivalents of \( L \) and \( C \). If the terminals are interchanged, the wave equivalent for a tank circuit connected in series branch can be obtained. Tab. 1 [2], [8] gives wave equivalents for all the series branch elements.

Proceeding in a similar manner, wave equivalents for shunt branch elements can also be derived. Tab. 2 [2], [8] lists the results for shunt branch elements.

3. OTRA Based Wave Active Filter

Fig. 4 shows an OTRA circuit symbol. Its transfer matrix is given in (15) [12]. It has low impedance input and output terminals. Ideally the trans-resistance gain \( R_m \) approaches infinity and when negative feedback is used then \( I_1 = I_2 \) [13].

\[
\begin{bmatrix}
V_i^1 \\
V_i^2 \\
V_o
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
R_m & -R_m & 0
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_o
\end{bmatrix}
\]

(15)

A closer study of (8), (9), Tabs. 1 and 2 would reveal that the realization of wave equivalents would require a summer, subtractor, a subtracting lossy integrator and an inverter.

An OTRA based summer is shown in Fig. 5. The circuit makes use of three resistors and one OTRA. Equation (16) can be obtained by equating the current at the inverting and the non-inverting terminal, which can be further simplified to (17).

\[
\frac{V_o}{R} = \frac{V_{IN1}}{R} + \frac{V_{IN2}}{R},
\]

(16)

\[
V_o = V_{IN1} + V_{IN2}.
\]

(17)

Similar analysis of a subtractor, shown in Fig. 6, gives

\[
V_o = V_{IN1} - V_{IN2}.
\]

(18)

Fig. 5. Summer.

Fig. 6. Subtractor.

Fig. 7 shows an inverter. In this case, since the non-inverting terminal has been left open, there would be no current flowing into the inverting terminal as well. It can be described by the following equation:

\[
V_o = -V_{IN}.
\]

(19)
Fig. 8 shows a subtracting lossy integrator. Its output can be described by:

\[ V_O = \frac{1}{1 + sCR} (V_{IN1} - V_{IN2}) \]  

(20)

![Fig. 8. Lossy integrator.](image)

The current differencing property of the OTRA makes it possible to implement the resistors connected to the input terminals of OTRA, using MOS transistors with complete non-linearity cancellation [13]. Fig. 9 shows the MOS based linear resistor using OTRA. Each resistor requires two matched n-MOSFETs connected in a manner as shown in Fig 9.

![Fig. 9. MOS based resistor.](image)

Symbols ‘+’ and ‘−’ represent the non-inverting and the inverting terminals of the OTRA. As shown in the figure, the voltages at the drain and the source terminals for both MOSFETs are equal. On taking the difference of the currents flowing in the two transistors, the non-linearity gets cancelled out. The following equation defines the resistor that has been realized.

\[ R = \frac{1}{K_N (V_d - V_b)} \]  

(21)

where \( R = \frac{1}{K_N (V_d - V_b)} \)

\( K_N \) needs to be determined for the transistors being used to implement the resistors. \( \mu, C_{OX} \) and \( W/L \) represent standard transistor parameters. The choice of voltages \( V_d \) and \( V_b \) is important. The circuit shown in Fig. 9 realizes a resistor of value expressed in (21) at the inverting terminal. If it is desired to realize a resistor of the same value at the non-inverting terminal, then \( V_d \) and \( V_b \) must be interchanged.

Using the blocks defined by (17), (18), (19) and (20), the wave equivalent for a series branch inductor, defined by (8) and (9), can be drawn and is shown in Fig. 10. The dashed blocks indicate the individual blocks which constitute the entire wave equivalent. It represents the symbol shown in Fig. 2. This can now be used as the elementary block to synthesize the wave equivalents for all the elements listed in Tab. 1 and Tab. 2.

By introducing inverters and interchanging output terminals, all other wave equivalents can be obtained. The circuit in Fig. 10 can be described by the following equations:

\[ B_1 = A_i - \frac{1}{1 + sCR} (A_i - A_2) \]  

(22)

\[ B_2 = A_2 + \frac{1}{1 + sCR} (A_i - A_2) \]  

(23)

![Fig. 10. Equivalent circuit for series branch inductance.](image)

Fig. 11 shows the circuit of Fig. 10 with MOS based linear resistors. The actual value of inductance \( L_i \) realized by circuit of Fig. 11 would be obtained by comparing (22) and (23) with (8) and (9). The realized value is

\[ L_i = 2R_CR. \]  

(24)

Similarly, comparing (22) and (23) with (12) and (13), the realized value of \( C \) is:

\[ C_i = \frac{CR}{2R_C}. \]  

(25)
Resistor $R$ can be controlled through voltages $V_d$ and $V_b$. If $C$ is assumed to be of some constant value, then the value of $L_A$ and $C_C$ can be controlled using $R$. This forms the basis of tunability of the circuit. Similarly, the actual values of $L$ and $C$ for wave equivalents of shunt branch elements, as presented in Tab. 2, are given by:

$$L_A = \frac{R_C R}{2}, \quad (26)$$

$$C_A = \frac{2CR}{R_n}. \quad (27)$$

For a filter, if $L_n$ and $C_n$ are the normalized inductor and capacitor values respectively, $\omega_0$ is the normalizing pole frequency and $R_n$ is the normalizing resistance, then to de-normalize $L_n$ and $C_n$ we make use of the following expressions:

$$L_A = \frac{R_L L_n}{\omega_0}, \quad (28)$$

$$C_A = \frac{1}{R_0 \omega_0} C_n. \quad (29)$$

Restating (24) and (25), such that different values of $C$ are used for $L_A$ and $C_A$, i.e. $C_L$ and $C_C$ respectively, we get:

$$L_A = 2R_C C_L R_n, \quad (30)$$

$$C_A = \frac{2R_C R}{2R_n}. \quad (31)$$

Equating (28) and (29) with (30) and (31) respectively, we get:

$$C_L R = \frac{1}{2\omega_0} L_n, \quad (32)$$

$$C_C R = \frac{2}{\omega_0} C_n. \quad (33)$$

The expression, for controlling $\omega_0$ using $R$, needs to be worked out for each circuit. A simple algorithm can be worked out to achieve the exact expression and range of tunability. For the frequency $\omega_0$, $C_L$ and $C_C$ are calculated in terms of $R$, as per the $L_n$ and $C_n$ values. For a suitable $R$ value, once $C_L$ and $C_C$ have been fixed, either (32) or (33) can be used to describe the relationship between $R$ and $\omega_0$ as:

$$\omega_0 = \frac{K}{R}. \quad (34)$$

Using (32) and (33), $K$ can be described as follows:

$$K = \frac{2C_L}{C_C} = \frac{L_n}{2C_L}. \quad (35)$$

Combining (21) and (34) one may get:

$$\omega_0 = KK_n(V_d - V_b). \quad (36)$$

Equation (34) describes how $\omega_0$ can be controlled using $R$. $R$ in turn is controlled by $V$. $\omega$ in turn is controlled by (21) and the overall relationship is described by (36).

4. Simulation Results

To demonstrate the wave filter approach using OTRA, a doubly terminated third order Butterworth low pass filter, as shown in Fig. 12, has been implemented. The wave equivalent circuit of the same is shown in Fig. 13 in which the reflected waves are available at $V_{OL}$ and $V_{OH}$. These outputs complement each other by virtue of wave theory [8]. Thus as the $V_{OL}$ represents the low pass filter response, its complementary high pass output is available at $V_{OH}$. The normalized values of components are $L_{nl} = 2$, $C_{nl} = 1$ and $C_{a2} = 1$. OTRA is realized using the CMOS circuit schematic given in Fig. 14 [17]. The filter specifications are as follows: $f_0 = 200$ kHz and maximum attenuation in pass band $\omega_{MAX}$ is 3 dB.

![Fig. 12. 3rd order low pass Butterworth filter.](image)

The value of normalizing resistance $R_n$ is chosen to be 2.5 kΩ. De-normalizing the values of $L_{nl}$, $C_{nl}$ and $C_{a2}$ we get:

$$L_A = 3.98 \text{ mH}, \quad (37)$$

$$C_{a1} = C_{a2} = 318.31 \text{ pF}. \quad (38)$$

Setting the value $R$ initially to 12 kΩ, we can calculate the value of $C_L$ for $L_A$ as 66.33 pF and $C_C$ for $C_{a1}$ and $C_{a2}$ as 132.66 pF. The value of $K_n$ is per (35) is $1.508 \times 10^6$. For this simulation exercise, the value of $K_n$ was found to be $5.25 \times 10^7$. The required $V_d$ and $V_b$ values for $R$ to be 12 kΩ were found to be 0.908 V and 0.75 V as per (21). Fig. 15 shows the low pass filter response of the circuit at $V_{OL}$. The complementary high pass output $V_{OH}$, as represented in Fig. 13, has been plotted in Fig. 16.

The performance of the proposed circuit is compared with the previous voltage mode structures [2], [8]-[10] in terms of power consumption, THD, output noise and electronic tunability. It may be noted from Tab. 3 that the topology presented in [9] shows best THD result, however the structure is not electronically tunable. Although the most recently reported literature [10] is having better THD performance its simulated power consumption is higher as compared to the proposed one. The relevant data for structures of [2], [8] which are designed using commercially available OPAMPs, is not available in the literature.
Fig. 13. Wave equivalent of circuit of Fig. 12.

<table>
<thead>
<tr>
<th>Ref</th>
<th>Active block and technology used</th>
<th>Filter structure</th>
<th>% THD</th>
<th>Power consumption (mW)</th>
<th>Output noise voltage (V/Hz^{1/2})</th>
<th>Electronic tunability</th>
</tr>
</thead>
<tbody>
<tr>
<td>[9]</td>
<td>CFOA (Commercially available IC AD844 with ± 5 V power supply)</td>
<td>3rd order elliptic Low pass</td>
<td>1 % for 1 Vpp signal</td>
<td>N/A</td>
<td>N/A</td>
<td>No</td>
</tr>
<tr>
<td>[10]</td>
<td>DVCCCTA (CMOS Technology 0.25 μm, Power supply ± 1.25 V)</td>
<td>4th order Butterworth Low pass</td>
<td>Less than 5 % up to 225 mVpp signal</td>
<td>59.2</td>
<td>8.36 × 10^{-5}</td>
<td>Yes</td>
</tr>
<tr>
<td>proposed</td>
<td>OTRA CMOS Technology (0.5 μm, Power supply ±1.5 V)</td>
<td>3rd order Butterworth Low pass</td>
<td>Less than 5 % up to 125 mVpp signal</td>
<td>10.7</td>
<td>7.26 × 10^{-6}</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Tab. 3. Comparison with the voltage mode filter structures.

Fig. 14. CMOS realization of OTRA.

Fig. 15. Low pass response \( V_{OL} \).

Fig. 16. Complementary high pass response \( V_{OH} \).

Fig. 17. Comparison curve between theoretical and observed frequency.
The proposed circuit can be tuned to different cut-off frequencies by controlling $V_a$ and $V_b$ as described by (36). Fig. 17 shows the comparison between theoretical and the observed frequencies, obtained by variation of control voltages $V_a$ and $V_b$. All simulations are done using PSPICE program using 0.5 μm CMOS technology parameters form MOSIS (AGILENT).

5. Conclusion

In this paper, the design of tunable wave active filter based on OTRA has been presented. It provides an alternative form of realizing ladders. Advantages of current mode approach have been exploited. The use of OTRA allows the simple implementation of linear resistor using only two MOSFETs. The controllability of the resistors value by a single voltage source allows the parameters of the proposed filters to be electronically tunable. On the downside, the circuit is slightly cumbersome to realize, though it is modular and can easily be implemented using the reference design tables. When compared with the previous wave active filters reported in [2], [8] and [9], the proposed one provides advantages of current mode design and is tunable as well. In comparison to the circuit presented in [10], OTRA as the basic building block is simpler to realize and also provides a low impedance voltage output, making it suitable for driving voltage input devices.

References


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Mayank BOTHRA was born in 1987 and received his B.E. degree in Electronics and Communication from Delhi Technological University (formerly Delhi College of Engineering, Delhi University) in 2009. He worked as a development engineer in the Embedded Group at Kritikal Solutions, Noida for two years. Currently he is working as a design engineer in the Memories’ team at ST Microelectronics. His research interests are in analog circuit design and microelectronics.

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