Research on 2×2 MIMO Channel with Truncated Laplacian Azimuth Power Spectrum

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Abstract. *Multiple-input multiple-output (MIMO) Rayleigh fading channel with truncated Laplacian azimuth power spectrum (APS) is studied. By using the power correlation matrix of MIMO channel model and the modified Jakes simulator, into which random phases are inserted, the effect of the azimuth spread (AS), angle of departure (AOD) and angle of arrival (AOA) on the spatial correlation coefficient and channel capacity are investigated. Numerical results show that larger AS generates smaller spatial correlation coefficient amplitude, while larger average AOD or AOA produces larger spatial correlation coefficient amplitude. The average capacity variation is comprehensively dominated by the average AOD, AOA and AS.*

Keywords

Multiple-input multiple-output (MIMO), Jakes simulator, power correlation matrix, spatial correlation.

1. Introduction

Multiple-input multiple-output (MIMO) technology can greatly improve the communication capacity and spectrum efficiency by exploiting space resources without increasing the total emission power and system bandwidth [1-2]. It becomes one of the chief methods to achieve high-speed and reliable data transmission for indoor wideband wireless communications. However, the superiority of MIMO communication depends overwhelmingly on the channel model, which is vital for the construction and evaluation of the communication system. Researchers have constructed different models in this field, such as IST METRA Model [8] and Maximum Eigenvalue Model [13].

The spatial correlation properties of a MIMO channel can greatly affect the performance of the MIMO system [3]. MIMO channel models can be grouped into the physical model and the analytical model [4]. For the latter, the MIMO channel matrix is characterized statistically in terms of the correlation between the matrix entries, such as the Kronecker model [5] and the Weichselberger model [6], which is suitable for the link level MIMO channel simulation. MIMO channel modeling based on the power correlation matrix [7-8] is a convenient and flexible method, and the required parameters are available in the open literatures.

The MIMO channel transmission coefficients are often modeled by applying stochastic approaches. Among the stochastic approaches, Jakes simulator, which allows an effective approximation of the desired analytical model by using a finite number of low frequency oscillators, is a widely used method to simulate the Rayleigh fading channel [9]. However, the correlated multipath components, which experiencing the same Doppler frequency shift in Jakes simulator, will cause the generated signal to be non-stationary. Nevertheless, by introducing random phases in each low-frequency oscillator, the generation of wide-sense stationary signals will be confirmed in Jakes simulator [10]. Consequently, in this letter, the power correlation matrix is combined with the modified Jakes simulator, which is modified by introducing random phases, to simulate the indoor MIMO Rayleigh channel. Thereafter, the spatial selective fading characteristic, spatial correlation and channel capacity are studied.

The rest of the article is organized as follows. In Section 2, the MIMO channel impulse response matrix based on the hybrid method is introduced in detail. In Section 3, the simulation results are provided and discussed. Finally, Section 4 ends the letter with some concluding remarks.

2. MIMO Channel Model

2.1 Correlated Channel Impulse Response Matrix

Channel characteristics are embodied in the channel impulse response matrix for MIMO channel modeling. The flat fading channel impulse response matrix can be expressed as

$$\mathbf{H}(\tau) = \mathbf{A}\delta(\tau - \tau_0) \tag{1}$$

where τ_0 is the delay of the only distinguishable multipath component, and the transmission coefficient matrix **A** is

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad (2)$$

where the element $a_{i,j}$ represents the transmission coefficient from the j^{th} transmitting antenna to the i^{th} receiving antenna. Generally speaking, between any two channel transmission coefficients, there exists spatial correlation, which has obvious impact on the system performance, especially on the channel capacity. So it is very important to indicate the spatial correlation in a MIMO channel model.

According to [7], the transmission coefficient matrix **A** can be obtained through the MIMO channel modeling based on the power correlation matrix, which is expressed as

$$vec(\mathbf{A}) = \sqrt{p} \cdot \mathbf{C} \cdot vec(\boldsymbol{\alpha}) \quad (3)$$

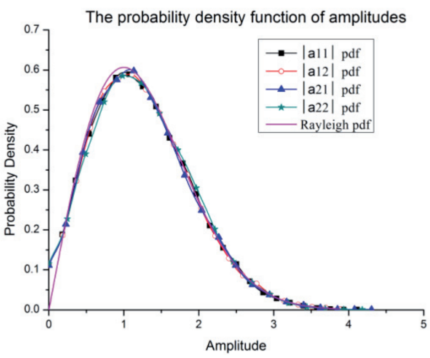
where $vec(\cdot)$ is the vectorization operator, p is the average power, which is normalized to 1 for simplicity, **C** is the spatial correlation mapping matrix, and the channel matrix without spatial correlation $\boldsymbol{\alpha}$ is as

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}. \quad (4)$$

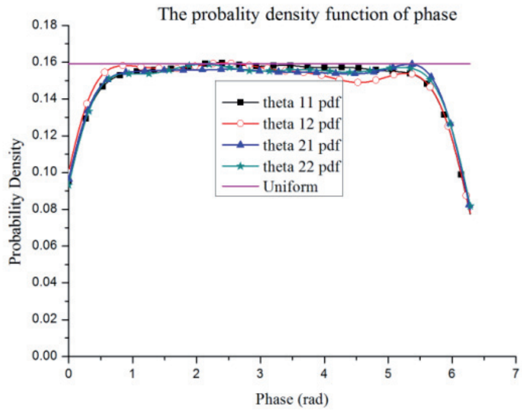
In this article, the channel matrix $\boldsymbol{\alpha}$ is generated by the modified Jakes simulator, which introduces random phases in each low-frequency oscillator. With this method, the wide sense stationarity of $\alpha_{i,j}$ can be achieved, and a 2×2 MIMO channel matrix $\boldsymbol{\alpha}$ without spatial correlation is obtained.

2.2 Statistical Characteristics of Uncorrelated Channel Impulse Responses

The statistical properties of envelopes and phases for the simulated α_{11}, α_{12}, α_{21} and α_{22} are shown in Fig. 1. It can be found that, when the number of low frequency oscillators N_0 is 20, all the envelopes of $\alpha_{i,j}$ are in good accordance with the standard Rayleigh distribution, which has standard deviation of 1, and the phases obey the uniform distribution in $[0, 2\pi)$.



(a)



(b)

Fig. 1. Probability density distribution functions of (a) envelopes and (b) phases, of the 2×2 MIMO channel matrix.

2.3 Spatial Correlation Mapping Matrix C

The spatial correlation mapping matrix **C** is acquired through the spatial correlation matrices of the 2×2 MIMO channel. The spatial correlation matrices $\mathbf{R}^{Tx}$ and $\mathbf{R}^{Rx}$ consist of the amplitudes of the spatial correlation coefficients at the transmitting (Tx) and receiving (Rx) ends respectively, and the spatial correlation coefficients are defined as

$$\begin{cases} \rho_{i,k} = \int_{\theta_0-\Delta}^{\theta_0+\Delta} p(\theta)\exp(j\frac{2\pi d}{\lambda}\sin\theta)d\theta & i \neq k \\ \rho_{i,k} = 1 & i = k \end{cases} \quad (5)$$

where θ is the angle of departure (AOD)/ angle of arrival (AOA) with average θ_0, which starts from the line between Tx and Rx, the Δ is the azimuth spread (AS), and d/λ is the normalized antenna spacing between two elements at Tx/Rx end, i and k represent the row number and column number respectively. The spatial correlation function obeys the truncated Laplacian azimuth distribution [11], [12], which is a typical indoor azimuth power spectrum (APS). $p(\theta)$ can be described as

$$\begin{cases} p(\theta) = \dfrac{Q}{\sqrt{2}\sigma}\exp[-\sqrt{2}\left|\theta-\theta_0\right|/\sigma], \\ where\ \theta \in [\theta_0-\Delta, \theta_0+\Delta] \end{cases} \quad (6)$$

where the normalized coefficient Q is $[1-\exp(-\sqrt{2}\Delta/\sigma)]^{-1}$, σ is the standard deviation of θ, and Δ represents the azimuth spread (AS). So the joint spatial correlation matrix **R** is the Kronecker product as follow

$$\mathbf{R} = \mathbf{R}^{Tx} \otimes \mathbf{R}^{Rx} \quad (7)$$

where $\otimes$ represents the Kronecker product, and the spatial correlation mapping matrix **C** is obtained by Cholesky decomposition of **R**, that is

$$\mathbf{R} = \mathbf{C}\mathbf{C}^T \tag{8}$$

where $[\cdot]^T$ denotes the transposition. By doing so, $\mathbf{C}$ is obtained.

3. Numerical Results

The simulation of a 2×2 flat fading channel transmission matrix **A** is configured as follow: the APSs at both ends follow truncated Laplacian distribution, and 10000 samples are taken in 1 second.

As a critical propagation characteristic of MIMO channel, the spatial correlation is investigated, and the results are shown in Fig. 2. It can be seen that the amplitudes of spatial correlation coefficients a_{11} and a_{12} are changing with the AS and d/λ. The simulation is performed with four AS values, which are 0°, 25°, 45° and 85°, respectively. Different average AOD/AOA values have different influences on the spatial correlation coefficient amplitudes.

It can also be found from Fig. 2 that larger average AOD/AOA brings larger mean correlation coefficient amplitudes. When the average AOD/AOA is 0°, the minimum correlation coefficient amplitude is 0, but it increases to 0.18 when the average AOD/AOA increases to 90°. It means that with the same element spacing and AS, the correlation coefficient amplitudes of the incident wave which comes from the broadside direction (average AOD/AOA is 0°) is the lowest one, and the correlation coefficient amplitude increases with the average AOD/AOA increasing.

Capacity is another significant propagation characteristic of the MIMO channel. The average capacity can be calculated by [2]

$$C = E\left\{\log_2\left[\det\left(\mathbf{I}_{n_R} + \frac{\rho}{n_T}\mathbf{H}\mathbf{H}^H\right)\right]\right\} \tag{9}$$

where n_T and n_R are the numbers of transmitting and receiving antennas, respectively, $\mathbf{I}_{n_R}$ is a n_R order identity matrix, ρ is the average signal to noise ratio (SNR) at each receiver, and $\mathbf{H}$ is a $n_R \times n_T$ matrix, representing the channel impulse response matrix.

Based on the simulation results of the 2×2 channel transmission coefficient matrix **A**, the relationship between the average channel capacity and the azimuth parameters is investigated, and the simulated results are shown in Fig. 3.

It can be found that the average capacity increases with the increasing of AS. When the AS is 0°, the capacities of all AODs/AOAs are nearly the same, about4.55 bit/s/Hz. With the increasing of AS, the channel capacities of all AODs/AOAs increase. When the AS is 90°, the capacities reach to the summit. Besides, the capacity with small AOD/AOA increases faster than that

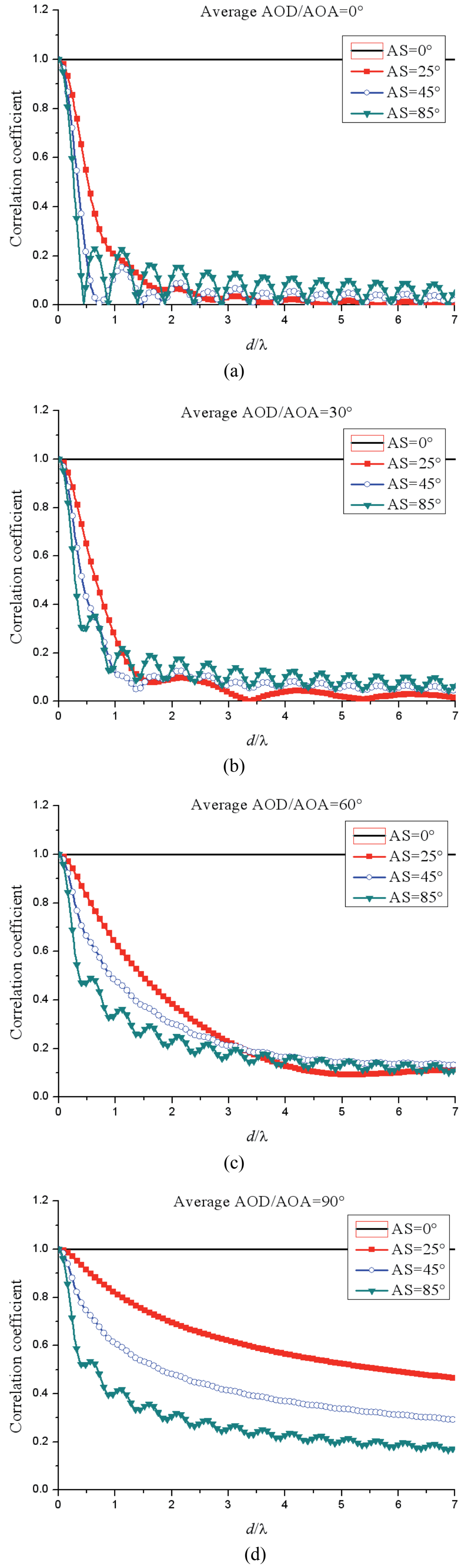


Fig. 2. The effects of normalized element spacing and ASs on spatial correlation coefficient amplitudes of a_{11} and a_{12}.

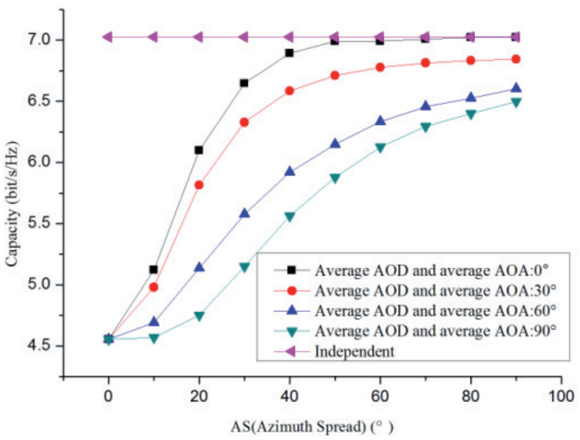


Fig. 3. The effect of average AOD, AOA and AS on the 2×2 MIMO capacity (SNR = 10 dB).

with larger AOD/AOA. As a result, for the same AS, the channel with small AOD/AOA has larger capacity. When the AS is 90°, the lowest capacity is 6.5 bit/s/Hz with the AOD/AOA of 90° and the highest capacity is nearly 7.01 bit/s/Hz, which is the capacity when the channel is ideally independent.

4. Conclusion

The Multiple-input multiple-output (MIMO) Rayleigh fading channel with truncated Laplacian azimuth power spectrum (APS) is studied with the modified Jakes simulator. Firstly, the combination of modified Jakes simulator with the insertion of random phases and power correlation matrix is used to calculate the indoor 2×2 flat fading channel transmission coefficient matrix **A**. Afterwards, the spatial correlation coefficient amplitudes and the channel capacity are investigated. The results show that the larger the AS, the smaller the correlation coefficient amplitude. Besides, the increasing of mean AOD/AOA will bring larger mean correlation coefficient amplitudes. Average capacities increase with the increasing of ASs. Meanwhile, larger mean AOD/AOA will bring higher mean capacities, provided that the ASs are the same. So the capacity variation is comprehensively dominated by the AS, average AOD and AOA.

Acknowledgements

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