# Simultaneous Perturbation Stochastic Approximation for Unambiguous Acquisition in Cosine-BOC Signals

Huihua CHEN, Jiawei REN, Weimin JIA, Minli YAO

Xi'an Research Institute of High Technology, Xi'an, China

chenhuihuachina@hotmail.com, wdrjw@yahoo.com.cn, jwm602@163.com, yaominli@sohu.com

Abstract. The binary offset carrier (BOC) proposed for the global navigation satellite systems (GNSS) will enhance navigation performance and spectrum compatibility. However, the acquisition process is made more complex, due to the ambiguity in the autocorrelation function (ACF) of BOC. This paper proposes an unambiguous acquisition technique for the new cosine phased BOC (cosine-BOC) modulated signals, which will most likely be used in both European Galileo system and Chinese Compass system. The test criterion employed in this technique is based on a synthesized correlation function which completely removes major positive side peaks while keeping the sharp main peak using the concept of simultaneous perturbation stochastic approximation (SPSA). Theoretical analysis and simulation results indicate that the proposed technique completely removes the ambiguity threat in acquisition process with some performance degradation. This technique is also suitable for arbitrary order cosine-BOC signals.

#### Keywords

Galileo, Compass, BOC, unambiguous acquisition, SPSA.

#### 1. Introduction

The modernization of American GPS, Russia GLONASS, European Galileo and Chinese Compass are the new generation of Global Navigation Satellite Systems (GNSS), which will use the novel Binary Offset Carrier (BOC) modulation [1]. This is because that the BOC modulation provides GNSS signals with enhanced robustness against multipath and thermal noise, and increases the precision of range measurement compared with BPSK modulation [2]. However, BOC modulation presents some drawbacks. The most severe of every BOC modulation signal is its multi-peak autocorrelation function (ACF) that implies possible false acquisition.

In order to get rid of the ambiguity from the ACF at acquisition stage, several techniques have been proposed [3-7]. Bump-Jumping (BJ) technique aims at determining

whether or not the peak being tracked is the correct one [3], [4]. It exhibits high tracking accuracy when locked on the main peak. However, the detection may have a high probability of false alarm when the signal-to-noise ratio is low. Moreover, this technique needs time to detect and recover from false lock, so it is inapplicable in some critical applications [7]. The BPSK-like technique only consists in considering the received BOC(m,n) signal as the sum of BPSK(n) with carrier frequency symmetrically positioned on each side of the BOC carrier frequency [4-6]. The main advantages of this technique are that it is unambiguous and allows the use of a higher searching step of time-uncertainty window compared with the traditional situation. However, the filtering and the sideband processing increase the implementation complexity and cause a loss of power. Besides, these two techniques make the correlation peak wider than the main peak of BOC ACF, if classical delay lock loop is used in tracking, an additional transition from acquisition to tracking is necessary, or false lock may occur [7]. Recently, a new technique presented in [7], namely general removing ambiguity via side-peak suppression (GRASS) can be used to realize unambiguous acquisition. This technique completely removes the ambiguity threat while keeping the sharp shape of the main peak with less complexity. However, this technique is only applicable to sine-BOC(kn,n) signals.

In order to overcome those limitations of the aforementioned methods, an unambiguous acquisition technique using the concept of simultaneous perturbation stochastic approximation (SPSA) for cosine-BOC signals is proposed in this paper. This method is convenient and flexible to implement by designing the modulated symbols of the local signal. Theoretical approximate analysis and simulation results obtained with cosine-BOC(n,n), cosine-BOC(1.5n,n) and cosine-BOC(2n,n) signals show that the proposed technique completely removes the ambiguity threat in acquisition process with less performance degradation compared with existing unambiguous acquisition techniques.

The remainder of this paper is organized as follows. In Section 2, the cosine-BOC signals model and ambiguous problem are given out. Section 3 proposes an unambiguous acquisition technique using the concept of SPSA. Section 4 presents the simulation results and discussions for the proposed technique. Section 5 analyzes the detection performance under noise for the proposed technique, followed by the conclusions in Section 6.

## 2. Cosine-BOC Signal Model and Ambiguous Problem

#### 2.1 Cosine-BOC Signal Model Definition

Using the terminology from [2], a cosine-BOC signal is denoted as cosine-BOC(*m*,*n*), where *m* means the ratio of the square wave frequency  $f_s$  to 1.023 MHz, and *n* denotes the ratio of the spreading code rate  $f_c$  to 1.023 MHz. *m* and *n* are both constrained to positive integer,  $m \ge n$ , and the ratio M = 2m/n is a positive integer. Similar with [8], cosine-BOC signal can be considered as a special case of step-shape code symbol (SCS) signals. A generic SCS signal is expressed as  $s(t) = \sum_i c_i p(t-iT_c)$ , where  $c_i$ represents pseudorandom code symbols, p(t) is chip waveform, which is non-zero only over the interval support [0,  $T_c$ ), and  $T_c$  refers to the chip waveform period. Different from [8], the chip waveform p(t) is divided into 2*M* segments, each with equal length  $T_s = T_c/2M$ . It can be expressed as  $p(t) = \sum_{k=0}^{2M-1} s_k \varphi_k(t)$ , where

$$\varphi_k(t) = \begin{cases} 1, & t \in [kT_s, (k+1)T_s] \\ 0, & \text{others} \end{cases}$$
(1)

and  $s_k$  can take any real number. We assume that  $s_k$  are energy normalized so that  $\sum_{k=0}^{2M-1} s_k^2 = 2M$ .

Every SCS waveform p(t) can be identified by shape vector  $\mathbf{s} = [s_0, s_1, \dots, s_{2M-1}]^T$  and chip rate  $f_c = 1/T_c$ . So a SCS waveform can be denoted by  $p(t; \mathbf{s}; f_c)$ . When the terms used for BOC signals, M is referred to as the order of SCS signal. The shape vector of cosine-BOC chip waveform is  $\mathbf{s} = (-1)^{\lceil k/2 \rceil}$ , where  $\lceil \cdot \rceil$  represents the ceiling operation. Note that when M is odd for cosine-BOC signal, the signal model should be changed as  $s_{BOC}(t) = \sum_i (-1)^i c_i p(t-iT_c)$ .

For the case of ideal pseudorandom code symbols with  $E[c_i c_j] = \delta_{ij}$  (or  $E[(-1)^i c_i (-1)^j c_j] = (-1)^{i+j} \delta_{ij}$ ), where  $E[\cdot]$  denotes the expectation operator. Hence, the cross-correlation function (CCF) of BOC and SCS signals using two chip waveforms  $p_{BOC}(t; \mathbf{s}^{BOC}; f_c)$ . and  $p_{SCS}(t; \mathbf{s}^{SCS}; f_c)$  can be expressed as

$$R_{B/S}^{CCF}(\tau) = E\Big[p_{BOC}(t)p_{SCS}(t+\tau)\Big] = \frac{1}{2M}\sum_{k=0}^{2M-1}\sum_{l=0}^{2M-1}s_{k}^{BOC}s_{l}^{SCS}Tri\Big[2Mf_{c}(t-(k-l)T_{s})\Big]$$
(2)

where

$$\operatorname{Tri}(x) = \begin{cases} 0 & , \ |x| > 1 \\ 1 - |x| & , \ |x| \le 1 \end{cases}$$
(3)

Fig. 1 shows the ACFs of the cosine-BOC(n,n) and cosine-BOC(2n,n) signals, it can be seen that the BOC modulation signal has a sawtooth-like, piecewise linear ACF which has one main peak and multiple positive and negative side peaks. Compared with the triangular ACF of BPSK signal with the same spreading code frequency, the ACF of BOC signals has sharper main peak, which means better tracking accuracy.



**Fig. 1.** Normalized ACFs of BPSK(*n*), cosine-BOC(*n*,*n*) and cosine-BOC(2*n*,*n*) signals.

#### 2.2 Ambiguous Problem in Acquisition

The acquisition process consists of detecting the incoming signal energy through a search for an approximate carrier frequency and code delay [7]. The detection criterion in the traditional acquisition scheme is given by

$$\Psi = \sum_{k=0}^{L-1} \left( I_{\mathrm{B},k}^2 + Q_{\mathrm{B},k}^2 \right) \tag{4}$$

where *L* denotes the number of successive correlator outputs used, or non-coherent summations.  $I_{B,k}$  and  $Q_{B,k}$  are in-phase and quadrature correlators outputs between the incoming cosine-BOC signal and local replica cosine-BOC signal.

The sequential approach tests each possible code delay and Doppler values one by one. Once the maximum correlation result is larger than a threshold, detection is declared. Due to the multiple side peaks in the BOC ACF, under the influence of noise it is quite likely that one of side peak magnitudes exceeds the main peak, and finally false acquisition will occur. If false acquisition occurs, the code tracking loop will initially lock on the side peak in transition to tracking mode. Fig. 2 shows the probability of false acquisition of the first side peak in the sequential approach for cosine-BOC(*n*,*n*), cosine-BOC(2*n*,*n*), and cosine-BOC(3*n*,*n*) signals. In the simulation, we assume that the probability of false alarm  $P_{fa} = 10^{-6}$ , L = 15, and the coherent integral time  $T_{coh} = 3$  ms.



Fig. 2. Probability of detection of first side peak for cosine-BOC(n,n), cosine-BOC(2n,n) signals, and cosine-BOC(3n,n) signals.

From the figure one can see that when carrier to noise ratio (CNR) is more than 40 dB-Hz, the probability of the first side peak acquisition for cosine-BOC(n,n) signal cannot be neglected, especially when CNR is more than 42 dB-Hz, the probability of false acquisition is approximated to 1. With the increase of M, the difference between the main peak and side peaks energy is getting smaller. Therefore, the traditional acquisition technique under the sequential approach has high probability of false acquisition, it is applicable to deal with BOC signals.

GRASS is a side-peaks cancellation technique which can be used to realize unambiguous acquisition with high resolution. However, it is only effective to sine-BOC(kn,n) signals. In the next section we formulate a family of unambiguous acquisition techniques for generic cosine-BOC(m,n) signals.

### 3. SPSA-Based Unambiguous Acquisition Technique

#### 3.1 Brief Overview of SPSA Algorithm

The simultaneous perturbation stochastic approximation (SPSA) algorithm was developed by J. C. Spall in 1987 [9], a complete discussion was presented in 1992 [10], adaptive stochastic approximation by the simultaneous perturbation method (2-SPSA) was presented in 2000 [11], and some recent works were introduced in 2009 [12]. By virtue of its generality, efficiency, and ease of use, it has been used in a number of different fields such as traffic management, neural network training, adaptive control, and antenna tracking [13-15].

SPSA is a method for optimization of multivariate stochastic systems. The goal of SPSA algorithm is to minimize a differentiable scalar valued cost function.

#### 3.2 Proposed Unambiguous Acquisition Technique

The main idea of the proposed technique is to remove the BOC ACF's undesired side peaks. In addition to the local in-phase and quadrature BOC signals, an in-phase and quadrature local auxiliary SCS signals  $p_{SCS}(t; \mathbf{s}; f_c)$ have to be generated. Thus, two correlation channels are generated here. On one channel, the received BOC signal is correlated with the local BOC signal, and on the other one the received signal is correlated with the local SCS signal. When these two correlation channels are combined, an unambiguous combined correlation function is obtained, which is used to the detection test. The test criterion is expressed as

$$\psi' = \sum_{k=0}^{L-1} \left[ \left( I_{\mathrm{B},k}^2 + Q_{\mathrm{B},k}^2 \right) - \beta \left( I_{\mathrm{B}/\mathrm{S},k}^2 + Q_{\mathrm{B}/\mathrm{S},k}^2 \right) \right]$$
(5)

where  $\beta$  denotes the weight coefficient,  $I_{B/S, k}$  and  $Q_{B/S, k}$  are the in-phase and quadrature cross-correlation results between the incoming BOC signal and the local SCS signal, respectively, which are expressed as

$$\begin{cases} I_{\text{B},k} = \sqrt{T_{\text{coh}}C/N_0}\operatorname{sinc}\left(\pi\Delta fT_{\text{coh}}\right)R_{\text{B}}\left(\Delta\tau\right)\cos\left(\Delta\varphi\right) + \eta_{I_{\text{B},k}} \\ \mathcal{Q}_{\text{B},k} = \sqrt{T_{\text{coh}}C/N_0}\operatorname{sinc}\left(\pi\Delta fT_{\text{coh}}\right)R_{\text{B}}\left(\Delta\tau\right)\sin\left(\Delta\varphi\right) + \eta_{\mathcal{Q}_{\text{B},k}} \\ I_{\text{B/S},k} = \sqrt{T_{\text{coh}}C/N_0}\operatorname{sinc}\left(\pi\Delta fT_{\text{coh}}\right)R_{\text{B/S}}\left(\Delta\tau\right)\cos\left(\Delta\varphi\right) + \eta_{I_{\text{BS},k}} \\ \mathcal{Q}_{\text{B/S},k} = \sqrt{T_{\text{coh}}C/N_0}\operatorname{sinc}\left(\pi\Delta fT_{\text{coh}}\right)R_{\text{B/S}}\left(\Delta\tau\right)\sin\left(\Delta\varphi\right) + \eta_{\mathcal{Q}_{\text{BS},k}} \end{cases}$$
(6)

where  $\operatorname{sinc}(x) = \operatorname{sin}(x) / x$ ,  $C/N_0$  is the CNR.  $\Delta f$ ,  $\Delta \tau$ , and  $\Delta \varphi$  refer the frequency wipe-off error, the code delay and carrier phase estimation errors, respectively.  $R_{\rm B}(\Delta \tau)$ denotes the ACF of BOC signal, and  $R_{\rm B/S}(\Delta \tau)$  represents the CCF between BOC signal and SCS signal. The noise power at the outputs of correlator has been normalized, i.e.

$$E(\eta_{I_{\rm B},k}^2) = E(\eta_{Q_{\rm B},k}^2) = E(\eta_{I_{\rm BS},k}^2) = E(\eta_{Q_{\rm BS},k}^2) = 1.$$
 (7)

Assuming the propagation delay and the carrier frequency are varying slowly in the process of non-coherent summations, the test criterion without noise is expressed as

$$\psi' \approx LT_{\rm coh}C / N_0 {\rm sinc} \left(\pi \Delta f T_{\rm coh}\right) R_{\rm SCF} \left(\Delta \tau\right)$$
(8)

where  $R_{\text{SCF}}(\Delta \tau)$  denotes the synthesized correlation function (SCF), which is modeled as

$$R_{\rm SCF}\left(\Delta\tau\right) = R_{\rm B}^2\left(\Delta\tau\right) - \beta R_{\rm B/S}^2\left(\Delta\tau\right). \tag{9}$$

Corresponding to (9), the principle of the proposed technique is shown in Fig. 3.



Fig. 3. The proposed technique principle.

Once the test criterion  $\psi'$  is determined, it is necessary to restrict the shape of the test criterion. Since the signal acquisition is a process of searching pronounced energy peak in a 2-dimentional space, the requirement to the test criterion in acquisition is relatively generous compared to code tracking. A test criterion having main peak without positive side peak is enough. Therefore, the objective of the proposed technique is to keep the main peak of BOC ACF envelop while remove all the positive side peaks (the negative side peaks do not interfere with the statistical test since only positive values could pass the threshold).

From (2) and (8), it can be seen that the test criterion totally depends on the CCF between the BOC signal and the SCS signal. Since the chip waveform shape of the received BOC signal is known, the CCF entirely depends on the shape of local SCS signal chip waveform. It is interesting to note that the SCS signal chip waveform corresponds to an unique point in 2*M* dimensional space whose coordinate is  $[s_0, s_1, ..., s_{2M-1}]^T$ . Therefore, changing the value of  $s_k$ , one can adjust the shape of CCF. That is, the CCF is a function of **s**.

After building the relationship between the shape of CCF and the value of  $\mathbf{s}$ , the search for good chip waveform can be equivalent to an optimization problem. By virtue of SPSA's generality, efficiency, and ease of use, SPSA is employed to optimize the shape of local SCS chip waveform. Therefore, set a target test criterion curve, which is depicted in Fig. 5 (the dashed line).

The goal is to minimize the loss of correlation function between the test criterion  $\psi'$  and the target test criterion curve  $\psi'_{target}$ , i.e.

$$Loss = \min_{s;\beta} (\psi' - \psi'_{target})^2 . \tag{10}$$

Thus, set  $\mathbf{s} = [s_0, s_1, \dots, s_{2M-1}]^T$  as the optimization variable.

So far the effect of noise has not been considered. In fact, the weight coefficient  $\beta$  amplifies noise components in  $R_{B/S}$ . Under a given pre-correlation SNR, the larger  $\beta$  is, the lower SNR in the SCF is [7]. Therefore, from the viewpoint of sensitivity, it is desired that  $\beta$  should be small as possible. So  $\beta$  is also considered as the optimization vari-

able. In summary, for a test criterion with multiple constraints, the loss of correlation function optimization can be expressed as

$$\begin{cases} \min \ Loss = \psi'(s_1, s_2, \cdots, s_{2M-1}; \beta) \\ s.t. \quad \sum_{k=0}^{2M-1} s_k^2 = 2M \\ \beta \ge 1 \& \beta \text{ is as small as possible} \end{cases}$$
(11)

Notice that the final iteration is not always the optimum one. Therefore, the optimum iteration in SPSA simulation can be searched and saved, and the corresponding SCS waveform can be obtained.

# 4. Simulated Performance of Proposed Method

In this section, the proposed test criterions are simulated via cosine-BOC(n,n), cosine-BOC(1.5n,n) and cosine-BOC(2n,n) signals. In our SPSA, the values of  $\alpha$ ,  $\gamma$  and A are set as 0.602, 0.101 and 1000, respectively [10]. The number of objective-function evaluations is set to 1000. The algorithms are run for 50 independent trials.

Fig. 4 shows the loss of correlation function versus number of iterations for cosine-BOC(n,n) signal. For comparison, the average characteristic of SPSA is also given out. It is interesting to note that the optimum characteristic of SPSA is better than the average one, and the corresponding characteristic of SPSA can convergent at about 500 iterations.



Fig. 4. Convergence characteristics of the SPSA for cosine-BOC(n,n) signal.

Fig. 5 shows the envelope of cosine-BOC(*n*,*n*) signal ACF, target test criterion curve  $\psi'_{\text{target}}$ , and the final test criterion by SPSA  $\psi'$ . From the figure one can see that no major positive side peaks exist. Although there are some pits in the correlation function, their magnitudes are all below zero, so they bring no threat to the acquisition. Therefore, it can be used to the test criterion for BOC

(14)

signals. In the simulation, the shape vector of the local SCS signal's waveform and the weight coefficient can be obtained

$$\begin{cases} \mathbf{s}^{\text{SCS}} = \begin{bmatrix} -1.351 & 0.149 & -0.026 & 1.467 \end{bmatrix}^{\text{T}} \\ \beta = 2 \end{cases}$$
(12)



**Fig. 5.** The envelope of cosine-BOC(n,n) signal ACF, target test criterion curve, and the final test criterion by SPSA.

The loss of correlation function and the test criterion for the case of cosine-BOC(1.5n,n) and cosine-BOC(2n,n) signals are shown in from Fig. 6 to Fig. 9, respectively. From the figures, we can discover that the unambiguous test criterion can be obtained by SPSA. The shape vector and the weight coefficients for SCS waveform for these signals can be given as follows, respectively

$$\begin{cases} \mathbf{s}^{\text{SCS}} = \begin{bmatrix} 2.015 & -0.873 & -0.304 & -0.063 & 0.778 & -2.108 \end{bmatrix}^{1} \\ \beta = 4 \end{cases}$$
(13)

 $\begin{cases} \mathbf{s}^{\text{SCS}} = \begin{bmatrix} 1.813 & -0.728 & -0.454 & -0.194 & 0.173 & 0.433 & 0.746 & -1.779 \end{bmatrix}^T \\ \beta = 5 \end{cases}$ 



Fig. 6. Convergence characteristics of the SPSA for cosine-BOC(1.5n,n) signal.



**Fig. 7.** The envelopes of cosine- BOC(1.5*n*,*n*) signal ACF, target test criterion curve, and the final test criterion by SPSA.







**Fig. 9.** The envelopes of cosine- BOC(2*n*,*n*) signal ACF, target test criterion curve, and the final test criterion by SPSA.

#### 5. Performance Analysis

In order to assess the proposed acquisition test criterion shown in (5), in this section, we calculate the detection and false alarm probabilities. Assuming a Gaussian incoming noise, it can be easily proved that the noise coming from the prompt correlators  $I_{\text{B},k}$ ,  $Q_{\text{B},k}$ ,  $I_{\text{B/S},k}$  and  $Q_{\text{B/S},k}$  are uncorrelated and can be assumed Gaussian. Therefore, in (5) both the first term  $\sum_{k=0}^{L-1} (I_{\text{B},k}^2 + Q_{\text{B},k}^2)$  and the second term  $\beta \sum_{k=0}^{L-1} (I_{\text{B/S},k}^2 + Q_{\text{B/S},k}^2)$  follow  $\chi^2$  distribution with 2L degrees of freedom (DOF), in which the noncentrality parameter of the first term is

$$\lambda_{1}^{2} = L(T_{\rm coh}C/N_{0})R_{\rm B}^{2}(\tau)\operatorname{sinc}^{2}\left(\frac{\varepsilon T_{\rm coh}}{2}\right)$$
(15)

and  $\sigma_1^2 = 1$ . The noncentrality parameter of the second term is

$$\lambda_2^2 = L(T_{\rm coh}C/N_0)R_{\rm B/S}^2(\tau){\rm sinc}^2\left(\frac{\varepsilon T_{\rm coh}}{2}\right)$$
(16)

and  $\sigma_2^2 = \beta$ . Here  $\varepsilon$  is the Doppler error.

For the case where there is no signal present,  $\lambda_1^2 = \lambda_2^2 = 0$ , the test criterion can be seen as the difference of two central  $\chi^2$  distributed variables with both 2*L* DOF. Utilizing the study results in noncoherent digital communication over Nakagami fading channels [16], the false alarm probability can be given as

$$P_{fa} = \frac{e^{-Th/2}}{(L-1)!} \left(\frac{1}{1+\beta}\right)^{L} \sum_{i=0}^{L-1} \sum_{j=0}^{i} \frac{\left(2(L-1)-i\right)!}{(i-j)!(L-1-i)!} \left(\frac{\beta}{1+\beta}\right)^{L-1-i} (17) \times \left(\frac{Th}{2}\right)^{i-j}$$

For signal present case, when the code delay is small, we have  $\lambda_1^2 \neq 0$  and  $\lambda_1^2 \approx 0$ . The detection probability can be calculated by

$$P_{d} = Q\left(\frac{Th - 2L + L\left(\lambda_{1}^{2} - 2\beta\right)}{2\sqrt{L\left(1 + \beta^{2} + \lambda_{2}^{2}\right)}}\right)$$
(18)

where Q(x) is the Gaussian Q-function. *Th* denotes the threshold of false alarm, which can be calculated by

$$P_{fa}\left(Th\right) = \exp\left(-\frac{Th}{2}\right)\sum_{n=0}^{L-1} \frac{1}{n!} \left(\frac{Th}{2}\right)^n.$$
 (19)

The detection probability versus CNR for both the proposed technique and traditional ambiguous acquisition with a fixed false alarm probability  $P_{fa} = 10^{-3}$  for cosine-BOC(*n*,*n*), cosine-BOC(1.5*n*,*n*) and cosine-BOC(2*n*,*n*) signals are shown in Fig. 10 to Fig. 12 with  $T_{\rm coh} = 3$  ms and L = 15, respectively. In the case of proposed technique, the values have been obtained by computation as well as by



Fig. 10. Theory (Th) and MC results for detection probability comparisons for cosine-BOC(*n*,*n*) signal.



Fig. 11. Theory (Th) and MC results for detection probability comparisons for cosine-BOC(1.5*n*,*n*) signal.



Fig. 12. Theory (Th) and MC results for detection probability comparisons for cosine-BOC(2n,n) signal.

Monte Carlo (MC) simulation with  $10^4$  runs. For comparison, the traditional ambiguous acquisition technique, BJ technique, BPSK-like technique and sub carrier phase cancellation (SCPC) technique are also given out.

Based on the figures, it can be seen that the approximate theoretical and simulation results of  $P_d$  are in good agreement. The proposed technique is worse than the BJ and traditional ambiguous acquisition techniques, because the local signal is changed. However, it still has a better performance than the SCPC and BPSK-like techniques. Noted that since  $\beta$  introduces more noise, when the ambiguous problem is negligible, compared with the traditional method, the sensitivity of the proposed technique is degraded.

#### 6. Conclusions

In this paper, we present a new unambiguous acquisition technique for cosine-BOC signals. The test criterion employed in this technique is based on a synthesized correlation function via optimizing the local SCS chip waveform and the coefficient  $\beta$  using SPSA. The detection performance of the proposed technique is also analyzed. Examples with respect to cosine-BOC(*n*,*n*), cosine-BOC(1.5*n*,*n*) and cosine-BOC(2*n*,*n*) signals demonstrate that the proposed technique completely removes the undesired positive side peaks, which would result false peak acquisition and achieves better detection performance than SCPC and BPSK-like techniques. The performance degradation can be kept relatively low by optimizing  $\beta$ .

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#### About Authors ...

**Huihua CHEN** was born in Fujian, China in 1983. He received his M.Sc. from Xi'an Research Institute of High Technology, Xi'an, China in 2009. He is currently working toward the Ph.D. degree at Xi'an Research Institute of High Technology. His research interests include satellite navigation signal structure design and signal processing.

**Jiawei REN** was born in Henan, China in 1985. He received his M.Sc. from Xi'an Research Institute of High Technology, Xi'an, China in 2010. He is currently working toward the Ph.D. degree at Xi'an Research Institute of High Technology. His research interests include satellite navigation signal structure design and signal processing.

Weimin JIA was born in Hebei, China in 1971. She received her Ph.D. degree from Xi'an Research Institute of High Technology, Xi'an, China in 2007. She is an associate professor at the Dept. of Communication Engineering, Xi'an Research Institute of High Technology, Xi'an, China. Her current research interests include Sat-COM, Spreading Communication. **Minli YAO** (\*Shanxi, China, 1966) obtained his M.Sc. from Xi'an Research Institute of High Technology, Xi'an, China in 1992 and Ph.D. degree from Xi'an Jiao Tong University, Xi'an, China in 1999. He is a professor at the

Dept. of Communication Engineering, Xi'an Research Inst. of High Technology. His research interests include satellite navigation signal structure design and signal processing, Sat-COM, and Spreading Communication.