Codes Cross-Correlation Impact on S-Curve Bias and Data-Pilot Code Pairs Optimization for CBOC Signals

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Abstract. *The aim of this paper is to analyze the impact of spreading codes cross-correlation on code tracking performance, and to optimize the data-pilot code pairs of Galileo E1 Open Service (OS) Composite Binary Offset Carrier (CBOC) signals. The distortion of the discriminator function (i.e., S-curve), due to data and pilot spreading codes cross-correlation properties, is evaluated when only the data or pilot components of CBOC signals are tracked, considering the features of the modulation schemes. Analyses show that the S-curve bias also depends on the receiver configuration (e.g., the tracking algorithm and correlator spacing). In this paper, two methods are proposed to optimize the data-pilot code pairs of Galileo E1 OS. The optimization goal is to obtain minimum average S-curve biases when tracking only the pilot components of CBOC signals for the specific correlator spacing. The S-curve biases after optimization processes are analyzed and compared with the un-optimized results. It is shown that the optimized data-pilot code pairs could significantly mitigate the intra-channel (i.e., data and pilot) codes cross-correlation, and then improve the code tracking performance of CBOC signals.*

Keywords

Code cross-correlation, CBOC, S-curve bias, data-pilot code pair optimization.

1. Introduction

In the context of Global Navigation Satellite System (GNSS) receivers, the interest in the new modulations that will be used for the modernized Galileo E1 Open Service (OS) and Global Positioning System (GPS) L1C civil signals grew rapidly in past years. These new signals result from an agreement between the European Commission and United States of America in order to use a common Multiplexed Binary Offset Carrier (MBOC) signal baseline, with the aim of assuring the compatibility and interoperability between GPS and Galileo system [1]. In particular, two different approaches to obtain a MBOC modulation have been proposed and studied: the Time Multiplex Binary Offset Carrier (TMBOC) [2] and the Composite Binary Offset Carrier (CBOC) [3]. This paper will address the CBOC that is the implementation selected for the Galileo E1 OS.

Pseudo Random Noise (PRN) codes (i.e. spreading codes) are an essential element in any Code Division Multiple Access (CDMA) system such as GPS, Galileo and Compass. In fact, these codes are the tool that enables a GNSS receiver to distinguish one satellite from another [4]. Galileo E1 OS will broadcast for the first time so-called random codes, which are codes optimized in a highly multidimensional space to make them look as random as possible. The idea of the patented random codes is presented in [5].

For the optimization of the Galileo codes, different metrics were employed to account for the different users Galileo will be targeting in the future, as discussed in [6]. Additionally, reference [6] analyzed the code performance during the signal acquisition and tracking phases separately. The selected PRN code sets of Galileo E1 OS, as well as the codes of GPS L1C, are presented in detail and its generation mechanisms are analyzed in [7]. The properties of both code families in terms of even and odd auto- and cross-correlation are shown and compared. Moreover, intersystem cross-correlation of Galileo E1 OS and GPS L1C has been presented in [7]. A new family of PRN codes that offers excellent even correlation properties has been introduced in [8]. The codes cross-correlation impact on the interference vulnerability of MBOC signals is analyzed in [9], based on a new family of curves, called Interference Error Envelope (IEE) [10].

In previous literatures, the code families of each particular band were optimized taking into account only code properties. This means that the real modulation characteristics of the signal, that is their particular spreading waveform and multiplex, were not considered in the code design. In fact, the derived codes of CBOC signals would only be optimal in the wide sense if the data and pilot signals were transmitted in quadrature [11]. However, as we know from the Galileo Interface Control Document (ICD) [3], the data and pilot components constituting the CBOC signals will be transmitted in phase [3] using a modified interplex modulation [12].

In this paper, the impact of data and pilot codes cross-correlation on CBOC code tracking performance will be measured by the S-curve bias. The analyses show that the currently published CBOC codes could still be further optimized to decrease the data and pilot codes cross-correlation, and thus a further improvement of performance is still achievable in this regard. Two approaches for optimizing the data-pilot code pairs, based on current CBOC PRN code sets in Galileo ICD, are introduced.

The remainder of this manuscript is organized as follows. Section 2 introduces CBOC signals and the cross-correlation function. The impact of the codes cross-correlation and the receiver setup (e.g., correlator spacings) on the S-curve bias is analyzed in Section 3. In Section 4, approaches to mitigate the effect of codes cross-correlation of data and pilot channels are given. Eventually in Section 5 some conclusions are drawn.

2. CBOC Signals and Cross-Correlation Function

Galileo E1 OS will transmit the CBOC(6,1,1/11) signal, where (6,1) refers to the BOC(6,1) part that is added with BOC(1,1) and 1/11 denotes the percentage of power of BOC(6,1) with respect to the total signal CBOC power [13]. The E1 OS signal is composed of two channels: the data channel and the pilot channel transmitted by splitting the power of 50%. It must be pointed out the different sign in combining the BOC(1,1) and BOC(6,1) components between the data channel (denoted as CBOC(+)) and the pilot channel (denoted as CBOC(-)), according to the Galileo ICD. The random codes for Galileo E1 OS, will be used in tiered code structures featuring different lengths, as summarized in Tab. 1 [3].

Signal	Channel	Code length (chips)		Code type	Tiered code Period (ms)
		Primary	Secondary		
CBOC	B(data)	4092	-	Random	4
	C(pilot)	4092	25		100

Tab. 1. Galileo E1 OS PRN code structures.

Because the impact of codes cross-correlation will be evaluated in the worst case scenario where the signs of the data message and the secondary code are the same, the message and the secondary code may not essentially affect the following analysis results, and will be ignored in this paper. The baseband spread spectrum signal can then be written as [14]

$$c(t)=\sum_{k=-\infty}^{\infty} a[k]p(t-kT_c) \tag{1}$$

where $a[k]$ is the spreading sequence with a period of N, and $p(t)$ is the spreading symbol with a duration time of T_c. Thus $c(t)$ is pseudorandom waveform with a period of NT_c. For CBOC(+)/CBOC(-) signal, $p(t)$ is the weighted sum/difference of the BOC(1,1) and the BOC(6,1) spreading symbols.

Now we focus on the Auto- and Cross-Correlation Function (ACF and CCF) of GNSS signals. In order to obtain the CCF expression, we introduce

$$\begin{aligned} c_1(t)&=\sum_{k=-\infty}^{\infty} a_1[k]p_1(t-kT_c)\\ c_2(t)&=\sum_{k=-\infty}^{\infty} a_2[k]p_2(t-kT_c) \end{aligned} \tag{2}$$

where $a_1[k]$ and $a_2[k]$ are the spreading sequences with the period of N, and $p_1(t)$ and $p_2(t)$ are the spreading symbols with the same duration time of T_c.

The CCF of $c_1(t)$ and $c_2(t)$ is defined as [14]

$$R_{c_1/c_2}(\tau)=\frac{1}{NT_c}\int_0^{NT_c} c_1(t+\tau)c_2(t)dt\,. \tag{3}$$

Substituting (2) into (3) yields

$$\begin{aligned} R_{c_1/c_2}(\tau)=&\frac{1}{NT_c}\sum_{k=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} a_1[k]a_2[n]\\ &\times\int_0^{NT_c} p_1(t+\tau-kT_c)p_2(t-nT_c)dt \end{aligned}\,. \tag{4}$$

By letting $k = n + m$, one can obtain

$$\begin{aligned} R_{c_1/c_2}(\tau)=&\frac{1}{NT_c}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} a_1[n+m]a_2[n]\\ &\times\int_0^{NT_c} p_1(t+\tau-(n+m)T_c)p_2(t-nT_c)dt \end{aligned}\,. \tag{5}$$

Considering that $p_1(t)$ and $p_2(t)$ are nonzero for $t\in[0, T_c]$, (5) becomes

$$\begin{aligned} R_{c_1/c_2}(\tau)=&\frac{1}{NT_c}\sum_{m=-\infty}^{\infty}\sum_{n=0}^{N-1} a_1[n+m]a_2[n]\\ &\times\int_0^{NT_c} p_1(t+\tau-(n+m)T_c)p_2(t-nT_c)dt \end{aligned}\,. \tag{6}$$

For $0\le n<N$, the integral of (6) can then be expressed as

$$\begin{aligned} &\frac{1}{T_c}\int_0^{NT_c} p_1(t+\tau-(n+m)T_c)p_2(t-nT_c)dt\\ &=\frac{1}{T_c}\int_{-\infty}^{\infty} p_1(t+\tau-(n+m)T_c)p_2(t-nT_c)dt\\ &=R_{p_1/p_2}(\tau-mT_c) \end{aligned} \tag{7}$$

where $R_{p_1/p_2}(\tau)$ is the normalized cross-correlation function of $p_1(t)$ and $p_2(t)$.

Substituting (7) into (6) yields

$$\begin{aligned} R_{c_1/c_2}(\tau)&=\frac{1}{N}\sum_{m=-\infty}^{\infty}\sum_{n=0}^{N-1}\begin{pmatrix} a_1[n+m]a_2[n]\\ \times R_{p_1/p_2}(\tau-mT_c)\end{pmatrix}\\ &=\sum_{m=-\infty}^{\infty} R_{a_1/a_2}[m]R_{p_1/p_2}(\tau-mT_c) \end{aligned} \tag{8}$$

where $R_{a_1/a_2}[m]$ is the discrete cross-correlation function of $a_1[k]$ and $a_2[k]$, i.e.

$$R_{a_1/a_2}[m] = \frac{1}{N}\sum_{n=0}^{N-1} a_1[n+m]a_2[n]. \quad (9)$$

Equation (8) shows that the CCF of $c_1(t)$ and $c_2(t)$ is determined by the CCF of the spreading sequences ($a_1[k]$ and $a_2[k]$) and the CCF of spreading symbols ($p_1(t)$ and $p_2(t)$). Obviously, $R_{c_1/c_2}(\tau)$ is periodic with period NT_c. In fact, that is why GNSS codes correlation properties are analyzed using circular not linear correlation operation [7].

From (8), the ACF of $c(t)$ can be written as

$$R_c(\tau) = \sum_{m=-\infty}^{\infty} R_a[m]R_p(\tau - mT_c) \quad (10)$$

with

$$\begin{aligned} R_a[m] &= \frac{1}{N}\sum_{n=0}^{N-1} a[n+m]a[n] \\ R_p(\tau) &= \frac{1}{T_c}\int_{-\infty}^{\infty} p(t+\tau)p(t)dt \end{aligned}. \quad (11)$$

In fact, a particular concern is the CCF $R_{c_1/c_2}(\tau)$ within the interval [$-T_c$,T_c], called the main lobe, which will affect the discriminator functions of GNSS signals. According to (8), the main lobe of $R_{c_1/c_2}(\tau)$ is determined by $R_{a_1/a_2}[-1]R_{p_1/p_2}(\tau+T_c)$, $R_{a_1/a_2}[0]R_{p_1/p_2}(\tau)$ and $R_{a_1/a_2}[1]R_{p_1/p_2}(\tau-T_c)$. As a preliminary conclusion, if $R_{p_1/p_2}(\tau)$ is even symmetric and $R_{a_1/a_2}[-1] = R_{a_1/a_2}[1]$, the main lobe of $R_{c_1/c_2}(\tau)$ will be even symmetric.

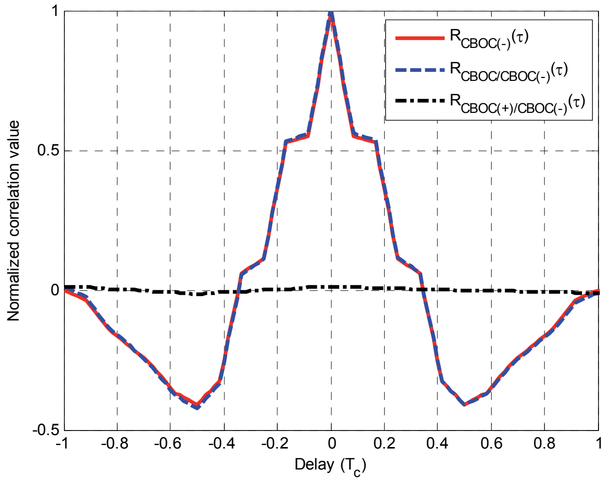


Fig. 1. Cross-correlation functions of CBOC signals.

According to the description above, we can obtain the CCFs of CBOC signals for Galileo E1 OS PRN1 shown in Fig. 1. It can be noted that the CCF of the CBOC (data and pilot together) signal and the pilot signal ($R_{CBOC/CBOC(-)}(\tau)$) is not symmetric, due to the asymmetry of the data/pilot CCF ($R_{CBOC(+)/CBOC(-)}(\tau)$). In the following section, the impact of the asymmetric CCF on the S-curve bias will be discussed in detail.

3. Codes Cross-Correlation Impact on Discriminator Functions

The impact of codes cross-correlation on the discriminator function (S-curve) for CBOC signals will be discussed in this Section. Several parameters and architectural choice, including the tracking algorithm and the correlator type and spacing, will be considered.

3.1 S-Curve Bias

The navigation receiver obtains the (noise-less) code delay by the zero-crossing of the code discriminator function (S-curve). Considering Coherent Early-Late Processing (CELP), the S-curve, based on the CCF $R_{c_1/c_2}(\tau)$, can be defined as [15]

$$Sc(\varepsilon,\Delta) = R_{c_1/c_2}(\varepsilon + \Delta/2) - R_{c_1/c_2}(\varepsilon - \Delta/2) \quad (12)$$

with its lock-point $\varepsilon_{bias}(\Delta)$ defined by

$$Sc(\varepsilon_{bias}(\Delta),\Delta) = 0 \quad (13)$$

where Δ is the correlator spacing (i.e., the early-late spacing), ε is the code delay, and $\varepsilon_{bias}(\Delta)$ represents the S-curve bias.

Considering that Δ is within the interval (0,T_c], $\varepsilon_{bias}(\Delta)$ is determined by the main lobe of $R_{c_1/c_2}(\tau)$. If the main lobe of $R_{c_1/c_2}(\tau)$ is even symmetrical (e.g., $R_c(\tau)$), we have the relationship

$$\begin{aligned} Sc(-\varepsilon,\Delta) &= R_{c_1/c_2}(-\varepsilon + \Delta/2) - R_{c_1/c_2}(-\varepsilon - \Delta/2) \\ &= R_{c_1/c_2}(\varepsilon - \Delta/2) - R_{c_1/c_2}(\varepsilon + \Delta/2) \\ &= -Sc(\varepsilon,\Delta) \end{aligned}. \quad (14)$$

In other words, the S-curve is an odd function, which guarantees that $\varepsilon_{bias}(\Delta)$ is zero for $0 < \Delta \le T_c$.

However, as mentioned above, the main lobe of $R_{c_1/c_2}(\tau)$ is even symmetric only when special requirements are satisfied. In practice, for example, tracking the Galileo E1 OS signal by using only the pilot component or the data component, to exactly satisfy these requirements is very difficult. The S-curve bias induced by codes cross-correlation for Galileo E1 OS is considered and analyzed in the following.

3.2 Impact of Codes Cross-Correlation on S-Curve Bias

For matched processing of CBOC signals (data and pilot channels together), the S-curve bias is zero. This is due to that the ACF is even symmetric. In order to evaluate the impact of the codes cross-correlation on the S-curve, let us consider tracking the Galileo E1 OS signals (data and pilot channels) by using only the pilot component (denoted as CBOC/CBOC(-)) or the data component (denoted as CBOC/CBOC(+)).

The corresponding S-curves using Galileo E1 OS PRN1 codes are depicted in Fig. 2, with CELP discriminator, early-late spacing of 1 chip and infinite bandwidth. In this case, it is assumed that the signs of the message and the secondary code are the same. It is clear that CBOC/CBOC(-) and CBOC/CBOC(+) methods induce the S-curve biases of 0.0225 chips and -0.0045 chips, respectively. The bias is due to the codes cross-correlation of data and pilot channels and the fact that data and pilot components are transmitted in phase. It is possible to notice that the codes cross-correlation properties can lead to asymmetric S-curves in case of receiving a single channel (e.g., the pilot channel).

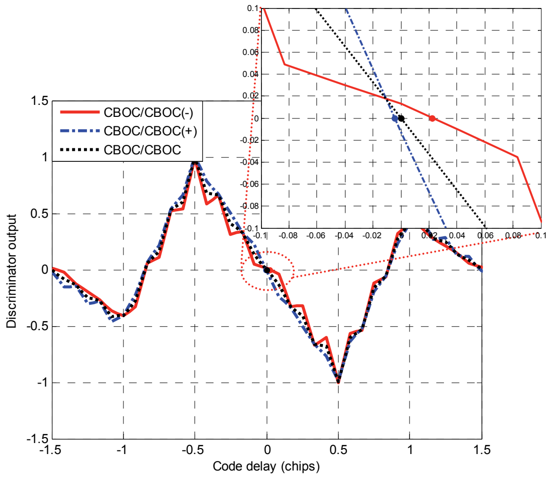


Fig. 2. Discriminator functions and its zoom for CBOC signals considering codes cross-correlation.

It is well known that the S-curve bias will result in the pseudorange error. According to the basic principle of GNSS positioning, if S-curve biases of satellites visible are the same, the positioning accuracy will not degrade. The S-curve biases induced by codes cross-correlation for all Galileo E1 OS signals are shown in Fig. 3 under the same conditions as Fig. 2. It can be clearly seen that the S-curve biases are not the same for different satellites and tracking methods. Thus, the codes cross-correlation effect cannot be neglected in positioning solution. The maximum bias is about 12.8 m for CBOC/CBOC(-), but 2.7 m for CBOC/CBOC(+). In this case, the CBOC/CBOC(+) method outperforms the CBOC/CBOC(-) method.

In order to fully analyze the impact of the codes cross-correlation, Fig. 4 shows the average S-curve biases of CBOC signals using arbitrary spacing, with CELP and Noncoherent Early-Late Processing (NELP) discriminators. The average S-curve bias is obtained from the absolute S-curve biases of all signals for specific early-late spacing. It can be observed from Fig. 4 that CBOC/CBOC(+) is potentially less sensitive to the early-late spacing than CBOC/CBOC(-). In other words, the CBOC(+) provides improvement of the resistance to codes cross-correlation, as compared to CBOC(-). Moreover, it can be noted that the S-curve biases of CELP are remarkably similar to NELP except for CBOC/CBOC(+) at the spacing near 0.7 chips.

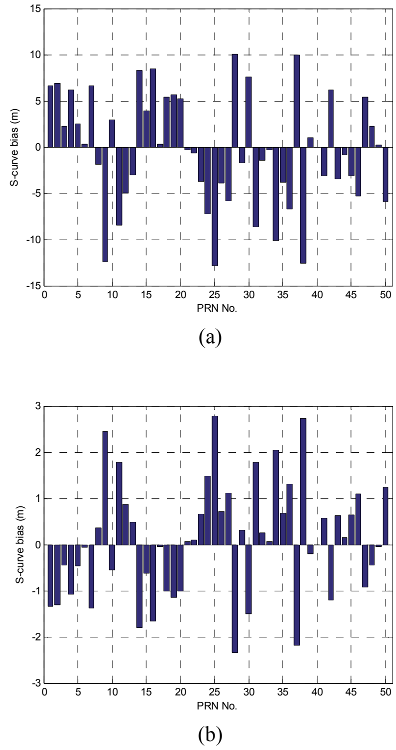


Fig. 3. S-curve biases induced by codes cross-correlation for Galileo E1 OS PRN1~50: (a) CBOC/CBOC(-), (b) CBOC/CBOC(+).

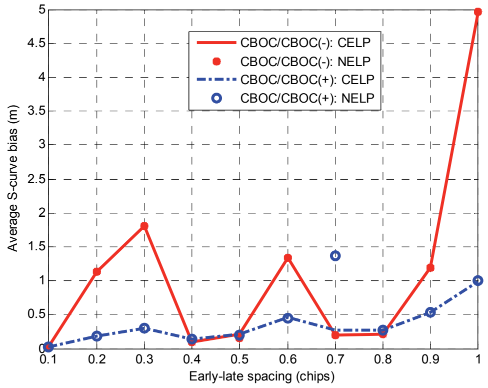


Fig. 4. Average S-curve biases of CBOC signals with CELP and NELP discriminators.

Except for the tracking methods mentioned above, CBOC signals can also be tracked with a BOC(1,1) receiver [16] or a TM61 receiver [17]. In Fig. 5, the impact of the codes cross-correlation on the average S-curve bias of CBOC signals with a BOC(1,1) receiver (denoted as CBOC/BOC(1,1)) is reported. In this case, CBOC(-)/BOC(1,1) and CBOC(+)/BOC(1,1) provide very similar performance for early-late spacings less than 0.9 chips. Compared to the results of CBOC/CBOC(-) (see Fig. 4), the CBOC(-)/BOC(1,1) method reduces the S-curve bias significantly.

Fig. 6 shows the impact of the codes cross-correlation on the S-curve average bias for CBOC/TM61 with Dot Product (DP) discriminator. It can be seen that the TM61 method will introduce very big biases for early-late spacings less than 0.9 chips. This is due to the fact that the power of BOC(6,1) component is very low with respect to the total CBOC power. Thus, the codes cross-correlation will significantly affect the ACF of BOC(6,1). At this point, the TM61 receiver may not suitable for CBOC signals.

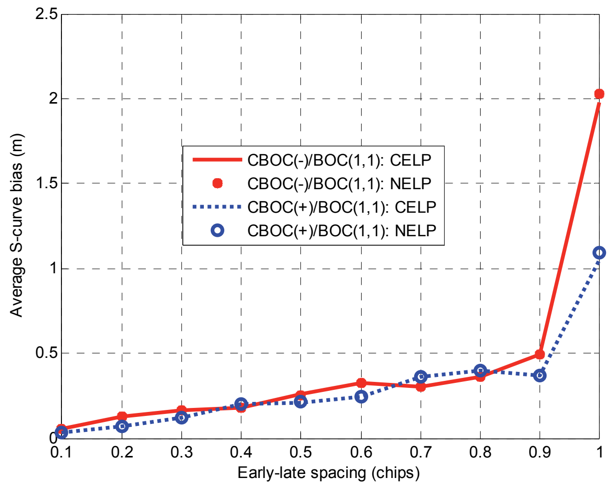


Fig. 5. Impact of codes cross-correlation on average S-curve biases for CBOC/BOC(1,1).

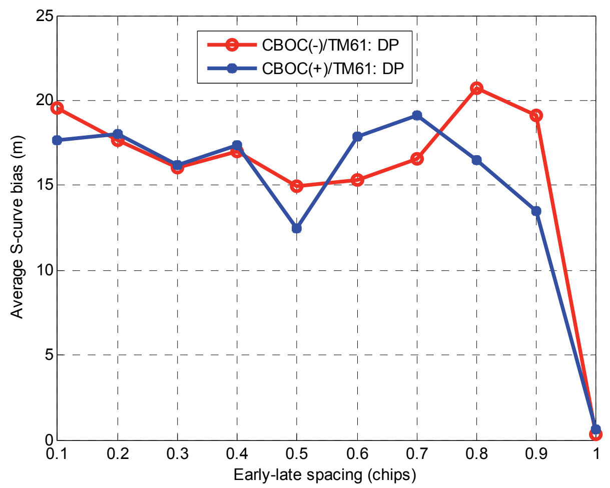


Fig. 6. Impact of codes cross-correlation on average S-curve biases for CBOC/TM61.

From the above discussion it is evident that the codes cross-correlation distortion on the S-curve can be magnified by an inappropriate choice of the early-late spacing, leading to noticeable worsening in receiver performance. Moreover, the tracking method for the data and pilot channels should be selected elaborately to decrease the S-curve bias induced by the codes cross-correlation.

4. Data-Pilot Code Pairs Optimization

As mentioned above, the data and pilot codes cross-correlation will result in the S-curve bias for Galileo E1 OS, which directly degrades code tracking performance. In this section, two methods to optimize the data-pilot code pairs of Galileo E1 OS, and then mitigate the codes cross-correlation, based on PRN code sets given by [3], are proposed and evaluated.

Method I: This method reassigns the pilot PRN code numbers, generating new data-pilot code pairs, but maintains the data and pilot PRN code groups given by [3].

Method II: This method regroups the PRN codes given by [3], generating new data-pilot code pairs.

In some studies, it has been shown that the modernized pilot channel would significantly improve the resistance of the code tracking loop to thermal noise [17], and improve the inherent multipath rejection capability of CBOC signals [18]. Thus the optimization criterion of proposed methods is to obtain minimum average S-curve bias when tracking only pilot components of CBOC signals for specific early-late spacing. We choose the early-late spacing of 1 chip as the optimization reference. As shown in Fig. 4, the maximum S-curve bias for CBOC/CBOC(-) occurs with the early-late spacing of 1 chip.

4.1 Method I

In this method, the data and pilot PRN code groups given by [3] are not changed, but the pilot PRN code numbers are reassigned. Then, new data-pilot code pairs are generated to obtain minimum average S-curve bias when tracking only the pilot component (i.e. CBOC/CBOC(-)) with the early-late spacing of 1 chips. That is to say, the data and pilot channel PRN code sequences are consistent with [3], but the PRN code numbers of the pilot channels will be changed.

For Galileo E1 OS, there are 50 data PRN code sequences and 50 pilot PRN code sequences, which can be modeled as

$$\begin{aligned}\mathbf{a}^{\mathrm{d}} &= \left[a_1^{\mathrm{d}}, a_2^{\mathrm{d}}, \cdots, a_{50}^{\mathrm{d}}\right] \\ \mathbf{a}^{\mathrm{p}} &= \left[a_1^{\mathrm{p}}, a_2^{\mathrm{p}}, \cdots, a_{50}^{\mathrm{p}}\right]^{\mathrm{T}}\end{aligned} \tag{15}$$

where a_k^d and $a_k^p, k = 1, 2, \cdots, 50$ represent data and pilot No. k code sequences defined in Galileo ICD, respectively.

In order to generate the optimized data-pilot code pairs, the S-curve biases induced by the possible data-pilot code pairs should be computed for tracking only the pilot component. It is convenient to introduce the bias matrix

$$\begin{aligned}\mathbf{B}_{N\times N} &= bias(\mathbf{a}^{\mathrm{d}}, \mathbf{a}^{\mathrm{p}}) \\ &= \begin{bmatrix} bias(a_1^{\mathrm{d}}, a_1^{\mathrm{p}}) & bias(a_1^{\mathrm{d}}, a_2^{\mathrm{p}}) & \cdots & bias(a_1^{\mathrm{d}}, a_N^{\mathrm{p}}) \\ bias(a_2^{\mathrm{d}}, a_1^{\mathrm{p}}) & bias(a_2^{\mathrm{d}}, a_2^{\mathrm{p}}) & \cdots & bias(a_2^{\mathrm{d}}, a_N^{\mathrm{p}}) \\ \vdots & \vdots & \ddots & \vdots \\ bias(a_N^{\mathrm{d}}, a_1^{\mathrm{p}}) & bias(a_N^{\mathrm{d}}, a_2^{\mathrm{p}}) & \cdots & bias(a_N^{\mathrm{d}}, a_N^{\mathrm{p}}) \end{bmatrix}\end{aligned} \tag{16}$$

where N=50, $(a_i^d, a_j^p), i, j = 1,2,\cdots,N$ represents the possible data-pilot PRN code pair (i.e. a_i^d as the data PRN code sequence and a_j^p as the pilot PRN code sequence), and $bias(a_i^d, a_j^p)$ is the S-curve bias for (a_i^d, a_j^p) when tracking only the pilot channel.

Mathematically, we can express the optimization problem as follows [19]. To minimize the cost function

$$z = \sum_{i=1}^{N}\sum_{j=1}^{N} v_{ij} \cdot |\, bias(a_i^d, a_j^p)\,| \qquad (17)$$

where
$$v_{ij} = \begin{cases} 1, & \text{if } (a_i^d, a_j^p) \text{ is chosen} \\ 0, & \text{if } (a_i^d, a_j^p) \text{ is not chosen} \end{cases}$$

with the restrictions

1) $\sum_{i=1}^{N} v_{ij} = 1, j = 1,2,\cdots,N$, i.e., only one element can be chosen in each column of **B**,

2) $\sum_{j=1}^{N} v_{ij} = 1, i = 1,2,\cdots,50$, i.e., only one element can be chosen in each row of **B**.

From the above description, the optimization problem can be modeled as the classic “assignment problem” in operation research, which can be solved using “Hungarian method” [20].

The optimized data-pilot code pairs are specified in Tab. 2. The “Pair/Data” rows in Tab. 2 show the new data-pilot code pair No. and the data PRN code No., which are the same as the original data PRN code No. given by [3]. The “Pilot” rows show the pilot PRN code No. given by [3], which is paired with the data PRN code No. of the row above in the same column, to generate a new data-pilot code pair. For example, the new pair No.1 is (1,47), where 1 represents the data code No. and 47 represents the pilot code No. given by [3]. It can be seen that the new pairs No. 6, 33 and 44 are the same as the original pairs defined in [3]. Comparison of the original and optimized data-pilot PRN code pairs is reported in Fig. 7, where the “green square” represents the original pair relationship defined by Galileo ICD, and the “red circle” represents the optimized pair relationship using method I. It is clear that the optimized pair relationship seems to be irregular, whereas the original pairs show linear relationship.

Type	Data and pilot PRN codes No.									
Pair/Data	1	2	3	4	5	6	7	8	9	10
Pilot	47	10	32	48	7	6	50	5	22	27
Pair/Data	11	12	13	14	15	16	17	18	19	20
Pilot	15	1	37	8	18	26	44	46	11	4
Pair/Data	21	22	23	24	25	26	27	28	29	30
Pilot	23	34	43	12	39	3	29	38	16	49
Pair/Data	31	32	33	34	35	36	37	38	39	40
Pilot	13	2	33	9	31	20	30	19	36	40
Pair/Data	41	42	43	44	45	46	47	48	49	50
Pilot	42	25	45	41	21	35	24	28	17	14

Tab. 2. Galileo E1 OS optimized data-pilot PRN code pairs: Method I.

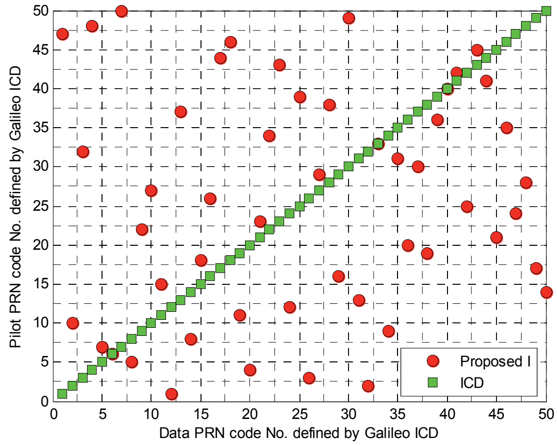
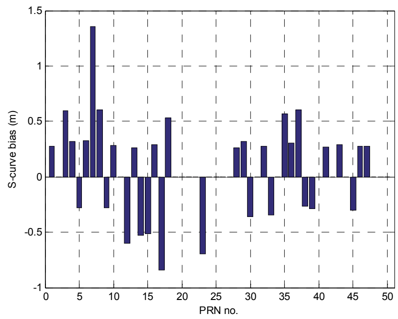
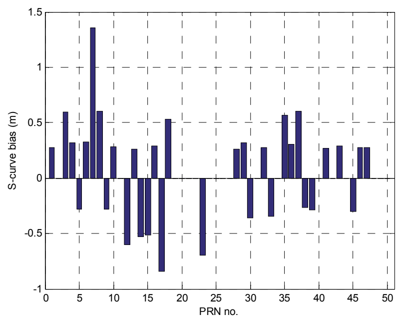


Fig. 7. Comparison of original and optimized data-pilot PRN code pairs of Galileo E1 OS.

The S-curve biases after optimization using Method I for all Galileo E1 OS signals are shown in Fig. 8 under the same conditions as Fig. 3. Both for CBOC/CBOC(-) and CBOC/CBOC(+), the S-curve biases reduce significantly, compared to un-optimized results (see Fig. 3).



(a)



(b)

Fig. 8. S-curve biases for Galileo E1 OS optimized by Method I: (a) CBOC/CBOC(-), (b) CBOC/CBOC(+).

In order to evaluate the performance of Method I comprehensively, the average S-curve biases after optimization for different tracking methods are shown in Fig. 9. For CBOC/CBOC(-), CBOC/CBOC(+), CBOC(-)/BOC (1,1) and CBOC(+)/BOC(1,1), method I provides about 25 dB improvement of the average S-curve bias, compared to Fig. 4 and Fig. 5. It should be noted that, although the optimization criterion of method I is directly related to the CBOC/CBOC(-) method with the early-late spacing of 1 chips, the optimized results are significantly improved for other tracking methods with different early-late spacings. However, for the TM61 method, the improvement is not so significant, and the average biases are still too big for CBOC receivers. Nevertheless, it is concluded that, for Galileo E1 OS, method I is very effective to mitigate data and pilot codes cross-correlation.

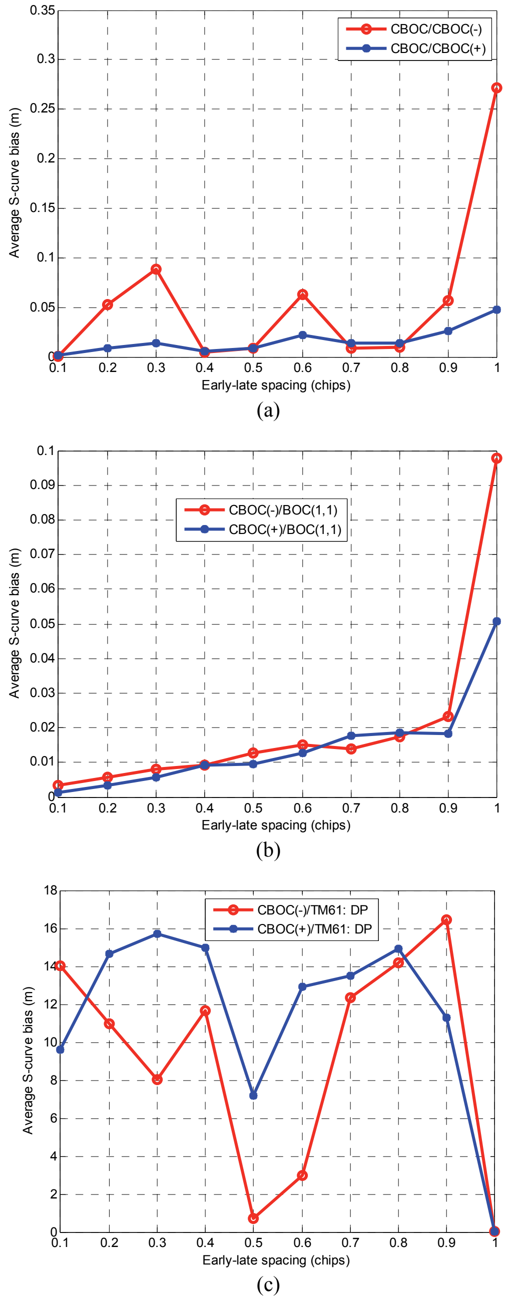


Fig. 9. Average S-curve biases for Galileo E1 OS optimized by Method I: (a) CBOC/CBOC(-) and CBOC/CBOC(+), (b) CBOC/BOC(1,1), (c) TM61.

4.2 Method II

In this method, the data and pilot PRN code groups given by [3] are changed, and the PRN code numbers will be reassigned. Then, new data-pilot code pairs are generated to obtain minimum average S-curve bias when tracking only the pilot components of CBOC signals with the specific early-late spacing of 1 chips. That is to say, the total PRN code sequences are consistent with [3], but the PRN code groups (i.e., data or pilot) and numbers may be changed.

For Galileo E1 OS, there are 100 PRN code sequences, which will be regrouped and renumbered. New data and pilot PRN code vectors are defined as

$$\begin{aligned} \mathbf{a}^{\mathrm{d}} &= \left[a_1^{\mathrm{d}}, a_2^{\mathrm{d}}, \cdots, a_{50}^{\mathrm{d}}, a_1^{\mathrm{p}}, a_2^{\mathrm{p}}, \cdots, a_{50}^{\mathrm{p}}\right] \\ \mathbf{a}^{\mathrm{p}} &= (\mathbf{a}^{\mathrm{d}})^{\mathrm{T}} \end{aligned} \tag{18}$$

where a_k^d and $a_k^p, k=1,2,\cdots,50$ represent data and pilot No. k code sequences defined in Galileo ICD.

Similarly to method I, the bias matrix is given by

$$\mathbf{B}_{100\times100} = (b_{ij})_{100\times100} = bias(\mathbf{a}^{\mathrm{d}}, \mathbf{a}^{\mathrm{p}})$$
$$= \begin{bmatrix} bias(a_1^{\mathrm{d}}, a_1^{\mathrm{d}}) & bias(a_1^{\mathrm{d}}, a_2^{\mathrm{d}}) & \cdots & bias(a_1^{\mathrm{d}}, a_{50}^{\mathrm{p}}) \\ bias(a_2^{\mathrm{d}}, a_1^{\mathrm{d}}) & bias(a_2^{\mathrm{d}}, a_2^{\mathrm{d}}) & \cdots & bias(a_2^{\mathrm{d}}, a_{50}^{\mathrm{p}}) \\ \vdots & \vdots & \ddots & \vdots \\ bias(a_{50}^{\mathrm{p}}, a_1^{\mathrm{d}}) & bias(a_{50}^{\mathrm{p}}, a_2^{\mathrm{d}}) & \cdots & bias(a_{50}^{\mathrm{p}}, a_{50}^{\mathrm{p}}) \end{bmatrix}. \tag{19}$$

In fact, the main diagonal elements of **B** are impossible to occur, because one code sequence cannot belong to data and pilot components simultaneously. Moreover, **B** is an antisymmetric matrix (i.e., $\mathbf{B}=-\mathbf{B}^{\mathrm{T}}$), which will be proved in the following.

Let a_1 and a_2 represent a_k^d or a_k^p, $k=1,2,\cdots,N$, and $a_1 \neq a_2$. For data-pilot PRN code pair (a_1,a_2), by considering (1), the CBOC(-) and CBOC(+) can be written as

$$\begin{aligned} c_{\mathrm{CBOC(+)},1}(t) &= \alpha c_{\mathrm{BOC(1,1)},1}(t) + \beta c_{\mathrm{BOC(6,1)},1}(t) \\ c_{\mathrm{CBOC(-)},2}(t) &= \alpha c_{\mathrm{BOC(1,1)},2}(t) - \beta c_{\mathrm{BOC(6,1)},2}(t) \end{aligned} \tag{20}$$

with $a=\sqrt{10/11}$, $\beta=\sqrt{1/11}$, and

$$\begin{aligned} c_{\mathrm{BOC(1,1)},i}(t) &= \sum_{k=-\infty}^{\infty} a_i[k] p_{\mathrm{BOC(1,1)}}(t-kT_{\mathrm{c}}), \quad i=1,2 \\ c_{\mathrm{BOC(6,1)},i}(t) &= \sum_{k=-\infty}^{\infty} a_i[k] p_{\mathrm{BOC(6,1)}}(t-kT_{\mathrm{c}}), \quad i=1,2 \end{aligned} \tag{21}$$

where $p_{\mathrm{BOC(1,1)}}(t)$ and $p_{\mathrm{BOC(6,1)}}(t)$ represent the spreading symbols of BOC(1,1) and BOC(6,1), respectively.

From (3), the CCF of CBOC (data and pilot together) and CBOC(-) can be written as

$$R_{\mathrm{CBOC/CBOC(-)},2}(\tau) = R_{c_{\mathrm{CBOC(+)},1}/c_{\mathrm{CBOC(-)},2}}(\tau) + R_{c_{\mathrm{CBOC(-)},2}}(\tau) \tag{22}$$

where $R_{c_{\mathrm{CBOC(+)},1}/c_{\mathrm{CBOC(-)},2}}(\tau)$ is the CCF of $c_{\mathrm{CBOC(+)},1}(t)$ and $c_{\mathrm{CBOC(-)},2}(t)$. Using (8), $R_{c_{\mathrm{CBOC(+)},1}/c_{\mathrm{CBOC(-)},2}}(\tau)$ can be expressed as

$$R_{c_{\mathrm{CBOC(+)},1}/c_{\mathrm{CBOC(-)},2}}(\tau) = \sum_{m=-\infty}^{\infty} R_{a_1/a_2}[m] \begin{pmatrix} \alpha^2 R_{p_{\mathrm{BOC}(1,1)}}(\tau - mT_c) \\ -\alpha\beta R_{p_{\mathrm{BOC}(1,1)}/p_{\mathrm{BOC}(6,1)}}(\tau - mT_c) \\ +\alpha\beta R_{p_{\mathrm{BOC}(6,1)}/p_{\mathrm{BOC}(1,1)}}(\tau - mT_c) \\ +\beta^2 R_{p_{\mathrm{BOC}(6,1)}}(\tau - mT_c) \end{pmatrix}. \quad (23)$$

Similarly, for data-pilot PRN code pair (a_2,a_1), the CCF of CBOC and CBOC(-) can be expressed as

$$R_{\mathrm{CBOC/CBOC(-)},1}(\tau) = R_{c_{\mathrm{CBOC(+)},2}/c_{\mathrm{CBOC(-)},1}}(\tau) + R_{c_{\mathrm{CBOC(-)},1}}(\tau) \quad (24)$$

with

$$R_{c_{\mathrm{CBOC(+)},2}/c_{\mathrm{CBOC(-)},1}}(\tau) = \sum_{m=-\infty}^{\infty} R_{a_2/a_1}[m] \begin{pmatrix} \alpha^2 R_{p_{\mathrm{BOC}(1,1)}}(\tau - mT_c) \\ -\alpha\beta R_{p_{\mathrm{BOC}(1,1)}/p_{\mathrm{BOC}(6,1)}}(\tau - mT_c) \\ +\alpha\beta R_{p_{\mathrm{BOC}(6,1)}/p_{\mathrm{BOC}(1,1)}}(\tau - mT_c) \\ +\beta^2 R_{p_{\mathrm{BOC}(6,1)}}(\tau - mT_c) \end{pmatrix}. \quad (25)$$

As shown in [7], the random codes of Galileo E1 OS fulfill the Autocorrelation Sidelobe Zero (ASZ) property (i.e. $R_{a_1}[-1] = R_{a_1}[1] = 0$ and $R_{a_2}[-1] = R_{a_2}[1] = 0$). From (10), the following relationship can be derived

$$R_{c_{\mathrm{CBOC(-)},1}}(\tau) = R_{c_{\mathrm{CBOC(-)},2}}(\tau), \quad -T_c \le \tau \le T_c . \quad (26)$$

Moreover, it can be easily proved that $R_{p_{BOC(1,1)}/p_{BOC(6,1)}}(\tau)$ is an even function.

Let ε_1=*bias*(a_1,a_2) and ε_2=*bias*(a_2,a_1). As mentioned above, ε_1 and ε_2 are determined by the main lobes of $R_{\mathrm{CBOC/CBOC(-)},2}(\tau)$ and $R_{\mathrm{CBOC/CBOC(-)},1}(\tau)$, respectively. Using (12) and (13), we can obtain

$$R_{\mathrm{CBOC/CBOC(-)},2}(\varepsilon_1 + \Delta/2) - R_{\mathrm{CBOC/CBOC(-)},2}(\varepsilon_1 - \Delta/2) = 0 . \quad (27)$$

Using (26) and (27), the following relationship can be derived

$$R_{\mathrm{CBOC/CBOC(-)},1}(-\varepsilon_1 - \Delta/2) - R_{\mathrm{CBOC/CBOC(-)},1}(-\varepsilon_1 + \Delta/2) = 0 . \quad (28)$$

Due to ε_2=*bias*(a_2,a_1), we obtain

$$R_{\mathrm{CBOC/CBOC(-)},1}(\varepsilon_2 + \Delta/2) - R_{\mathrm{CBOC/CBOC(-)},1}(\varepsilon_2 - \Delta/2) = 0 . \quad (29)$$

Combining (28) and (29) gives

$$\varepsilon_2 = -\varepsilon_1 . \quad (30)$$

That is *bias*(a_1,a_2) = -*bias*(a_2,a_1). Therefore, **B** is an anti-symmetric matrix. In other words, for two PRN code sequences, the absolute S-curve bias is the same, no matter which one is defined as the data PRN code.

Obviously, the "Hungarian method" cannot be directly applied to |**B**| to generate optimized data-pilot PRN code pairs. In order to ensure that the diagonal elements of **B** will not be chosen, we introduce a modified bias matrix

$$\mathbf{B}' = |\mathbf{B}| + \lambda \mathbf{I} \quad (31)$$

where **I** is the identity matrix, and λ is a positive big enough (e.g., $\lambda > \sum_{i=1}^{2N}\sum_{j=1}^{2N} |b_{ij}|$).

Then, using the "Hungarian method", we can obtain 100 elements from **B′**, which are symmetric along the main diagonal of **B′**, due to that **B′** is a symmetric matrix. If arbitrarily choose one from two elements symmetric along the main diagonal, we will obtain 50 elements, which correspond the new data-pilot PRN code pairs. Clearly, the optimized data-pilot PRN code pairs will not be unique.

For simplicity, we choose the 50 elements from the upper triangular part of **B′**. The results are reported in Tab. 3. The "Pair" rows show the new data-pilot pairs number. The "Data" rows show the PRN code No. in [3], and the corresponding PRN code belongs to the data channel after optimization. The "Pilot" rows show the PRN code No. in [3], which is paired with the data PRN code No. of the row above in the same column, to generate a new data-pilot code pair. And the corresponding PRN code belongs to the pilot channel after optimization. The postfix "d" represents the original code sequence belongs to the data channel, and "p" represents the original code sequence belongs to the pilot channel in [3]. For example, the new pair No.3 is (3d,8p), where "3d" indicates the data PRN code No. and "8p" indicates the pilot PRN code No. in [3]. Similarly to Tab. 2, the new pairs No. 6, 33 and 44 are the same as the original pairs defined in [3].

Type	Data and pilot PRN codes No.									
Pair	1	2	3	4	5	6	7	8	9	10
Data	1d	3p	3d	4d	5d	**6d**	7d	8d	9d	10d
Pilot	2d	5p	8p	41d	29d	**6p**	15d	16d	13p	50d
Pair	11	12	13	14	15	16	17	18	19	20
Data	11d	12d	13d	14d	4p	7p	17d	18d	19d	20d
Pilot	15p	11p	39d	22d	26p	44p	32d	30d	46p	49p
Pair	21	22	23	24	25	26	27	28	29	30
Data	21d	16p	23d	24d	25d	26d	27d	28d	17p	20p
Pilot	23p	50p	14p	12p	39p	1p	29p	38p	30p	31p
Pair	31	32	33	34	35	36	37	38	39	40
Data	31d	22p	**33d**	34d	24p	36d	37d	38d	32p	**40d**
Pilot	19p	47p	**33p**	35d	27p	49d	2p	10p	42p	**40p**
Pair	41	42	43	44	45	46	47	48	49	50
Data	34p	42d	43d	44d	45d	46d	47d	48d	36p	37p
Pilot	48p	25p	9p	41p	21p	35p	18p	28p	43p	45p

Tab. 3. Galileo E1 OS optimized data-pilot PRN code pairs: Method II.

Comparison of average S-curve biases optimized using method I and method II, for different tracking methods, are shown in Fig. 10. For CBOC/CBOC(-) and CBOC/CBOC(+), the improvement of method II has been shown to be around 8 dB in Fig. 10(a), as compared to method I. The similar performance improvement can also be observed for CBOC/BOC(1,1) (see Fig. 10(b)). Compared to the results of method I, the average S-curve biases of method II for CBOC/TM61 further decrease, but they are also too big in contrast with other methods. As regards to the principles of proposed methods, these results were expected.

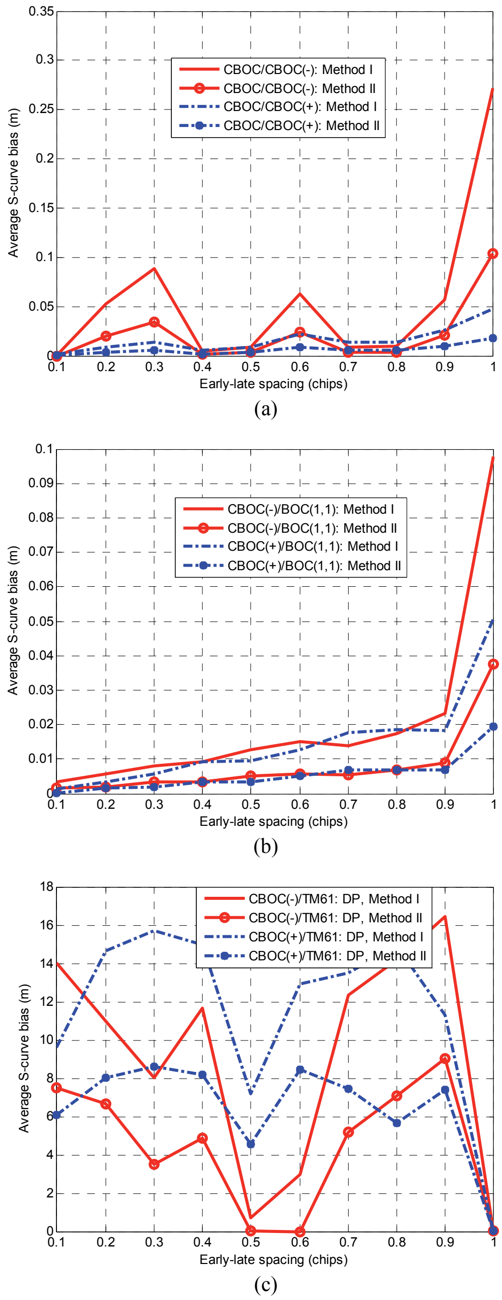


Fig. 10. Comparison of average S-curve biases optimized by using method I and method II: (a) CBOC/CBOC(-) and CBOC/CBOC(+), (b) CBOC/BOC(1,1), (c) TM61.

As described above, method I and method II mitigate data and pilot PRN codes cross-correlations significantly by optimizing the data-pilot PRN code pairs of Galileo E1 OS. It should be noted that the optimization criterion may not be unique (e.g. changing the early-late spacing). Similar results could be expected upon using other optimization criteria for CBOC signals. It can be concluded that the currently published Galileo E1 OS codes could still be further optimized to mitigate the effect of the codes cross-correlation, and thus a further increase of performance is still achievable in this regard.

5. Conclusion

The impact of the codes cross-correlation on the S-curve bias in case of receiving a single channel (e.g., the pilot channel) of CBOC signals has been presented in this paper. The S-curve bias can be magnified by an inappropriate choice of the early-late spacing, leading to noticeable worsening in receiver performance. It can be noted that the data channel provides improvement of the resistance to the codes cross-correlation with respect to the pilot channel. Considering the S-curve bias, CBOC reception with a BOC(1,1) receiver is recommendable, especially for the mass-market applications. However, it seems inappropriate to apply the TM61 method to CBOC signals tracking.

Two methods are proposed to optimize the data-pilot PRN code pairs, hence, mitigating data and pilot codes cross-correlation for CBOC signals. As compared to un-optimized results, the S-curve biases induced by the optimized data-pilot PRN code pairs decrease significantly. As for the average S-curve bias, method II outperforms method I for all tracking algorithms and early-late spacings considered. As regards to the principles of method I and method II, these results were expected. It should be noted that the optimization criterion may not be unique (e.g. changing the early-late spacing). Similar results could be expected upon using other optimization criteria. Analyses of this paper show that the currently published Galileo E1 OS codes could still be further optimized to mitigate the codes cross-correlation, and thus a further improvement in code tracking performance is still achievable in this regard.

Finally, it can be concluded that the intra-channel (data and pilot) codes cross-correlation would be an important criterion for PRN codes design, especially when data and pilot components are transmitted in phase. Furthermore, the modulation characteristics of data and pilot signals should also be considered. As for the complexity in PRN codes design, it is advisable to transmit the data and pilot components in quadrature (e.g., the GPS L5 signal), or multiplex the data and pilot components in time domain (e.g., the GPS L2C signal) for future GNSS signals.

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