# Channel Parameters Estimation Algorithm Based on the Characteristic Function under Impulse Noise Environment

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**Abstract.** *Under communication environments, such as wireless sensor networks, the noise observed usually exhibits impulsive as well as Gaussian characteristics. In the initialization of channel iterative decoder, such as low density parity check codes, it is required in advance to estimate the channel parameters to obtain the prior information from the received signals. In this paper, a blind channel parameters estimator under impulsive noise environment is proposed, which is based on the empirical characteristic function in MPSK/MQAM higher-order modulation system. Simulation results show that for various MPSK/MQAM modulations, the estimator can obtain a more accurate unbiased estimation even though we do not know which kind of higher-order modulation is used.*

## Keywords

Parameter estimation, impulsive noise, symmetric alpha stable distribution, characteristic function.

## 1. Introduction

Employing the iterative codes, such as turbo codes and low-density parity check codes (LDPC), for channel coding is in a growing trend. In standard iterative soft decoder, an important initialization is to estimate the channel parameters to obtain the soft prior information from the received symbols. The performance of the decoder relies on the Gaussian assumption for the additive noise. However, impulsive noise arises as results of automobile spark plugs, lightning discharges, underwater sonar, etc. Moreover, studies [1]-[2] show that, in a multi-user network with power-law path loss, the multiple access interference results in a Symmetric Alpha Stable (S$\alpha$S) distribution. Therefore, the received signals in wireless networks are corrupted by noise that is a mixture of both Gaussian and $\alpha$-stable components. As a matter of fact, an increasing number of applications require consideration of impulsive and non-Gaussian noise in wireless communications [3]. Thus, it is of paramount importance to incorporate the effect of impulsive noise in the initialization of standard iterative soft decoder.

Summers and Wilson [4] proposed a signal to noise ratio (SNR) estimation algorithm called online SNR estimator (OSNRE), and studied the performance of the turbo decoder using OSNRE. Li [5] generalized OSNRE from binary phase-shift keying (BPSK) to 8-PSK modulation symbols, and proposed a blind SNR estimation algorithm (8PSK-BSNRE) using the moment theory and function fitting method. The authors claimed that the proposed algorithm can be applied to other higher-order modulation systems. Seen from the derivation steps of the 8PSK-BSNRE algorithm, however, different modulation types need different fitting functions. That is why the proposed blind SNR estimation algorithm is not universal. Moreover, the SNR scope of the estimator is narrow. To solve these disadvantages, Xu [6] proposed a low complexity SNR estimator based on the empirical characteristic function (ECF) for both $M$-ary PSK (MPSK) and $M$-ary quadrature amplitude modulation (MQAM), which provided a good unbiased performance even without knowing the modulation type.

Meanwhile, a common disadvantage of these works is to consider only the white Gaussian noise (WGN), regardless of the presence of the actual impulsive noise. As mentioned before, the impulsive noise (or interference) can not be negligible in many communication scenarios. The performance of the iterative decoder based on the Gaussian assumption is deteriorated sharply because of the incorrect soft prior information due to the presence of the impulsive noise. In order to ensure that the iterative decoding algorithm is still applicable, a new prior information estimator is needed to take account of such impulsive noise environment.

The alpha stable distribution encompasses an important class of distributions which can successfully model a number of impulsive noise processes [7], because it satisfies the generalized center extreme limit theorem. In particular, the Gaussian distribution can be considered as a special case of the S$\alpha$S distribution [8]. In this paper, the impulsive noise is modeled as a S$\alpha$S because in most of the practical communication systems, the impulsive noise is assumed to be symmetrical. Unfortunately, explicit expressions for the probability density function (PDF) of the SαS distribution in terms of *elementary functions* are still unknown except for the Gaussian and Cauchy laws [9]. This limits the application of the SαS distribution in practice. Nevertheless, it is still an open question on how to obtain a simple yet accurate estimator of the prior information with the impulsive noise.

Brcich et al use the weighted least squares to increase the estimation accuracy compared to using the moment function method, and propose a parameter estimator based on the nonlinear weighted least squares (NWLS) in the characteristic function domain [10]. However, this estimator ignores the effect of modulation types of channel parameter estimation. Swami et al use the normalized correlations, moments, and cumulants to obtain consistent estimates of the autoregressive moving average parameters of the linear *SαS* processes. This method works even under an additive noise (Gaussian or non-Gaussian, white or colored) with finite variance [11]. However, the ARMA model does not have an explicit PDF expression, and is not suitable for iterative decoding.

For the received samples modeled by (8) in Section 2, taking BPSK as an example, the optimal log-likelihood ratio (LLR) of initialization in iterative decoding is given by [12]

$$LLR(r_k) = \ln \frac{f(r_k|s_k = 1)}{f(r_k|s_k = -1)} \tag{1}$$

where $f$ is the PDF of the mixed noise. Notice that $LLR(r_k)$ represents the contribution from the $k$-th channel observation and is also called as the *intrinsic information*.

As the simplest suboptimal solution, LLR is simplified by assuming $\alpha = 2$ (*i.e.*, the noise is assumed to be Gaussian), and the intrinsic information of the standard sum-product algorithm is given by

$$LLR(r_k) = \frac{2}{\sigma^2} r_k \tag{2}$$

where $\sigma^2$ is the variance of Gaussian noise, and the constant of proportionality $\frac{2}{\sigma^2}$ is called the *channel reliability*.

However, the Gaussian assumption makes poor decoding performance compared to the optimal one when $\alpha < 2$. Meanwhile, the optimal LLR is still impractical because it requires complex computations to evaluate. To overcome this disadvantage, Sureka *et al* proposed the alternative PDF to approximate the actual one. According to [13], this alternative PDF is given by

$$f(v) = \frac{1}{I}\left(c_1 g_0 e^{-\frac{v^2}{4\gamma_s g}} + \frac{\hat{\alpha}\hat{\gamma}_s C_{\hat{\alpha}}}{|v|^{\hat{\alpha}+1} + c_2}\right) \tag{3}$$

where all the parameters are functions of channel parameters $(\alpha, \sigma_\alpha, \sigma_g)$, respectively. The proposed performance is almost indistinguishable from that of the intractable optimum receiver over a wide range of noise parameters. However, the parameters estimation of $f(v)$ is too complicated. Anyway, it is possible to obtain the intrinsic information in the condition that the channel parameters are estimated in advance.

As we know, the characteristic function (CF) and the PDF of the same random variable form a pair of Fourier transforms. Fortunately, the CF of the SαS distribution has a simple analytical expression. Based on the CF, we propose a new channel parameters estimator in the paper. Specifically, the CFs of the transmitted symbol sequences for the various normalized higher-order modulation type are derived. Then, the variance $\sigma_g^2$ of the Gaussian noise and the characteristic exponent $\alpha$, the divergence $\sigma_\alpha$ of the SαS distribution noise are estimated simultaneously based on the ECF.

The rest of the paper is organized as follows. The SαS distribution and system model is briefly introduced in Section 2. We derive the CF of the transmitted higher-order modulation symbols and point out that the transmitted symbols obey the Gaussian distribution since both of them have the same CF expression under appropriate simplified assumptions in Section 3. An ECF-based channel parameters estimator is proposed in Section 4. In Section 5, we show the performances of our proposed estimator and verify its effectiveness and robustness. The results show that the proposed estimator obtains a better performance and is very robust with low complexity.

# 2. SαS Distribution and System Model

## 2.1 SαS Distribution Model

In 1925, Lévy put forward to the conception of alpha stable distribution. As it satisfies the generalized central limit theorem, the alpha stable distribution has been confirmed on theory and experiments to describe the impulsive noise perfectly. According to [9], the SαS distribution can be completely determined by two parameters: (1) a characteristic exponent $\alpha \in (0, 2]$, which indicates the characteristic of the tail of the SαS distribution. Specifically, a smaller $\alpha$ leads to a greater probability of existing larger pulse and a thicker (or heavier) tail compared to the Gaussian distribution which decays exponentially; (2) a divergence (or scale) parameter $\sigma$ (or $\gamma = \sigma^\alpha$)$\in (0, \infty)$, which is analogous to the variance of the Gaussian distribution. Despite of its simple CF shown in (4), there is yet no explicit expression for the PDF $f_\alpha(x)$ of the SαS distribution except for $\alpha = 1$ (*i.e.*, the Cauchy distribution) and $\alpha = 2$ (*i.e.*, the Gaussian distribution). A real valued SαS random variable, $w \sim S_\alpha(\sigma)$, has a characteristic function $\phi_w(t)$ given by

$$\phi_w(t) = \mathbb{E}[\exp(\imath wt)] = \int_{-\infty}^{\infty} f_\alpha(w) e^{\imath wt} \mathrm{d}w = e^{-|\sigma t|^\alpha}. \tag{4}$$

In particular, $G \sim S_2(\sigma)$ is Gaussian distribution with zero mean and variance $2\sigma^2$, which can be considered as a special case of the SαS distribution. The CF of the Gaussian distribution is given by

$$\phi_G(t) = e^{-|\sigma t|^2}. \tag{5}$$

BPSK signals in real domain and MPSK/MQAM signals in complex domain (I and Q components in Constellation respectively) are used to investigate the performances in this study. We now define *complex isotropy* SαS random variables, which we will use to model the noise in next section. If both $w_I := \Re\{w\}$ and $w_Q := \Im\{w\}$, distributed as $S_\alpha(\sigma)$, are independent and identically distribution (i.i.d.), then

$$w = w_I + \imath w_Q \tag{6}$$

is said to follow complex isotropy SαS distribution denoted by $w \sim \mathcal{C}S_\alpha(\sigma)$. Specially, if $w \sim \mathcal{C}S_\alpha(\sigma)$, it means that the characteristic exponent $\alpha_I = \alpha_Q = \alpha$ and the divergence $\sigma_{\alpha_I} = \sigma_{\alpha_Q} = \sigma_\alpha$. Since both $w_I$ and $w_Q$ are independent, the characteristic function of the complex isotropy SαS distribution random variable, *i.e.* $w$, can be expressed as follows

$$\begin{aligned}\phi_w(t_1,t_2) &= \mathbb{E}[e^{\imath(t_1 w_I + t_2 w_Q)}] = \mathbb{E}[e^{\imath t_1 w_I}]\cdot\mathbb{E}[e^{\imath t_2 w_Q}] \\ &= e^{-|\sigma t_1|^\alpha - |\sigma t_2|^\alpha}.\end{aligned} \tag{7}$$

## 2.2 System Model

We make the following assumptions throughout this paper: first, our algorithms operate on discrete sequences of observations; second, the discrete-time additive noise components in the observation-process are independent. The system model under consideration is the complex, discrete, baseband-equivalent, band-limit model of coherent MPSK/MQAM in complex domain (which is also a suitable model of BPSK in real domain with minor modifications). Perfect carrier recovery and symbol timing recovery are also assumed. We introduce SαS into a statistical model of impulsive noise, and consider that the received interference is a mixture of complex impulsive noise and Gaussian noise. Therefore, the signal presented to the receiver is described by the following relation:

$$r_k = \sqrt{E}s_k + w_{\alpha,k} + w_{G,k} = \sqrt{E}s_k + w_k\ ,k = 1,2,\ldots,N \tag{8}$$

where $E$ is the signal's average power. Without loss of generality, we assume $E = 1$. The transmitted signal $s_k$ is chosen from a normalized MPSK/MQAM constellation; $w_{\alpha,k}$ follows complex isotropy SαS distribution; similarly, $w_{G,k}$ follows complex isotropy Gaussian distribution; the complex noise $w_k$ is referred to as the complex mixed noise.

For simplicity, (8) with the vector form is expressed as

$$R = S + W = S + W_\alpha + W_G \tag{9}$$

# 3. CF of the Received Signal

From (9), it is easy to obtain the corresponding characteristic function of the received signal, $\phi_R(t_1,t_2)$, which is given by

$$\phi_R(t_1,t_2) = \phi_S(t_1,t_2)\cdot\phi_W(t_1,t_2) \tag{10}$$

since the transmitted signal $S$ and the complex mixed noise $W$ are assumed to be mutually independent. To simplify the calculation, we assume that $t_1 = t_2 = t > 0$ in the next sections. Under this assumption, (10) is simplified to

$$\phi_R(t) = \phi_S(t)\cdot\phi_W(t). \tag{11}$$

We have noticed that the complex Gaussian $W_G$ also follows $\mathcal{C}S_\alpha(\sigma)$. Therefore, reusing (7) with the assumption $t_1 = t_2 = t$, the CF of $W_\alpha, W_G$ and $W$ are also simplified and given by

$$\phi_{W_\alpha}(t) = e^{-2(\sigma_\alpha t)^\alpha}, \tag{12a}$$

$$\phi_{W_G}(t) = e^{-2(\sigma_g t)^2}, \tag{12b}$$

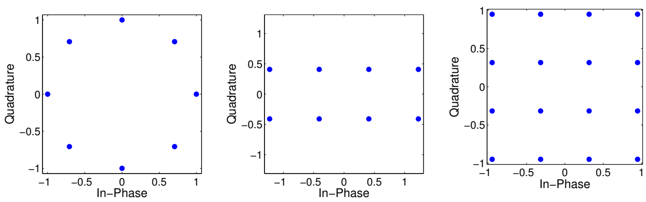
$$\phi_W(t) = \phi_{W_\alpha}(t)\cdot\phi_{W_G}(t) = e^{-2(\sigma_\alpha t)^\alpha - 2(\sigma_g t)^2} \tag{12c}$$

where $\sigma_\alpha$ is the divergence of $\mathcal{C}S_\alpha(\sigma)$ noise, and $\sigma_g$ is the standard variance of the Gaussian noise.

The following is to derive the CF of the transmitted symbol $S$, denoted as $\phi_S(t)$. The $\phi_S(t)$ can be obtained by the direct use of the definition of discrete random variable characteristic function, i.e.

$$\phi_S(t) = \sum_{m=0}^{M-1} \Pr(s_m) e^{\imath t(s_{m,I} + s_{m,Q})} \tag{13}$$

where $s_m$ is a symbol from the normalized MPSK/MQAM constellation; $\Pr(s_m)$ is the probability of the symbol $s_m$. In a communication system, it is reasonable to assume that each symbol in a fixed constellation is equiprobable. That is to say $\Pr(s_m) = 1/M$; $s_{m,I}$, $s_{m,Q}$ are the components of the symbol $s_m$ in the I and Q axis respectively.




**(a)** 8PSK **(b)** 8QAM **(c)** 16QAM

**Fig. 1.** Normalized MPSK/MQAM constellation.

In Fig. 1(a), for example, the symbols in the normalized 8PSK constellation are $(1,0)$, $(1/\sqrt{2},1/\sqrt{2})$, $(0,1)$, $(-1/\sqrt{2},1/\sqrt{2})$, $(-1,0)$, $(-1/\sqrt{2},-1/\sqrt{2})$, $(0,-1)$, $(1/\sqrt{2},-1/\sqrt{2})$ respectively. According to (13), $\phi_S(t)$ can be expressed as

$$\begin{aligned}\phi_S(t) &= \frac{1}{8}\Big[2e^{\imath t(0+1)} + 2e^{\imath t(-1+0)} + 2e^{\imath t(1/\sqrt{2}-1/\sqrt{2})} \\ &\quad + e^{\imath t(1/\sqrt{2}+1/\sqrt{2})} + e^{\imath t(-1/\sqrt{2}-1/\sqrt{2})}\Big] \\ &= \frac{1}{4}\Big[1 + 2\cos(t) + 2\cos(\sqrt{2}t)\Big].\end{aligned} \tag{14}$$

Similarly, it is easy to derive the CFs of the other higher-order MPSK/MQAM modulations, listed in Tab. 1.

| Modulation | $\phi_S(t)$ |
|---|---|
| BPSK | $\cos(t)$ |
| QPSK | $\cos(t)$ |
| 8PSK | $\frac{1}{4}[1+2\cos(t)+\cos(\sqrt{2}t)]$ |
| 4QAM | $\frac{1}{2}[1+\cos(\sqrt{2}t)]$ |
| 8QAM | $\frac{1}{4}[1+2\cos(\frac{2t}{\sqrt{6}})+\cos(\frac{4t}{\sqrt{6}})]$ |
| 16QAM | $\frac{1}{8}[2+\cos(\frac{6t}{\sqrt{10}})+2\cos(\frac{4t}{\sqrt{10}})+3\cos(\frac{2t}{\sqrt{10}})]$ |
| 64QAM | $\frac{1}{32}[4+7\cos(\frac{2t}{\sqrt{42}})+6\cos(\frac{4t}{\sqrt{42}})+5\cos(\frac{6t}{\sqrt{42}})$ $+4\cos(\frac{8t}{\sqrt{42}})+3\cos(\frac{10t}{\sqrt{42}})+2\cos(\frac{12t}{\sqrt{42}})$ $+\cos(\frac{14t}{\sqrt{42}})]$ |

**Tab. 1.** The CFs of the higher-order MPSK/MQAM modulations

In practice, we use the ECF to estimate the CF of mixed random variables and extract information about their parameters. Let the vector $\{x_k, k=1,\dots,N\}$ be a set of random observations of an i.i.d random variable $X$. Then the ECF $\hat{\phi}_X(t)$ is given by

$$\hat{\phi}_X(t) = \frac{1}{N}\sum_{k=1}^{N} e^{\iota t x_k} \tag{15}$$

where $N$ is the length of the observation vector and $t \in \mathbb{R}$. Thus, $\hat{\phi}_X(t)$ is computable for all values of $t$. For a given $t$, $\hat{\phi}_X(t)$ is a random variable, and $\{\hat{\phi}_X(t), t \in \mathbb{R}\}$ is a stochastic process. The ECF of $\hat{\phi}_X(t)$ is a sufficient accurate approximation of the CF $\phi_X(t)$ when $N$ is large enough. Moreover, it is an unbiased estimator. The ECF associated with the observation vector $\{x_k, k=1,\dots,N\}$ evaluated at $m$ points $\mathbf{t}=[t_1,\dots,t_m]$ is the complex random vector $\hat{\phi}_X(\mathbf{t}) = [\hat{\phi}_X(t_1),\dots,\hat{\phi}_X(t_m)]$. The $\hat{\phi}_X(\mathbf{t})$ converges weakly to a complex Gaussian random vector with the mean $\phi_X(\mathbf{t})$ and the covariance matrix $\mathbf{C}$ [14], whose each element is given by

$$\mathbf{C}_{j,k} = \mathrm{cov}\left(\hat{\phi}(t_j), \hat{\phi}(t_k)\right) = \frac{1}{N}\left[\phi(t_j+t_k) - \phi(t_j)\phi(t_k)\right]. \tag{16}$$

For a given $N$, we have a 'good' estimation accuracy of the CF if $t$ is close to zero, and has a 'poor' accuracy if $t$ is large. Specially, the covariance matrix $\mathbf{C}$ is smaller as $t$ approaches to zero, and the estimation performance of the ECF is better. On the contrary, the numerical calculation of the ECF is unstable, and the covariance matrix $\mathbf{C}$ is larger as $t$ becomes too large. That means the estimation performance of the ECF is worse. Therefore, a value close to zero is chosen for $t$ to get the ECF of the observation vector $\{x_k, k=1,\dots,N\}$.

Based on the above discussion, a value close to zero is chosen for $t$ to calculate the ECF. In this condition, the CF $\phi_S(t)$ of the higher-order MPSK/MQAM modulations can be approximated by a fitting function $\hat{\phi}_S(t)$, which is defined as

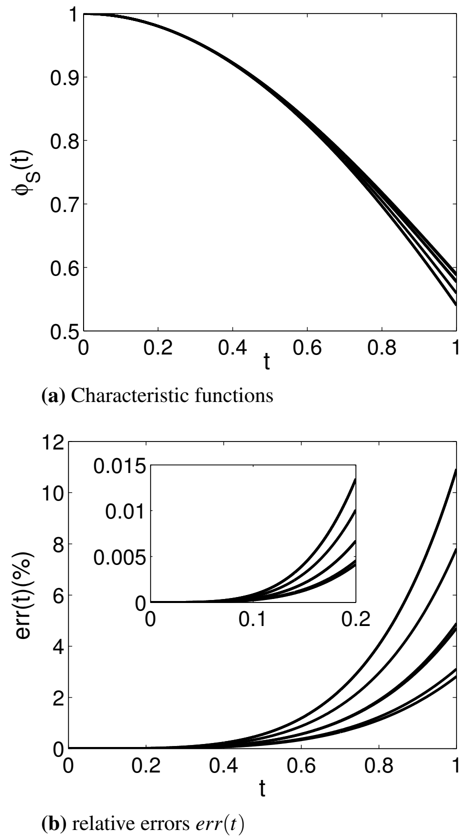
$$\hat{\phi}_S(t) \triangleq e^{-\frac{1}{2}t^2}. \tag{17}$$

Equation (17) may be explained by the central limit theorem. The constellation shown in Fig. 1 is seen as a discrete random variable. Due to the symmetry and normalization of the constellation, the mean of this random variable is 0, and its variance is 1. Therefore, this random variable converges to a standard normal distribution according to the central limit theorem.

In order to verify the approximation effect, we define a relative error $err(t)$ as

$$err(t) = \left(1 - \frac{\hat{\phi}_S(t)}{\phi_S(t)}\right) * 100\% \tag{18}$$

Combining (17), (18) with Tab. 1, for various higher-order MPSK/MQAM modulations, the CFs are shown in Fig. 2(a), and the relative errors $err(t)$ are shown in Fig. 2(b). We observe that the relative errors $err(t)$ are an increasing function of $t \in (0,1)$ with $\lim_{t\to 0} err(t) = 0$. In addition, the top curve corresponds to the BPSK and QPSK since they have the same CF, and the bottom curve corresponds to the 16QAM in Fig. 2(b). That is to say, the largest approximation error is BPSK/QPSK, and the smallest is 64QAM, which is in consistent with the central limit theorem. Further, the $err(t)$ is smaller than 0.015 % when $t \le 0.2$. Therefore, $\hat{\phi}_S(t)$ is accurate enough when $t \le 0.2$.




**(a)** Characteristic functions

**(b)** relative errors $err(t)$

**Fig. 2.** The CFs and relative errors of various higher-order modulation (listed in Tab. 1) symbols.

Combining (11), (12) with (17), the CF $\phi_R(t)$ of the received sequences $R$ is given by

$$\phi_R(t) = e^{-t^2/2-2(\sigma_\alpha t)^\alpha-2(\sigma_g t)^2}, \ t \leq 0.2 \tag{19}$$

since the transmitted symbols $S$ and the complex mixed noise $W$ are independent.

# 4. ECF-based Parameter Estimation

We obtain the CF $\phi_R(t)$ of the received symbol $R$ in the previous section. In this section, we discuss procedures to estimate parameters of the complex mixed noise, which is the sum of two independent SαS random variables with the CF given in (19). The estimation procedures proposed in this section are important for the initiation of the iterative soft decoder, as we state before.

The estimation of parameters of the stable laws is severely hampered by the lack of closed-form expressions for the PDF, and the problem is even more complicated for the complex mixed noise. Fortunately, the moment-type method does not rely on its PDF, which means that we do not have to know the PDF of the complex mixed noise in advance. Let

$$\theta_1 = 2\sigma_\alpha^\alpha, \tag{20a}$$

$$\theta_2 = 2\sigma_g^2 + \frac{1}{2}. \tag{20b}$$

Taking the logarithm transformation to both sides of (19), we obtain

$$\psi_R(t) = -\ln(\phi_R(t)) = \theta_1 t^\alpha + \theta_2 t^2 \tag{21}$$

where $\psi_R(t)$ denotes the second characteristic function. In practical estimation process, the $\psi_R(t)$ is replaced by its estimated value $\hat{\psi}_R(t)$ which is given by

$$\hat{\psi}_R(t) = -\ln\left(\frac{1}{N}\sum_{k=1}^{N} e^{\iota t(r_{I,k}+r_{Q,k})}\right). \tag{22}$$

Here $r_{I,k}, r_{Q,k}$ are the I and Q component of the $k$-th received signals, respectively.

Forming a similar set of equations for $\varepsilon^{-1}t, t$, $\varepsilon t \in (0, 0.2)$, replacing $\psi_R(\cdot)$ by its estimate $\hat{\psi}_R(\cdot)$, and after simple mathematical derivation, we obtain the estimated parameters $\hat{\alpha}, \hat{\sigma}_\alpha, \hat{\sigma}_g$ of the complex mixed noise. They are given by

$$\varepsilon^{\hat{\alpha}} = \frac{\varepsilon^{-2}\hat{\psi}_R(\varepsilon t) - \hat{\psi}_R(t)}{\varepsilon^{-2}\hat{\psi}_R(t) - \hat{\psi}_R(\varepsilon^{-1}t)} \tag{23}$$

and

$$\begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \end{bmatrix} = \frac{1}{\varepsilon^2 - \varepsilon^{\hat{\alpha}}} \begin{bmatrix} \varepsilon^2 t^{-\hat{\alpha}} & -t^{-\hat{\alpha}} \\ -\varepsilon^{\alpha} t^{-2} & t^{-2} \end{bmatrix} \begin{bmatrix} \hat{\psi}_R(t) \\ \hat{\psi}_R(\varepsilon t) \end{bmatrix} \tag{24a}$$

$$\hat{\sigma}_\alpha = (\frac{1}{2}\hat{\theta}_1)^{1/\hat{\alpha}} \tag{24b}$$

$$\hat{\sigma}_g = \sqrt{\frac{1}{2}(\hat{\theta}_2 - \frac{1}{2})} \tag{24c}$$

where $\hat{\alpha}$ is the estimate obtained in (23). The estimates of $\alpha$ and $\sigma_\alpha, \sigma_g$ given in (23) and (24) are consistent since they are based on $\hat{\psi}_R(\cdot)$, which is consistent. However, the rate of convergence with the number of samples $N$ to the true values will depend on the choice of $\varepsilon$ and $t$. In the following section, simulations are carried out to test and verify the proposed estimate algorithm.

# 5. Simulation Results and Conclusions

The simulation platform is constructed as shown in Fig. 3. The "Bernoulli random binary generator" block generates random binary numbers using a Bernoulli distribution that produces '1's with the probability $p = \frac{1}{2}$. In order to improve the bit error rate performance of a traditional communication system, there must exist a channel codec. As an example, a "convolutional encoder" with a constraint length of 7, code generator polynomials of 171 and 133 (in octal numbers), and a feedback connection of 171 (in octal) is used for channel coding. The modulator is used to map the binary data to a MPSK/MQAM symbol, and chooses 'binary' (*i.e.*, binary mapping) choice for mapping symbols to ideal constellation points. The "normalize" block is to ensure that the symbols transmitted average power is 1, regardless of the transmitted symbols modulation. The MATLAB code of the SαS distribution is obtained from [16].

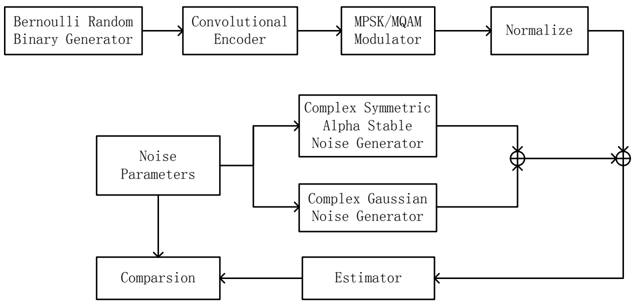



**Fig. 3.** Simulation system.

The estimation procedure requires solving (23) and (24) at various values of $t$ and $\varepsilon$. However, the optimum choice of these variables is not clear. The performance of the estimation strongly depends on the number of samples (*i.e.*, $N$) available and the choice of these variables. The "estimator" block uses (23) and (24) with $t = 0.1, \varepsilon = 2$ in the simulations. The estimates maybe further refined by averaging over various combinations of these variables.

To cover a broad range of parameters $(\alpha, \sigma_\alpha)$, we conduct six experiments: three with $\alpha = 1.5$, and three with $\alpha = 0.5$. We keep the dispersion of the Gaussian component constant ($\sigma_g = const$), and for each value of $\alpha$, we consider the following three cases: Case (i) the level of impulsive component is smaller than that of Gaussian ($\sigma_\alpha = 0.5\sigma_g$); Case (ii) the level of impulsive component is comparable to that of Gaussian ($\sigma_\alpha = \sigma_g$); and Case (iii) the level of impulsive component is higher than that of Gaussian ($\sigma_\alpha = 2\sigma_g$). We give the average and standard deviation values (in parentheses) of Monte Carlo simulation results based on 100 independent runs with $5 \times 10^4$ i.i.d. samples.

| modulator | Experiment 1 | | | Experiment 2 | | | Experiment 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 1.5$ | $\sigma_\alpha = 0.5$ | $\sigma_g = 1$ | $\alpha = 1.5$ | $\sigma_\alpha = 1$ | $\sigma_g = 1$ | $\alpha = 1.5$ | $\sigma_\alpha = 2$ | $\sigma_g = 1$ |
| BPSK | 1.5046 | 0.6557 | 0.9615 | 1.504 | 1.0401 | 0.9560 | 1.5049 | 2.0594 | 1.0687 |
| | (0.1445) | (1.0473) | (0.1701) | (0.0710) | (0.2138) | (0.2208) | (0.0544) | (0.3205) | (0.3010) |
| QPSK | 1.5139 | 0.5837 | 0.9447 | 1.4833 | 0.9873 | 1.0058 | 1.5013 | 2.0458 | 1.0525 |
| | (0.1421) | (0.2499) | (0.1740) | (0.0844) | (0.2345) | (0.2402) | (0.0600) | (0.3530) | (0.3496) |
| 8PSK | 1.4995 | 0.5508 | 0.9626 | 1.5093 | 1.0756 | 0.9355 | 1.5021 | 2.0387 | 1.0033 |
| | (0.1373) | (0.2370) | (0.1780) | (0.0894) | (0.3006) | (0.2599) | (0.0493) | (0.2908) | (0.3291) |
| 4QAM | 1.5284 | 0.6088 | 0.9493 | 1.5030 | 1.0483 | 0.9775 | 1.4985 | 2.0193 | 1.0740 |
| | (0.1351) | (0.2991) | (0.1565) | (0.0921) | (0.2694) | (0.2431) | (0.0531) | (0.3147) | (0.3015) |
| 8QAM | 1.5112 | 0.5280 | 0.9676 | 1.5055 | 1.0686 | 0.9552 | 1.4964 | 1.9980 | 1.0618 |
| | (0.1301) | (0.2008) | (0.1722) | (0.0704) | (0.2623) | (0.2398) | (0.0551) | (0.2990) | (0.3469) |
| 16QAM | 1.4867 | 0.5300 | 0.9661 | 1.5046 | 1.0494 | 0.9591 | 1.5035 | 2.0446 | 1.0293 |
| | (0.1385) | (0.2193) | (0.1816) | (0.0810) | (0.2384) | (0.2213) | (0.0548) | (0.3194) | (0.3401) |
| 64QAM | 1.4707 | 0.4944 | 1.0080 | 1.5029 | 1.0474 | 0.9774 | 1.4978 | 2.1068 | 1.0792 |
| | (0.1299) | (0.2012) | (0.1125) | (0.0807) | (0.2935) | (0.2147) | (0.0534) | (0.3135) | (0.2872) |

**Tab. 2.** Simulation results for $\alpha = 1.5$.

| modulator | Experiment 4 | | | Experiment 5 | | | Experiment 6 | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\alpha = 0.5$ | $\sigma_\alpha = 0.5$ | $\sigma_g = 1$ | $\alpha = 0.5$ | $\sigma_\alpha = 1$ | $\sigma_g = 1$ | $\alpha = 0.5$ | $\sigma_\alpha = 2$ | $\sigma_g = 1$ |
| BPSK | 0.4999 | 0.5039 | 0.9887 | 0.5007 | 1.0157 | 0.9725 | 0.4991 | 2.0019 | 0.9843 |
| | (0.0210) | (0.0663) | (0.1408) | (0.0228) | (0.1375) | (0.2022) | (0.0210) | (0.2555) | (0.2660) |
| QPSK | 0.4936 | 0.4786 | 1.0359 | 0.4985 | 1.0004 | 0.9907 | 0.4977 | 1.9968 | 0.9988 |
| | (0.0222) | (0.0649) | (0.1283) | (0.0229) | (0.1402) | (0.2096) | (0.0234) | (0.2870) | (0.2837) |
| 8PSK | 0.5000 | 0.5064 | 0.9789 | 0.5028 | 1.0270 | 0.9500 | 0.5022 | 2.0516 | 0.9784 |
| | (0.0255) | (0.0778) | (0.1568) | (0.0227) | (0.1455) | (0.2213) | (0.0261) | (0.3297) | (0.3071) |
| 4QAM | 0.5007 | 0.5064 | 0.9882 | 0.5056 | 1.0436 | 0.9229 | 0.4979 | 1.9973 | 1.0173 |
| | (0.0260) | (0.0790) | (0.1604) | (0.0234) | (0.1476) | (0.2378) | (0.0229) | (0.2877) | (0.2476) |
| 8QAM | 0.4971 | 0.4932 | 1.0029 | 0.4990 | 1.0023 | 0.9889 | 0.5013 | 2.0363 | 1.0015 |
| | (0.0228) | (0.0693) | (0.1437) | (0.0211) | (0.1253) | (0.1677) | (0.0256) | (0.3192) | (0.2619) |
| 16QAM | 0.4977 | 0.4969 | 0.9925 | 0.5034 | 1.0328 | 0.9472 | 0.4979 | 2.0024 | 1.0265 |
| | (0.0240) | (0.0735) | (0.1512) | (0.0239) | (0.1501) | (0.2141) | (0.0259) | (0.3141) | (0.2754) |
| 64QAM | 0.4994 | 0.5035 | 0.9942 | 0.4964 | 0.9920 | 1.0139 | 0.4985 | 2.0069 | 0.9856 |
| | (0.0259) | (0.0783) | (0.1565) | (0.0261) | (0.1598) | (0.2100) | (0.0208) | (0.2463) | (0.2403) |

**Tab. 3.** Simulation results for $\alpha = 0.5$.

Tables 2 and 3 list the simulation results of the moment-type estimator for different pairs $(\alpha, \sigma_\alpha)$ when $\sigma_g = 1$. Simulation results show that for various MPSK/MQAM modulators, the proposed estimator can obtain a good performance with low complexity even without any knowledge of modulation type. Especially, under the condition of enough samples, the ECF is very close to the CF. Progressively, seen from Fig. 2, all CFs of different modulation types are basically the same when $t$ is small enough, *i.e.*, the CF has nothing to do with the type of modulation. Therefore, the proposed estimator has similar performance in different modulation methods. Furthermore, the estimator performance of $\alpha = 0.5$ (refer Tab. 3) is better than that of $\alpha = 1.5$ (Tab. 2). The reasons are as follows. Apparently, these two kinds of noise are easy to distinguish if $\alpha$ is far from 2, and we can get a more accurate estimation of $\alpha$. According to our proposed algorithm, the estimation performance of the other two parameters depends on the accuracy of the estimated characteristic exponent $\alpha$. Therefore, the result in Tab. 3 is more close to the actual value with smaller variance than that in Tab. 2.

However, when $\alpha$ approaches to 2, $\varepsilon^{-2}\psi_R(\varepsilon t) \simeq \psi_R(t) \simeq \psi_R(\varepsilon^{-1}t)$. This makes (23) numerical instability and leads to a 'poor' estimation performance. On the contrary, when $\alpha$ is far from 2, the estimation performance is quite better. This is consistent with our intuitions. The characteristic of the impulsive noise is different from the Gaussian noise if its characteristic exponent $\alpha$ is far from 2. Therefore, these two kinds of noise are easy to distinguish, and we can get a more accurate estimation. On the other hand, when $\alpha$ approaches to 2, their characteristics are similar and difficult to distinguish. Hence, the estimation performance becomes worse.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