Polarization Decomposition Algorithm for Detection Efficiency Enhancement

Chao-Zhu ZHANG, Jing ZHANG, Cheng-Yuan LIU, Lin LI

Dept. of Information and Communication Engineering, University of Harbin Engineering University,
Street Nantong 145, 150001, Harbin, China

zhangchaozhu@hrbeu.edu.cn, zhangjing_heu@hrbeu.edu.cn, liuchengyuan@hrbeu.edu.cn, lilin@hrbeu.edu.cn

Abstract. *In the paper, a new polarization decomposition of the optimal detection algorithm in the partially homogeneous environment is presented. Firstly, the detectors Matched Subspace Detector (MSD) and Adaptive Subspace Detector (ASD) are adopted to deal with detection problems in the partially homogeneous environment. Secondly, the fitness function with polarization parameters is equivalently decomposed to enhance time detection efficiency in the algorithm.*

It makes the multiplication number of the fitness function from square to a linear increase along with the increase in parameters. Simulation results indicate that the proposed decomposition is much more efficient than direct use of the fitness function.

Keywords

Adaptive subspace detector (ASD); matched subspace detector (MSD); optimal detection algorithm; polarization decomposition; fitness function.

1. Introduction

The problem of detecting a target signal in the presence of noise is often encountered in radar signal processing. In a homogeneous environment, the noise in test data (primary data) is considered to be scaled the same as that in training data (secondary data). Many classic algorithms, such as generalized likelihood ratio (GLR) [1], [2] and adaptive matched filter (AMF) [3], [4], have been developed for this ideal case in the past. In many realistic scenarios, however, the noise power in the test data does not remain the same as that in the training data. For example, in a partially homogeneous environment, the noise covariance matrix in the test and training data has the same structure but may differ in a scaling factor [5]-[8]. The ASD and MSD are proposed to deal with detection problems in the partially homogeneous environment [9].

As well known, a diversely polarized antenna (DPA) possesses several advantages over a scalar sensor due to its ability to handle signals based on their polarization characteristics. The tripole antenna is a common DPA [10].

Usually, the performance of a system is associated with polarization of its transmitted signals [10]. Therefore, the system performance can be enhanced by optimally selecting the transmitted signals to max. the target response. In [9], a waveform design algorithm to enhance the detection performance of ASD and MSD was developed in partially homogeneous environment. Its detection performance was improved by optimally choosing the polarization of the transmitted pulses to maximize a fitness function.

In this paper, we study the detection problem based on a DPA in partially homogeneous environment. The ASD and MSD are employed and the fitness function in [9] with polarization parameters is equivalently decomposed to enhance time detection efficiency in the proposed method. As increase with the parameters, the multiplication number of the fitness function in the proposed method (using the fitness function decomposition) is a linear increase, while in the previous method (direct use of the fitness function expression) [9], [11] it is a square increase.

Next, Taguchi optimization algorithm [12]-[15] is used in this paper to get the optimum solution because of its high efficiency. Now, a brief introduction about the ASD and MSD are explained in order to study their performance.

2. Signal Model

In this section, we consider a detection problem in partially homogeneous environments. The received Q-dimensional complex vector $\mathbf{x}$, commonly called primary data or test data, is constrained to be of the form [9]

$$\mathbf{x} = \boldsymbol{\Sigma}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\boldsymbol{\Sigma}$ is a known $Q \times q$ dimension signal subspace matrix representing the system response associated with the characteristics of the transmitted signals (e.g., polarization) and suppose that $Q > q$ and $rank(\boldsymbol{\Sigma}) = q$; $\mathbf{s}$ is a q-dimensional deterministic but unknown complex vector accounting for the target reflectivity and the channel propagation effects; $\mathbf{n}$ is a noise data vector and is assumed to have a complex circular Gaussian distribution with zero mean and covariance matrix $\gamma\mathbf{R}$, i.e. $\mathbf{n} \sim \mathcal{CN}(0, \gamma\mathbf{R})$, where $\mathbf{R}$

denotes the noise covariance matrix structure and γ is an unknown scaling of the noise in the test data. Notice that the scaling factor accounts for the noise power mismatch between the primary and secondary data. The arbitrary scaling between the primary and secondary data is important in some realistic scenarios [5].

Suppose that $K(K > Q)$ secondary data samples free of the target signal, i.e. $\{\mathbf{y}_k, k=1,...,K | \mathbf{y}_k \sim \mathcal{CN}(0,\mathbf{R})\}$, are available. The problem of detection is to decide whether the target signal is present or not in the range cell under test. This problem can be posed in terms of a binary hypotheses test. We let the null hypothesis (H_0) be that no target signal is present and the alternative hypothesis (H_1) be that the data contains target signal. Hence, the detection problem is to decide between the null hypothesis and the alternative one and can be stated as a parameter test:

$$H_0 : \begin{cases} \mathbf{x} \sim \mathcal{CN}(0,\gamma\mathbf{R}) \\ \mathbf{y}_k \sim \mathcal{CN}(0,\mathbf{R}), k=1,...,K \end{cases} \tag{2}$$

$$H_1 : \begin{cases} \mathbf{x} \sim \mathcal{CN}(\mathbf{\Sigma s},\gamma\mathbf{R}) \\ \mathbf{y}_k \sim \mathcal{CN}(0,\mathbf{R}), k=1,...,K \end{cases} \tag{3}$$

3. Detection Performance of ASD and MSD

We begin by considering the detection problem in the case of known $\mathbf{R}$. According to [9], the detector referred to as MSD can be expressed as

$$\Phi = \frac{\mathbf{u}^H \mathbf{P}_\Theta \mathbf{u}}{\mathbf{u}^H \mathbf{P}_\Theta^\perp \mathbf{u}} \underset{H_1}{\overset{H_0}{<>}} \varsigma \tag{4}$$

where ς is the detection threshold; $\mathbf{u} = \mathbf{R}^{-1/2}\mathbf{x}$; $\mathbf{\Theta} = \mathbf{R}^{-1/2}\mathbf{\Sigma}$; $\mathbf{P}_\Theta = \mathbf{\Theta}(\mathbf{\Theta}^H\mathbf{\Theta})^{-1}\mathbf{\Theta}^H$; $\mathbf{P}_\Theta^\perp = \mathbf{I} - \mathbf{P}_\Theta$ with $\mathbf{I}$ denoting the identity matrix. The probability of false alarm can be obtained as [9]

$$P_{FA}^{MSD} = \sum_{j=1}^{q} C_{Q-j-1}^{q-j} \varsigma^{q-j} (1+\varsigma)^{-(Q-j)} . \tag{5}$$

Then, the probability of detection can be obtained by

$$\begin{aligned} P_D^{MSD} &= 1 - \varsigma^{q-1}(1+\varsigma)^{-(Q-1)} \sum_{j=1}^{Q-q} C_{Q-1}^{q+j-1} \varsigma^j \\ &\times \exp\left(-\frac{\Gamma}{1+\varsigma}\right) \sum_{m=0}^{j-1} \frac{1}{m!} \left(\frac{\Gamma}{1+\varsigma}\right)^m \end{aligned} \tag{6}$$

$$\Gamma = \mathbf{s}^H \left[\mathbf{\Sigma}^H (\gamma\mathbf{R})^{-1} \mathbf{\Sigma}\right] \mathbf{s} . \tag{7}$$

In practice, a prior knowledge on the covariance matrix structure is usually unknown. According to [9], the detector used to handle the detection problem with unknown $\mathbf{R}$, which is referred to as ASD, is

$$\Psi = \frac{\mathbf{x}^H \hat{\mathbf{R}}^{-1} \mathbf{\Sigma}(\mathbf{\Sigma}^H \hat{\mathbf{R}}^{-1} \mathbf{\Sigma})^{-1} \mathbf{\Sigma}^H \hat{\mathbf{R}}^{-1} \mathbf{x}}{\mathbf{x}^H \hat{\mathbf{R}}^{-1} \mathbf{x}} \underset{H_1}{\overset{H_0}{<>}} \xi \tag{8}$$

where ξ is the detection threshold and $\hat{\mathbf{R}} = \sum_{k=1}^{K} \mathbf{y}_k \mathbf{y}_k^H$.

The probability of false alarm of the ASD can be written as

$$P_{FA}^{ASD} = \int_0^1 P_{FA|\rho} f_\rho(\rho) d\rho \tag{9}$$

where ρ denotes a loss factor whose distribution is

$$f_\rho(\rho) = \frac{K!\rho^{K-Q+q}(1-\rho)^{Q-q-1}}{(Q-q-1)!(K-Q+q)!}, 0<\rho<1 \tag{10}$$

and

$$P_{FA|\rho} = \left(\frac{1-\xi}{1-\xi\rho}\right)^{K-Q+1} \sum_{j=1}^{q} C_{K-Q+q-j}^{q-j} \left[\frac{\xi(1-\rho)}{1-\xi\rho}\right]^{q-j} . \tag{11}$$

Furthermore, the probability of detection is

$$P_D^{ASD} = \int_0^1 P_{D|\rho} f_\rho(\rho) d\rho \tag{12}$$

where

$$\begin{aligned} P_{D|\rho} &= 1 - \left[\frac{\xi(1-\rho)}{1-\xi\rho}\right]^{q-1} \left(\frac{1-\xi}{1-\xi\rho}\right)^{K-Q+1} \\ &\times \sum_{j=1}^{K-Q+1} C_{K-Q+q}^{q+j-1} \left[\frac{\xi(1-\rho)}{1-\xi}\right]^j \exp\left[\frac{-\Gamma\rho(1-\xi)}{1-\xi\rho}\right] \\ &\times \sum_{m=0}^{j-1} \frac{1}{m!} \left[\frac{\Gamma\rho(1-\xi)}{1-\xi\rho}\right]^m \end{aligned} \tag{13}$$

with Γ defined as that in (7). In [9] it is concluded the greater the value of $\Gamma(\Gamma > 0)$, the better detection performance. The detection performance of the MSD and ASD can be enhanced by designing the system response $\mathbf{\Sigma}$ to maximize the parameter Γ. The system response matrix can be parameterized as $\mathbf{\Sigma} = \mathbf{\Sigma}(\varepsilon)$. The fitness function can be written as

$$\Gamma(\varepsilon) = \mathbf{s}^H [\mathbf{\Sigma}^H(\varepsilon)(\gamma\mathbf{R})^{-1}\mathbf{\Sigma}(\varepsilon)]\mathbf{s} . \tag{14}$$

From (5) and (9) we can see that the MSD and ASD have the desirable constant false alarm rate (CFAR) property with respect to the optimization parameter.

4. Polarization Decomposition Algorithm Theory Analysis

Let $\mathbf{V}$ denotes a response of the diversely polarized sensor array acts as a detector [10]. If the array is a tripole antenna, it can be written as

$$\mathbf{V} = \begin{bmatrix} -\sin\varphi & -\cos\varphi\sin\psi \\ \cos\varphi & -\sin\varphi\sin\psi \\ 0 & \cos\psi \end{bmatrix} \tag{15}$$

where φ and ψ denote the elevation and azimuth angles of the target return with $\phi \in [0,\pi]$ and $\psi \in [-\pi,\pi]$.

The vector $\mathbf{z}_p(t)$ is the p-th pulse of the narrowband transmitted signal which can be represented by

$$\mathbf{z}_p(t)=\begin{bmatrix} z_{1p} \\ z_{2p} \end{bmatrix} a_p(t)=\begin{bmatrix} \cos\alpha_p & \sin\alpha_p \\ -\sin\alpha_p & \cos\alpha_p \end{bmatrix}\begin{bmatrix} \cos\beta_p \\ j\sin\beta_p \end{bmatrix} a_p(t) \quad (16)$$

where z_{1p} and z_{2p} are the signal components on the polarization basis of transmitter; α_p and β_p are the orientation and ellipticity angles of polarization ellipse with $\alpha_p \in [-\pi/2, \pi/2]$ and $\beta_p \in [-\pi/4, \pi/4]$; $a_p(t)$ ($p = 1,\ldots,P$) is the complex envelope of the p-th transmitted signal pulse and each element of $\mathbf{a}_p = [a_p(t_{1p}),...,a_p(t_{Mp})]^T$ ($p = 1,\ldots,P$) with $t_{mp} (m=1,...,M)$ denoting the m-th sampling instant within the p-th pulse.

The polarization matrix of each diversely polarized pulse ($p=1,...,P$) is given by

$$\mathbf{E}_p=\begin{bmatrix} z_{1p} & 0 & z_{2p} \\ 0 & z_{2p} & z_{1p} \end{bmatrix}. \quad (17)$$

So the system response matrix can be written as

$$\mathbf{\Sigma}=\begin{bmatrix} \mathbf{a}_1 \otimes \mathbf{VE}_1 \\ \vdots \\ \mathbf{a}_P \otimes \mathbf{VE}_P \end{bmatrix} \quad (18)$$

and matrix $\mathbf{\Sigma}$ has dimension $3MP \times 3$, where P is the number of the transmitted pulses.

The noise covariance matrix is supposed to be $\mathbf{R}=\sigma_n^2(\mathbf{I}_P \otimes \mathbf{C}_{3M\times 3M})$, where σ_n^2 is the noise power of each sample, $\mathbf{I}_P$ denotes the P-dimensional identity matrix, and $\mathbf{C}_{m\times n}$ is Gaussian shaped with one-lag correlation coefficient $\rho_c = 0.9$ [9] and it represents correlation between the samples within a pulse. That is to say

$$\mathbf{C}_{m\times n}=\begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix} \quad (19)$$

where $c_{ij}=0.9^{(i-j)^2}, i=1,...,m; j=1,...,n$.

Suppose

$\mathbf{R}' = \mathbf{I}_P \otimes \mathbf{C}_{3M\times 3M}$ and $\Gamma = \frac{1}{\gamma\sigma_n^2}\mathbf{s}^H\left[\mathbf{\Sigma}^H(\mathbf{R}')^{-1}\mathbf{\Sigma}\right]\mathbf{s}$. Note that $\mathbf{R}'$ is real symmetric matrix, and $(\mathbf{R}')^{-1}$ is also real symmetric matrix. So $(\mathbf{R}')^{-1}$ can be decomposed into $(\mathbf{R}')^{-1}=\mathbf{G}^T\mathbf{G}$ uniquely, where $\mathbf{G}$ is $3MP\times 3MP$ dimension real upper triangular matrix. Therefore, the fitness can be written as

$$\Gamma=\frac{1}{\gamma\sigma_n^2}\mathbf{s}^H\mathbf{\Sigma}^H\mathbf{G}^T\mathbf{G}\mathbf{\Sigma}\mathbf{s}=\frac{1}{\gamma\sigma_n^2}\left(\mathbf{G\Sigma s}\right)^H\mathbf{G\Sigma s}. \quad (20)$$

Suppose $\mathbf{H}=\mathbf{G\Sigma s}$, and $\mathbf{H}$ is $3MP\times 1$ dimension complex vector. Then finding the maximum value of Γ is equivalent to finding the maximum modulus value of $\mathbf{H}$.

Note that $\mathbf{C}_{3M\times 3M}$ is real symmetric matrix, and $\mathbf{C}_{3M\times 3M}^{-1}$ is also real symmetric matrix. So it can be decomposed into $\mathbf{C}_{3M\times 3M}^{-1}=\mathbf{g}^T\mathbf{g}$ uniquely, where $\mathbf{g}$ is $3M\times 3M$ dimension real upper triangular matrix. Now we have an important discovery: $\mathbf{G}=\mathbf{I}_P\otimes\mathbf{g}$.

Proof: From the above analysis, we can get

$$\begin{aligned} & \mathbf{C}_{3M\times 3M}^{-1}=\mathbf{g}^T\mathbf{g} \\ \Leftrightarrow\ & \mathbf{I}_P\otimes\mathbf{C}_{3M\times 3M}^{-1}=\mathbf{I}_P\otimes\mathbf{g}^T\mathbf{g} \\ \Leftrightarrow\ & \mathbf{I}_P^{-1}\otimes\mathbf{C}_{3M\times 3M}^{-1}=\left(\mathbf{I}_P\otimes\mathbf{g}^T\right)\left(\mathbf{I}_P\otimes\mathbf{g}\right) \\ \Leftrightarrow\ & \left(\mathbf{I}_P\otimes\mathbf{C}_{3M\times 3M}\right)^{-1}=\left(\mathbf{I}_P\otimes\mathbf{g}\right)^T\left(\mathbf{I}_P\otimes\mathbf{g}\right). \end{aligned} \quad (21)$$

Due to $\left(\mathbf{I}_P\otimes\mathbf{C}_{3M\times 3M}\right)^{-1}=\mathbf{G}^T\mathbf{G}$, we can get

$$\mathbf{G}=\mathbf{I}_P\otimes\mathbf{g}. \quad (22)$$

The proof is completed.

Then $\mathbf{H}$ can be written as

$$\begin{aligned}\mathbf{H}=\mathbf{G\Sigma s}=\left(\mathbf{I}_P\otimes\mathbf{g}\right)\mathbf{\Sigma s}=diag[\mathbf{g},...,\mathbf{g}]\begin{bmatrix} \mathbf{a}_1 \otimes \mathbf{VE}_1 \\ \vdots \\ \mathbf{a}_P \otimes \mathbf{VE}_P \end{bmatrix}\mathbf{s} \\ =\begin{bmatrix} \mathbf{ga}_1 \otimes \mathbf{VE}_1\mathbf{s} \\ \vdots \\ \mathbf{ga}_P \otimes \mathbf{VE}_P\mathbf{s} \end{bmatrix}=[\mathbf{h}_1,\mathbf{h}_2,...,\mathbf{h}_P]^T\end{aligned} \quad (23)$$

where $\mathbf{h}_p=\mathbf{ga}_p\otimes\mathbf{VE}_p\mathbf{s}, p=1,...,P$ is a P-dimensional complex vector group and each one of it is a $3M\times 1$ dimension complex vector.

It is considered in our system that the polarization parameters of different transmitted signal pulses are independent of each other, i.e. when $i \neq j$ ($i = 1,\ldots, P$; $j = 1,\ldots, P$) (α_i, β_i) and (α_j, β_j) are independent of each other. Thus, we can get a conclusion that finding the maximum modulus value of $\mathbf{H}$ is equivalently decomposed into finding the maximum modulus value of every vector in the complex vector group: $\mathbf{h}_p$, $p = 1,\ldots, P$.

Now we analyze the complex vector group: $\mathbf{h}_p=\mathbf{ga}_p\otimes\mathbf{VE}_p\mathbf{s}, p=1,...,P$, where real upper triangular matrix $\mathbf{g}$ is fixed; when transmitted signal pulses and the sampling form are fixed, the complex envelope of the p-th transmitted signal pulse $\mathbf{a}_p$ is fixed; when the target is deterministic, the target reflectivity vector $\mathbf{s}$ is fixed; in the same pulse interval, we assume that the elevation and azimuth angles of the target fixed, i.e. $\mathbf{V}$ is fixed. Thus, there are two variable parameters (α_p, β_p) to be optimized in each vector $\mathbf{h}_p$, $p = 1,\ldots, P$. Therefore, the proposed algorithm is to optimally choose the parameters (α_p, β_p) to meet the maximum modulus value of every vector in the complex vector group: $\mathbf{h}_p$, $p = 1,\ldots, P$.

The optimization detection algorithm in [9] is to find the maximum fitness function value: $\Gamma(\varepsilon)=\mathbf{s}^H[\mathbf{\Sigma}^H(\varepsilon)(\gamma\mathbf{R})^{-1}\mathbf{\Sigma}(\varepsilon)]\mathbf{s}$, and there are $N_1 = 9M^2P^2 + 36MP + 3$ multiplications in the fitness. The proposed algorithm is the equivalently decomposed of previous method. There are p fitness functions: using (23) $\mathbf{h}_p = \mathbf{g}\mathbf{a}_p \otimes \mathbf{V}\mathbf{E}_p\mathbf{s}, p = 1,...,P$ and they totally have $N_2 = 9M^2P + 30MP$ multiplications. The multiplication number of the proposed method is a linear increasing as the parameters increase, while it is a square increasing in the previous method. From Fig. 1 we can see that the proposed method is much more efficient than the previous method.

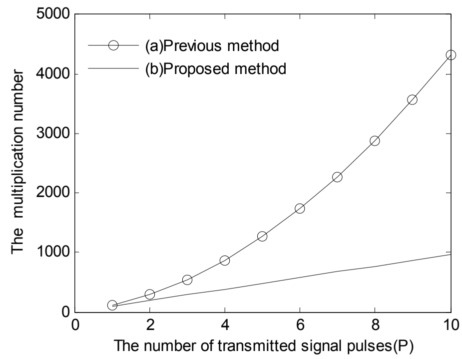


Fig. 1. The multiplication numbers of two methods.

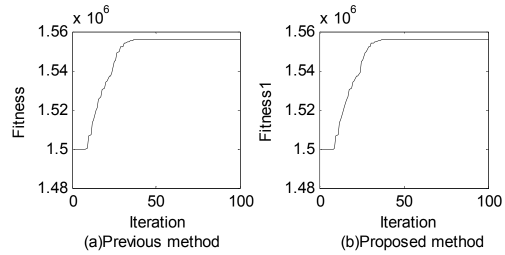
In a special circumstance, $\lambda_1\mathbf{a}_1 = \lambda_2\mathbf{a}_2 = \ldots = \lambda_P\mathbf{a}_P = \mathbf{a}$ ($\lambda_k \in R$, $k = 1,\ldots,P$), i.e. $\mathbf{a}_1$, $\mathbf{a}_1,\ldots,\mathbf{a}_P$ are linear correlation, e.g. rectangular pulses [11]. Thus, we get a conclusion that finding the maximum modulus value of every vector in the complex vector group: $\mathbf{h}_p$, $p = 1,\ldots, P$ is degraded equivalent to finding the maximum modulus value of any vector.

5. Simulation Results and Discussions

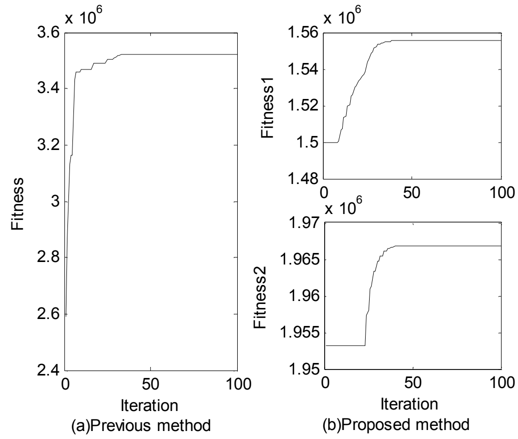
The experiment results are done by MATLAB program in a PC computer with CPU: inter I3-2100, 3.1 GHz dual-core processor, and 2 GB memory.

5.1 The Simulation Results and Discussions in Normal Circumstance

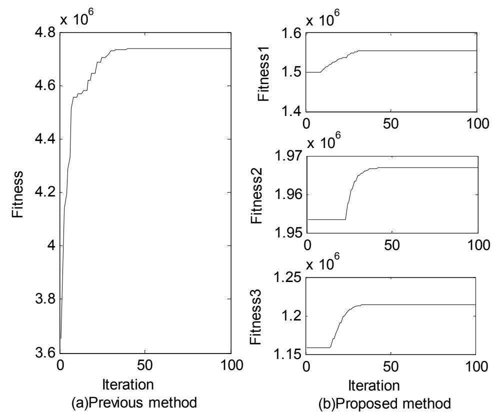
We validated the analytical performance of the algorithms by computer simulations. In the following simulations, we select $P = 1,2,3,4,$; $M = 2$; $\sigma_n^2 = 1/3$; $\gamma = 3$; $\varphi = 0.2\pi$; $\psi = 0.5\pi$; $\mathbf{s} = [2\mathrm{i},\text{-}1\mathrm{i},0.5]^T$; $\mathbf{a}_1 = [7+8\mathrm{i},8\text{-}2\mathrm{i}]^T$; $\mathbf{a}_2 = [5+3\mathrm{i},6\text{-}9\mathrm{i}]^T$; $\mathbf{a}_3 = [3+7\mathrm{i},4\text{-}4\mathrm{i}]^T$; $\mathbf{a}_4 = [1+5\mathrm{i},2\text{-}8\mathrm{i}]^T$ in normal circumstance. The analytical solution can be solved by the proposed method theoretically, but the solution procedure is very complex. Therefore, we use Taguchi optimization algorithm to solve this problem.



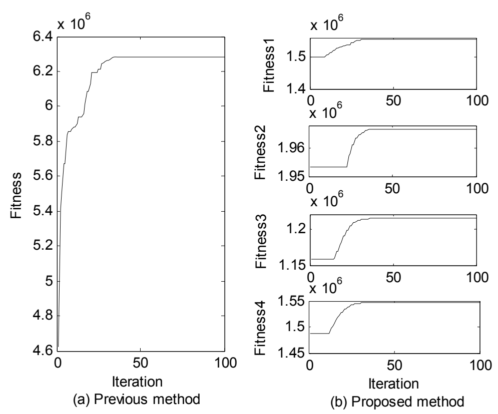
(A) P=1



(B) P=2



(C) P=3



(D) P=4

Fig. 2. The fitness curves of two methods.

As shown in Fig. 2 and Tab. 1, the sum of the maximum fitness function values in the proposed method is the same as the maximum fitness function value in the previous method. And from Tab. 2 we can see that the two methods get the same optimal polarization parameters. There is no doubt that the numerical simulations are conducted to attest to the validity of the above theoretical equivalence relation.

From Tab. 3 we can see that the proposed method costs less time than the previous method. The numerical simulations confirm the truth that the multiplication number of the proposed method is a linear increasing as the parameters increase, while it is a square increasing in the previous method.

From the above theoretical analysis and simulation experiments we can get a conclusion that the proposed method can get the same detection performance as the previous method, but it is more efficient than the previous method.

Methods	P	1	2	3	4
Proposed method	Fitness 1	1.5560	1.5560	1.5560	1.5560
	Fitness 2		1.9668	1.9668	1.9668
	Fitness 3			1.2143	1.2143
	Fitness 4				1.5471
	Sum	1.5560	3.5228	4.7371	6.2842
Previous method	Fitness	1.5560	3.5228	4.7371	6.2842

Tab. 1. The maximum fitness function values got by two methods ($\times 10^6$).

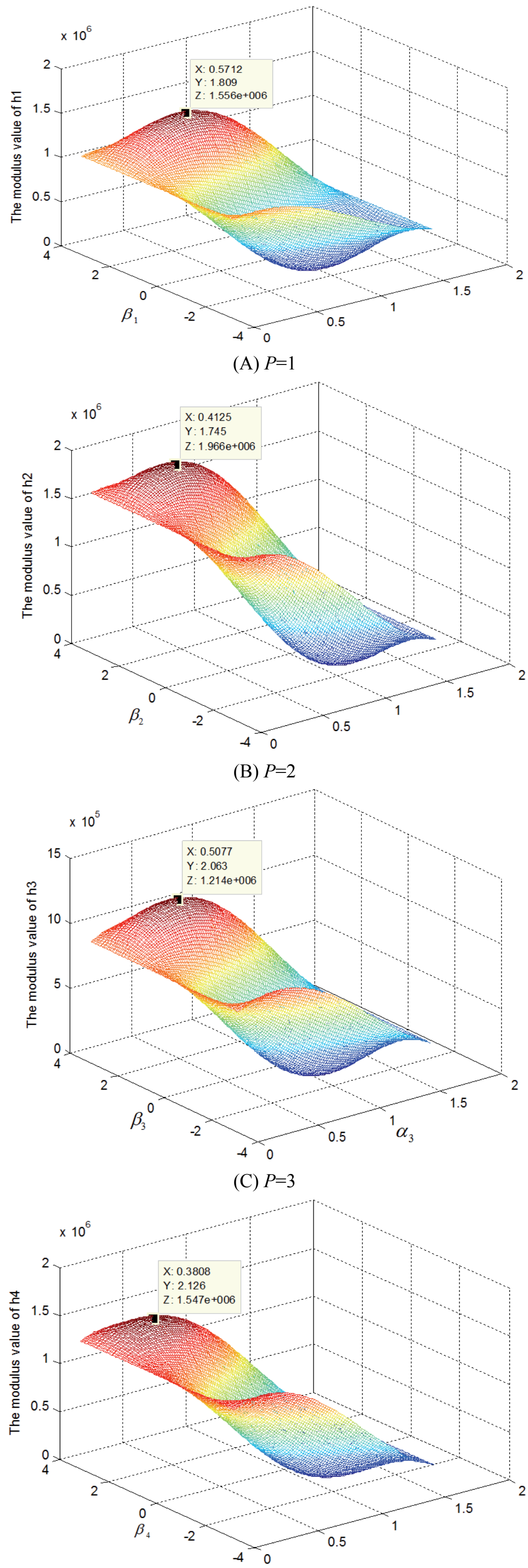
(α_p, β_p)	Previous method	Proposed method
α_1	0.5648	0.5648
β_1	1.7805	1.7805
α_2	0.4070	0.4070
β_2	1.7996	1.7996
α_3	0.5013	0.5013
β_3	2.0538	2.0538
α_4	0.3776	0.3776
β_4	2.1467	2.1467

Tab. 2. The optimal polarization parameters got by two methods (rad).

P	Previous method	Proposed method
1	47.139	43.702
2	164.46	87.175
3	354.16	130.52
4	457.63	178.48

Tab. 3. The time cost by two methods (ms).

Fig. 3 depicts a three-dimensional distribution of modulus value of h_p when the orientation angle α_p and the ellipticity angle β_p of polarization ellipse are valued within the range: $\alpha_p \in [-\pi/2, \pi/2]$ and $\beta_p \in [-\pi/4, \pi/4]$. Comparing Tab. 2 and Tab. 3 with Fig. 3, we can prove that the proposed algorithm is reliable.



(A) P=1

(B) P=2

(C) P=3

(D) P=4

Fig. 3. The distribution of modulus value of h.

5.2 The Experiment Results and Discussions in a Special Circumstance

In this section, we select K = 2Q; P = 4; M = 2; $\gamma = 3$; $\mathbf{s} = [2i,-1i,0.5]^T$; $\mathbf{a}_1 = [7+8i,8-2i]^T$; $\mathbf{a}_2 = 2\mathbf{a}_1$; $\mathbf{a}_3 = 3\mathbf{a}_1$; $\mathbf{a}_4 = 4\mathbf{a}_1$ in special circumstance. From Fig. 4 and Tab. 5 we can see that the following experiment results get the same conclusion as Section 5.1 in detection performance analysis. However, from Tab. 6 we can see that the efficiency of the proposed method is 9 times more than the efficiency of the previous method. The numerical simulations are conducted to attest to the validity of the above theoretical analysis.

Methods	P	4
Proposed method	Fitness 1	0.1556
	Fitness 2	0.6224
	Fitness 3	1.4004
	Fitness 4	2.4896
	Sum	4.6880
Previous method	Fitness	4.6880

Tab. 4. The maximum fitness function values got by two methods in a special circumstance ($\times 10^7$).

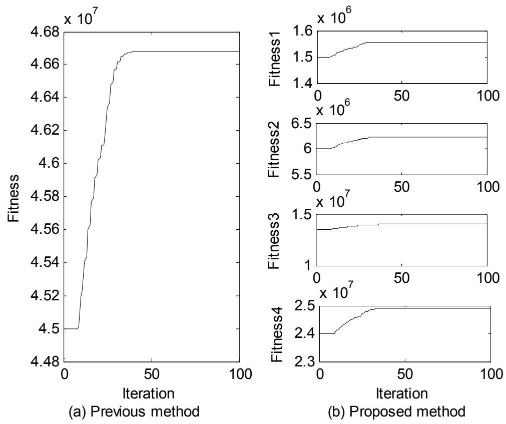


Fig. 4. The fitness curves of two methods in special circumstance.

(α_p, β_p)	Previous method	Proposed method
α_1	0.5648	0.5648
β_1	1.7805	1.7805
α_2	0.5648	0.5648
β_2	1.7805	1.7805
α_3	0.5648	0.5648
β_3	1.7805	1.7805
α_4	0.5648	0.5648
β_4	1.7805	1.7805

Tab. 5. The optimal polarization parameters got by two methods in a special circumstance (rad).

Previous method	Proposed method
443.72	43.542

Tab. 6. The time cost by two methods in a special circumstance (ms).

6. Conclusions

In this paper, a new optimal polarization decomposition algorithm was developed for detection efficiency enhancement in the partially homogeneous environment. Firstly, the two detectors, i.e. MSD and ASD which have CFAR property were adopted to deal with detection problems in the partially homogeneous environment. Secondly, the fitness function with polarization parameters was equivalently decomposed to enhance detection efficiency in the proposed method, while it was maintaining high detection performance. The improvement was achieved by decomposing the fitness function with polarization parameters to make the multiplication number of fitness function from a square increasing to a linear increasing as the parameters increase. The theoretical analyses and the numerical simulations were conducted to attest to the equivalence relationship between the previous method and the proposed method in detection performance. More importantly, the proposed method was much more efficient than the previous method.

Acknowledgements

This work is supported by a grant from the National Natural Science Fund of China (No. 61172159) and the Fundamental Research Funds for the Central Universities (HEUCFT1101).

References

[1] KELLY, E. J. An adaptive detection algorithm. *IEEE Transactions on Aerospace and Electronic Systems*, 1986, vol. 22, no.1, p. 115 to 127.

[2] PARK, H., LI, J., WANG, H. Polarization-space-time domain generalized likelihood ratio detection of radar targets. *Signal Processing*, 1995, vol. 41, p. 153-164.

[3] ROBEY, F. C., FUHRMANN, D. R., KELLY, E. J., NITZBERG, R. A CFAR adaptive matched filter detector. *IEEE Transactions on Aerospace and Electronic Systems*, 1992, vol. 28, no. 1, p. 208 to 216.

[4] MAIO, A. D., RICCI, G. A polarimetric adaptive matched filter. *Signal Processing*, 2001, vol. 81, p. 2583-2589.

[5] LIU, J., ZHANG, Z. J., YANG, Y. A CFAR adaptive subspace detector for first-order or second-order Gaussian signals based on a single observation. *IEEE Transactions on Signal Processing*, 2011, vol. 59, no. 11, p. 5126-5140.

[6] JIN, Y., FRIEDLANDER, B. A CFAR adaptive subspace detector for second-order Gaussian signals. *IEEE Transactions on Signal Processing*, 2005, vol. 53, no.3, p. 871-884.

[7] SCHARF, L. L., FRIEDLANDER, B. Matched subspace detectors. *IEEE Transactions on Signal Processing*, 1994, vol. 42, no. 8, p. 2146-2157.

[8] KRAUT, S., SCHARF, L. L., MCWHORTER, L. T. Adaptive subspace detectors. *IEEE Transactions on Signal Processing*, 2001, vol. 49, no. 1, p. 1-16.

[9] LIU, J., ZHANG, Z. J., YANG, Y. Performance enhancement of subspace detection with a diversely polarized antenna. *IEEE Signal Processing Letters*, 2012, vol. 19, no. 1, p. 4-7.

[10] HURTADO, M., NEHORAI, A. Polarimetric detection of targets in heavy inhomogeneous clutter. *IEEE Transactions on Signal Processing*, 2008, vol. 56, no. 4, p. 1349-1361.

[11] LIU, J., ZHANG, Z. J., YANG, Y. Optimal waveform design for generalized likelihood ratio and adaptive matched filter detectors using a diversely polarized antenna. *Signal Processing*, 2012, vol. 92, no. 2012, p. 1126-1131.

[12] HEDAYAT, A. S., SLOANE, N. J. A., STUFKEN, J. *Orthogonal Arrays. Theory and Applications.* New York: Springer-Verlag, 1999.

[13] TAGUCHI, G., CHOWDHURY, S., WU, Y. *Taguchi's Quality Engineering Handbook*. New Jersey: John Wiley & Sons Inc., 2005.

[14] WENG, W. C., YANG, F., ELSHERBENI, A. *Electromagnetics and Antenna Optimization Using Taguchi's Method*. Morgan and Clay-pool Publishers, 2007.

[15] WANG, L. G., ZHANG, J., LIU, C. H., ZHANG, C. Z. Construction and solution of a new spectral unmixing model. *Journal of Optoelectronics Laser*. 2011, vol. 22, no. 11, p. 937-941.

About Authors ...

Chao-Zhu ZHANG was born in 1970. He received his B.S. degree in Electronics and Information Engineering from Harbin Institute of Technology in 1993, M.S. degree in Communications and Information Systems and Ph.D. degree in Signal and Information Processing from Harbin Engineering University in 2002 and 2006 respectively. Now he is a professor for Harbin Engineering University, China. He is a member of IEEE, academician of Chinese Aerospace Society and Heilongjiang Biomedical Engineering Society. His research interests include signal processing applications in radar and communications and image processing.

Jing ZHANG was born in 1985. She received her B.S. degree in Electronics and Information Engineering and M.S. degree in Signal and Information Processing from Harbin Engineering University in 2008 and 2011 respectively. Now she is a Ph.D. Candidate in Harbin Engineering University, China. Her research interests include radar signal processing and image processing.

Cheng-yuan LIU was born in 1984. He received his B.S. degree in Electrical and Information Engineering and M.S. degree in Electromagnetic Field and Microwave Technology from Harbin Engineering University in 2006 and 2011 respectively. Now he is a Ph.D. Candidate in Harbin Engineering University, China. His research interests include microwave theory, UWB antenna, UWB filters and radar signal processing.

Lin LI was born in 1987. She received her B.S. degree in Electrical and Information Engineering from Anhui Agricultural University in 2009. Now she is a Ph.D. Candidate in Harbin Engineering University, China. Her research interests include radar signal processing and target tracking.