Novel Empirical Equations to Calculate the Impedance of a Strip Dipole Antenna

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Abstract. *This paper investigates the input impedance of strip dipoles since they are the basic elements of folded strip dipole antennas. A novel, simple and accurate design algorithm is presented. Compared to state-of-the art design equations, the new proposed equations are more accurate than those found in the literature and take into consideration the antenna feeding width. These equations reduce the calculation time, when compared to commercial electromagnetic simulation (EM) software, allowing for fast antenna designs with very high accuracy. So one can usually obtain results relatively quickly when compared to EM simulation software. Based on the novel equations, a strip dipole antenna is designed, simulated, manufactured and measured. The simulation results are validated by measurements.*

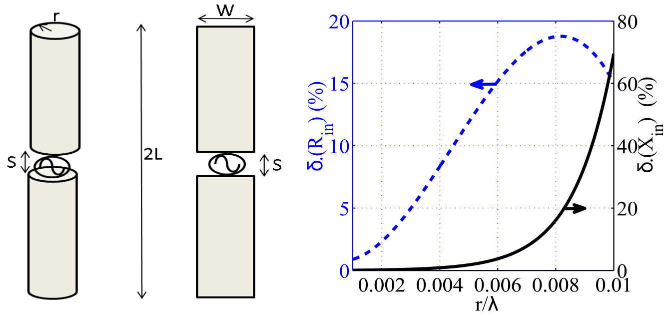
Keywords

Analytical solution, dipole antenna, input impedance, power waves, reflection coefficients.

1. Introduction

Wireless power transmission (WPT) is an attractive powering method for wireless sensor nodes, battery-less sensors, and radio-frequency identification (RFID) tags [1]. The key element on the receiving side of a WPT system is the rectifying antenna (rectenna) [2] which captures electromagnetic power and converts it to electric power. For the design of the rectenna, the input impedance of the rectifier circuit should be analyzed, and for maximum power transfer between the antenna and the rectifier, the antenna input impedance should be equal to the complex conjugate of that of the rectifier circuit [2].

To be able to design an antenna with a specific input impedance, the geometry of the antenna should have enough parameters to be able to tune the input impedance. One such an antenna is the folded dipole array antenna [3]. The impedance tuning ability eliminates the need for a matching network between the antenna and the rectifier, which makes the system more compact, power-efficient and cheaper to produce.



(a) (b)

Fig. 1. (a) Cylindrical and strip dipole configurations, (b) relative difference between the real and imaginary parts of the input impedance as a function of frequency for different dipole radii.

The basic element of the folded dipole antenna is the single dipole. A folded dipole antenna is commonly analyzed by recognizing a transmission line mode and a dipole antenna mode, see e.g. [2]. This paper presents strip dipoles, using accurate and easy-to-use design equations to calculate their input impedances.

The input impedance of a dipole antenna can be calculated using full wave analysis techniques, e.g. the method of moments (MoM) [4], and the finite integration technique (FIT) [5]. These methods are potentially very accurate but in general are time consuming. An alternative way is to use dedicated analytical-equation-methods. Analytical equations may be derived by employing, among other methods, the induced EMF method, the Hallen's integral equation (HIE) method, and the King-Middleton Second-Order method [6]. While the resulting equations are easy-to-use, they appear to be insufficiently accurate to design antennas with a specific non-standard input impedance, especially when the radius of the cylindrical dipole increases. In this work, we present a simple but highly accurate design method that improves upon the known analytical equations.

2. Strip Dipole Antennas

From a realization perspective, strip dipole antennas are more convenient than cylindrical wire dipole antennas.

Strip dipole antennas may be applied to foil, e.g. in RFID tags and even on-chip realizations are feasible. A printed dipole antenna of width W can be analyzed through treating it as an equivalent cylindrical dipole antenna with a radius $r = \frac{W}{4}$ [7]. In the following subsection we will investigate the accuracy of this equivalence, especially for larger dipole radii.

2.1 Equivalent Radius of a Strip Dipole Antenna

Cylindrical and strip dipole configurations are shown in Fig. 1a. To investigate the accuracy of the equivalence $W = 4*r$, the width of the strip dipole W is set to four times the radius of a cylindrical dipole r, and using CST microwave studio [8] the input impedance of the strip dipole and its equivalent cylindrical dipole are calculated. This procedure is repeated for different dipole radii. Fig. 1b shows the relative difference errors $\delta R_{in} = \left(R_{Cylindrical} - R_{Strip}\right)/R_{Cylindrical}$ (dashed curve) and $\delta X_{in} = \left(X_{Cylindrical} - X_{Strip}\right)/X_{Cylindrical}$ (solid curve) as a function of dipole radius. It is clear that for larger dipole radii, the deviation increases which makes the quasi-static approach ($W = 4*r$) not accurate enough specially for higher dipole radii and it should be used only for very thin cylindrical radii ($\frac{r}{\lambda} \leq 0.004$ for $\varepsilon_{\delta_{R_{in},X_{in}}} \leq 10\,\%$).

2.2 Input Impedance of a Strip Dipole Antenna

Since the quasi-static approach ($W = 4*r$) is not accurate enough to calculate the input impedance of a strip dipole, new analytical solution to calculate the impedance of a strip dipole of length of $2L$ and a width of W as shown in Fig. 1a are derived. The input impedance of the strip dipole antenna is calculated using the finite integration technique for different lengths and widths at a frequency of 300 MHz. Different data fitting methods including: polynomial models, exponential models, Fourier series and power series have been investigated in order to describe the antenna input impedance. It has been found that the polynomial models best match the simulated results. Matlab has been used to investigate the optimum fitting method. Performing surface fitting on the calculated input impedances, the real and imaginary parts of the input impedance, are described by (1) and (2) respectively,

$$R_{in}\left(\frac{l}{\lambda},\frac{W}{\lambda}\right) = \sum_{m=0}^{5}\sum_{n=0}^{5} R_{mn}\left(\frac{l}{\lambda}\right)^{m}\left(\frac{W}{\lambda}\right)^{n}, \qquad (1)$$

$$X_{in}\left(\frac{l}{\lambda},\frac{W}{\lambda}\right) = \sum_{m=0}^{5}\sum_{n=0}^{5} X_{mn}\left(\frac{W}{\lambda}\right)^{m}\left(\frac{l}{\lambda}\right)^{n} \qquad (2)$$

where the coefficients R_{mn} and X_{mn} are listed in Tab. 1 and Tab. 2 respectively. The feeding width S is set to $0.002\,\lambda$. These equations are valid for $1.0 \leq \left(\frac{2\pi L}{\lambda}\right) \leq 2.0$ and $0.003 \leq \left(\frac{W}{\lambda}\right) \leq 0.04$, and show an error less than 10 % for both real and imaginary parts. Compared to King-Middleton Second-Order equations [6], which are valid only for

$m\backslash n$	0	1	2	3
0	211.4	-3.456e4	7.96e5	-5.796e6
1	-1486	5.072e5	-9.993e6	4.49e7
2	-1.544e4	-2.582e6	4.235e7	-5.274e7
3	2.047e5	4.834e6	-6.439e7	0
4	-7.511e5	-1.251e6	0	0
5	9.655e5	0	0	0

$m\backslash n$	4	5
0	2.535e7	2.089e8
1	-2.478e8	0
2	0	0
3	0	0
4	0	0
5	0	0

Tab. 1. R_{mn} coefficients used in (1) to calculate the input impedance of a strip dipole.

$m\backslash n$	0	1	2	3
0	1022	-4.528e4	4.563e5	-2.015e6
1	4.256e4	4.843e5	-6.868e6	2.478e7
2	-4.147e6	2.13e7	-2.259e7	5.027e6
3	9.215e7	-3.885e8	2.088e8	0
4	-8.8e8	2.709e9	0	0
5	1.873e9	0	0	0

$m\backslash n$	4	5
0	4.277e6	-3.472e6
1	-3.084e7	0
2	0	0
3	0	0
4	0	0
5	0	0

Tab. 2. X_{mn} coefficients used in (2) to calculate the input impedance of a strip dipole.

$1.3 \leq \left(\frac{2\pi L}{\lambda}\right) \leq 1.7$ and $0.001 \leq \left(\frac{W}{\lambda}\right) \leq 0.01$, the novel empirical equations can model longer and wider dipole antennas. The main limitations of the presented equations is that the feeding gap S is fixed and is set to $0.002\,\lambda$, and doesn't account for conductor losses. Fig. 2 shows the real and imaginary parts of the input impedance for a strip dipole as a function of frequency using the finite integration technique (FIT) and the analytical equation for strip dipoles (AE). The electrical length of the dipole is set to kL = 1.5, the width W is set to $0.003\,\lambda$. It is clear from Figs. 2a and 2b that the results obtained by the new empirical equations overlap the results obtained by the electromagnetic simulation software, which shows the accuracy of these equations.

The relative difference error is calculated for both real and imaginary parts. For the real part, the highest relative difference error is at a frequency of 800 MHz and it is equal to 8 %. For the imaginary part the highest relative difference error is at a frequency of 400 MHz and it is equal to 6 %.

2.3 Feeding Gap S Dependency

The presented equations are valid for a fixed feeding width S of $0.002\,\lambda$. This constrain can be too specific for

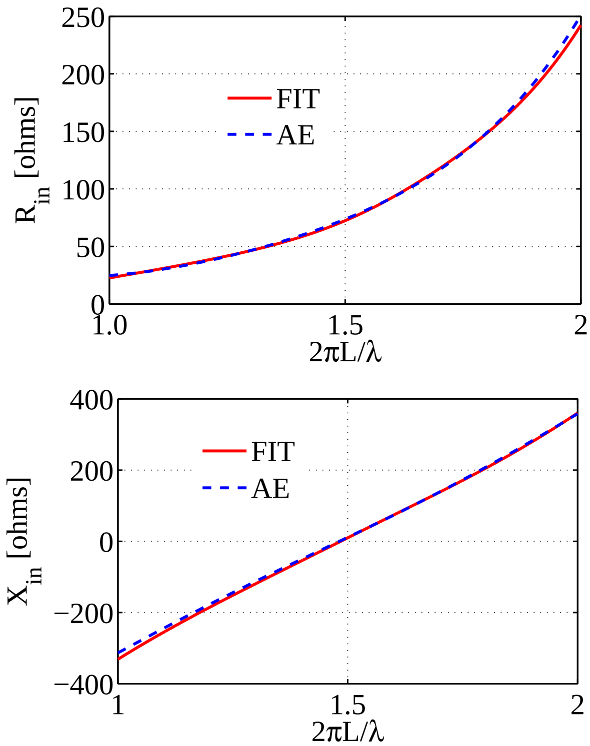


Fig. 2. Real (top) and imaginary (bottom) parts of the input impedance of a strip dipole antenna as a function of frequency calculated by the Finite Integration Technique (FIT) and by the novel Analytical equations (AE). $\frac{2\pi L}{\lambda} = 1.5$, $\frac{W}{\lambda} = 0.003$, $\frac{S}{\lambda} = 0.002$, central frequency = 600 MHz.

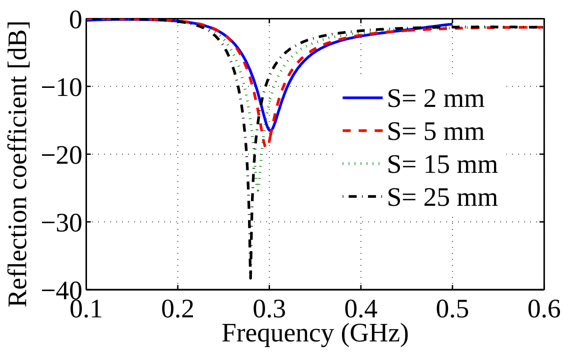


Fig. 3. Reflection coefficients as a function of frequency for different feeding width S. $L = 225$ mm, $W = 20$ mm, operating frequency = 300 MHz.

general use. In order to investigate the effect of the feeding width, the FIT is used to calculate the reflection coefficients and the input impedance of an strip dipole with a length $L = 225$ mm and a width $W = 20$ mm. Fig. 3 shows the reflection coefficients as a function of frequency for different feeding width S. It is clear from the figure that the antenna resonance is sifted when the feeding width is changed. Consequently, the input impedance is changed.

A new Figure of Merit (FoM) is presented to investigate the validity range of the presented formulas for different feeding widths S. The input impedance of a strip dipole is calculated using the presented empirical equations and using the FIT for different feeding width. The figure of merit is

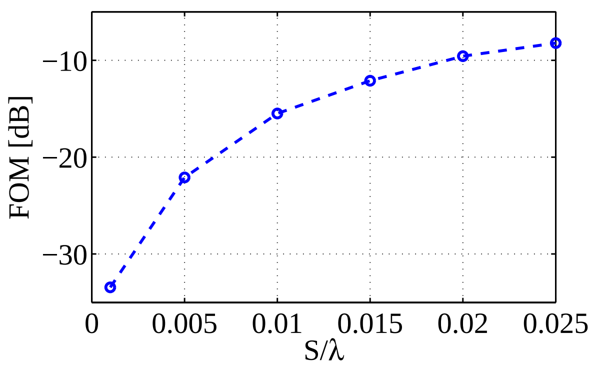


Fig. 4. Figure of merit to calculate the deviations between the presented equations and the FIT for different feeding width. $L = 225$ mm, $W = 20$ mm, operating frequency = 300 MHz.

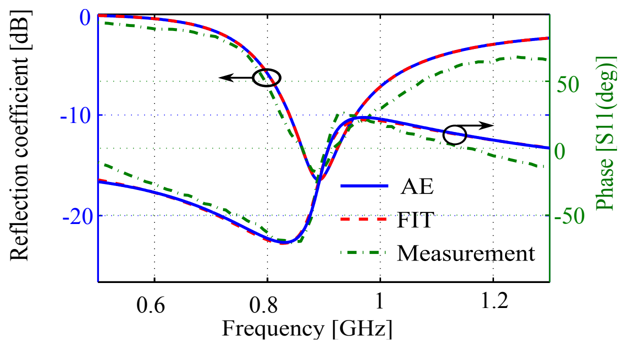


Fig. 5. Simulated and measured reflection coeffitients as a function of frequency.

calculated using the following equation:

$$FOM = S11 = 20\log_{10}\left(\frac{Z_{AE} - Z_{FIT}}{Z_{AE} + Z_{FIT}}\right). \tag{3}$$

It is clear from the figure that at the resonance, the presented equations are valid for a feeding width up to 0.02 λ ($S11 \leq -10$ dB) which demonstrates the accuracy of the presented empirical equations.

3. Fabricated Antenna

To verify the accuracy of the novel empirical equations, an antenna is designed, simulated, manufactured and tested. The antenna is designed to cover GSM signals at a frequency of 900 MHz, $2 * L2 = 15$ cm, $W2 = 1$ cm. Fig. 5 shows the simulated and the measured reflection coefficient as a function of frequency. It is clear from the full wave analysis results and the measurement results that the novel empirical equations can accurately predict the impedance behavior and the resonance of the strip dipoles.

4. Conclusions

In this paper, new design equations to calculate the input impedance of strip dipole antennas are presented. The development is driven by the necessity to meet the requirements of ultra-low power applications, especially when a specific input impedance is required to match the input impedance of the rectifier for maximum power transfer. Simulation and measurement results have shown the proposed equations to be very accurate and can be easily implemented in standard computing tools. Due to their simplicity, high accuracy and fast computation time, these equations can be used as a starting point to quickly design a folded dipole array with the desired input impedance when coupled with computational algorithms like genetic or gradient descent ones. Compared to the state-of-the-art analytical solutions, the validity range is extended by 150 %, which allows for the modeling of longer and wider strip dipole antennas.

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